CHAPTER 24

Answer to Checkpoint Questions

1. (a) $+EA$; (b) $-EA$; (c) 0; (d) 0

2. (a) 2; (b) 3; (c) 1

3. (a) equal; (b) equal; (c) equal

4. $+50e$; (b) $-150e$

5. 3 and 4 tie, then 2, 1

Answer to Questions

1. (a) $8\,\mathrm{N{\cdot}m^2/C}$; (b) 0

2. (a) a^2; (b) πr^2; (c) $2\pi rh$

3. (a) all tie (zero); (b) all tie

4. (a) all four; (b) neither (they are equal)

5. $+13q/\epsilon_0$

6. (a) less (zero); (b) greater; (c) equal; (d) less (zero)

7. all tie

8. (a) S_3, S_2, S_1; (b) all tie; (c) S_3, S_2, S_1; (d) all tie

9. all tie

10. (a) impossible for E to be zero there; (b) $q_B = -3q_0$, $q_c =$ any value; (c) any values giving $q_B + q_C = -3q_0$

11 2σ, σ, 3σ; or 3σ, σ, 2σ

12. (a) 2, 1, 3; (b) all tie $(+4q)$

13. (a) all tie $(E = 0)$; (b) all tie

14. (a) $50e/4\pi\epsilon_0 r^2$, inward; (b) 0; (c) $150e/4\pi\epsilon_0 r^2$, inward

15. (a) same ($E = 0$); (b) decrease to zero; (c) decrease (to zero); (d) same

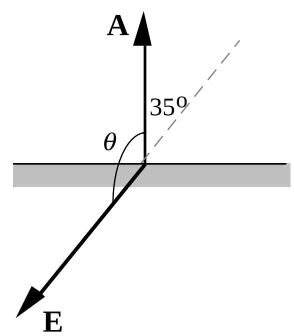
Solutions to Exercises & Problems

1E

(a) The mass flux is $wd\rho v = (3.22\,\text{m})(1.04\,\text{m})(1000\,\text{kg/m}^3)(0.207\,\text{m/s}) = 693\,\text{kg/s}$.
(b) Since water flows only through an area wd, the flux through a larger area is still $693\,\text{kg/s}$.
(c) Now the mass flux is $(wd/2)v = (693\,\text{kg/s})/2 = 347\,\text{kg/s}$.
(d) In this case the water flows through an area $(wd/2)$, so the flux is $347\,\text{kg/s}$.
(e) Now the flux is $(wd\cos\theta)v = (693\,\text{kg/s})(\cos 34°) = 575\,\text{kg/s}$.

2E

The vector area $\mathbf{A}$ and the electric field $\mathbf{E}$ are shown on the diagram to the right. The angle θ between them is $180° - 35° = 145°$ so the electric flux through the area is $\Phi = \mathbf{E}\cdot\mathbf{A} = EA\cos\theta = (1800\ \text{N/C})(3.2\times10^{-3}\ \text{m})^2\cos 145° = -1.5\times10^{-2}\,\text{N}\cdot\text{m}^2\,/\text{C}$.



3E

Use $\Phi = \mathbf{E}\cdot\mathbf{A}$, where $\mathbf{A} = A\,\mathbf{j} = (1.40\,\text{m})^2\,\mathbf{j}$.
(a) $\Phi = (6.00\,\text{N/C})\,\mathbf{i}\cdot(1.40\,\text{m})^2\,\mathbf{j} = 0$.
(b) $\Phi = (-2.00\,\text{N/C})\,\mathbf{j}\cdot(1.40\,\text{m})^2\,\mathbf{j} = -3.92\,\text{N}\cdot\text{m}^2/\text{C}$.
(c) $\Phi = [(-3.00\,\mathbf{i} + 4.00)(\,\text{N/C})]\,\mathbf{k}\cdot(1.40\,\text{m})^2\,\mathbf{j} = 0$.
(d) The total flux of a uniform field through an enclosed surface is always zero.

4P

Use $\Phi = \int \mathbf{E}\cdot d\mathbf{A}$. Note that the side length of the cube is $3.0\,\text{m} - 1.0\,\text{m} = 2.0\,\text{m}$.
(a) On the top face of the cube $y = 2.0\,\text{m}$ and $d\mathbf{A} = (dA)\,\mathbf{j}$. So $\mathbf{E} = 4\,\mathbf{i} - 3(2.0^2 + 2)\,\mathbf{j} = 4\,\mathbf{i} - 18\,\mathbf{j}$. Thus the flux is

$$\begin{aligned}\Phi &= \int_{\text{top}} \mathbf{E}\cdot d\mathbf{A} = \int_{\text{top}} (4\,\mathbf{i} - 18\,\mathbf{j})\cdot(dA)\,\mathbf{j} \\ &= -18\int_{\text{top}} dA = (-18)(2.0)^2\,\text{N}\cdot\text{m}^2/\text{C} = -72\,\text{N}\cdot\text{m}^2/\text{C}\,.\end{aligned}$$

(b) On the bottom face of the cube $y = 0$ and $d\mathbf{A} = (dA)(-\mathbf{j})$. So $\mathbf{E} = 4\,\mathbf{i} - 3(0^2 + 2)\,\mathbf{j} = 4\,\mathbf{i} - 6\,\mathbf{j}$. Thus the flux is

$$\begin{aligned}\Phi &= \int_{\text{bottom}} \mathbf{E}\cdot d\mathbf{A} = \int_{\text{bottom}} (4\,\mathbf{i} - 6\,\mathbf{j})\cdot(dA)(-\mathbf{j}) \\ &= 6\int_{\text{bottom}} dA = 6(2.0)^2\,\mathrm{N\cdot m^2/C} = +24\,\mathrm{N\cdot m^2/C}\,.\end{aligned}$$

(c) On the left face of the cube $d\mathbf{A} = (dA)(-\mathbf{i})$. So

$$\begin{aligned}\Phi &= \int_{\text{left}} \mathbf{E}\cdot d\mathbf{A} = \int_{\text{left}} (4\,\mathbf{i} + E_y\,\mathbf{j})\cdot(dA)(-\mathbf{i}) \\ &= -4\int_{\text{bottom}} dA = -4(2.0)^2\,\mathrm{N\cdot m^2/C} = -16\,\mathrm{N\cdot m^2/C}\,.\end{aligned}$$

(d) On the back face of the cube $d\mathbf{A} = (dA)(-\mathbf{k})$. But since $\mathbf{E}$ has no z component $\mathbf{E}\cdot d\mathbf{A} = 0$. Thus $\Phi = 0$.
(e) We now have to add the flux through all the six faces. You can easily verify that the flux throught the front face is zero, while that through the right face is the opposite of that through the left one, or $+16\,\mathrm{N\cdot m^2/C}$. Thus the net flux through the cube is $\Phi = (-72 + 24 - 16 + 0 + 0 + 16)\,\mathrm{N\cdot m^2/C} = -48\,\mathrm{N\cdot m^2/C}$.

5E

Use $\Phi = q_{\text{enclosed}}/\epsilon_0$.
(a) A surface which encloses the charges $2q$ and $-2q$, or all four charges.
(b) A surface which encloses the charges $2q$ and q.
(c) The maximum amount of negative charge you can enclose by any surface which encloses the charge $2q$ is $-q$, so it is impossible to get a flux of $-2q/\epsilon_0$.

6E

Use $\Phi = q_{\text{enclosed}}/\epsilon_0$ and the fact that the amount of positive (negative) charges on the left (right) side of the conductor is $q(-q)$. Thus $\Phi_1 = q/\epsilon_0$, $\Phi_2 = -q/\epsilon_0$, $\Phi_3 = q/\epsilon_0$, $\Phi_4 = (q - q)/\epsilon_0 = 0$, and $\Phi_5 = (q + q - q)/\epsilon_0 = q/\epsilon_0$.

7E

Use Gauss' law: $\epsilon_0\Phi = q$, where Φ is the total flux through the cube surface and q is the net charge inside the cube. Thus

$$\Phi = \frac{q}{\epsilon_0} = \frac{1.8\times 10^{-6}\,\mathrm{C}}{8.85\times 10^{-12}\,\mathrm{C^2/N\cdot m^2}} = 2.0\times 10^5\,\mathrm{N\cdot m^2/C}\,.$$

8E

Let $\Phi_0 = 10^3\,\mathrm{N\cdot m^2/C}$. The net flux through the entire surface of the dice is given by

$$\Phi = \sum_{i=1}^{6} \Phi_i = \sum_{i=1}^{6} (-1)^i i \Phi_0 = \Phi_0(-1+2-3+4-5+6) = 3\Phi_0\,.$$

Thus the net charge enclosed is

$$\begin{aligned} q &= \epsilon_0 \Phi = 3\epsilon_0 \Phi_0 = 3(8.85\times 10^{-12}\,\mathrm{C^2/N\cdot m^2})(10^3\,\mathrm{N\cdot m^2/C}) \\ &= 2.66\times 10^{-8}\mathrm{C}\,. \end{aligned}$$

9E

Imagine if we place the charge q at the center of a cube of side length d. Then the surface in question is just the bottom side of that cube. Now, by symmetry the flux through all the six surfaces of the cube should be the same, so the flux through the bottom side is

$$\Phi = \frac{1}{6}\Phi_{\mathrm{total}} = \frac{q}{6\epsilon_0}\,.$$

10E

The flux through the flat surface encircled by the rim is given by $\Phi = \pi a^2 E$. Thus the flux through the netting is $\Phi' = -\Phi = -\pi a^2 E$.

11E

Choose a coordinate system whose origin is at the center of the flat base, such that the base is in the xy plane and the rest of the hemisphere is in the $z>0$ half space.

(a) $\Phi = \pi R^2(-\mathbf{k})\cdot E\,\mathbf{k} = -\pi R^2 E$.

(b) Since the flux through the entire hemisphere is zero, the flux through the curved surface is $\Phi_{\mathbf{c}} = -\Phi_{\mathrm{base}} = \pi R^2 E$.

12P

Let $A = (1.40\,\mathrm{m})^2$. Then

(a)

$$\begin{aligned} \Phi &= (3.00y\,\mathbf{j})\cdot(-A\,\mathbf{j})|_{y=0} + (3.00y\,\mathbf{j})\cdot(A\,\mathbf{j})|_{y=1.40\,\mathrm{m}} \\ &= (3.00)(1.40\,\mathrm{N/C})(1.40\,\mathrm{m})^2 = 8.23\,\mathrm{N\cdot m^2/C}\,. \end{aligned}$$

(b) The electric field can be re-written as $\mathbf{E} = 3.00y\,\mathbf{j} + \mathbf{E}_0$, where $\mathbf{E}_0 = -4.00\,\mathbf{i} + 6.00\,\mathbf{j}$ is a constant field which does not contribute to the net flux through the cube. Thus Φ is still $8.23\,\mathrm{N\cdot m^2/C}$.

(c) The charge is given by

$$q = \epsilon_0 \Phi = (8.85 \times 10^{-12}\,\mathrm{C^2/N{\cdot}m^2})(8.23\,\mathrm{N{\cdot}m^2/C}) = 7.29 \times 10^{-11}\,\mathrm{C}$$

in each case.

13P

The net charge q enclosed is given by

$$q = \epsilon_0 \Phi = (8.85 \times 10^{-12}\,\mathrm{C^2/N{\cdot}m^2})(-48\,\mathrm{N{\cdot}m^2/C}) = -4.2 \times 10^{-10}\,\mathrm{C}\,.$$

14P

Let A be the area of one face of the cube, E_u be the magnitude of the electric field at the upper face, and E_ℓ be the magnitude of the field at the lower face. Since the field is downward the flux through the upper face is negative and the flux through the lower face is positive. The flux through the other faces is zero, so the total flux through the cube surface is $\Phi = A(E_\ell - E_u)$. The net charge inside the cube is given by Gauss' law:

$$\begin{aligned} q &= \epsilon_0 \Phi = \epsilon_0 A(E_\ell - E_u) \\ &= (8.85 \times 10^{-12}\,\mathrm{C^2\,/N{\cdot}m^2})(100\,\mathrm{m})^2(100\,\mathrm{N/C} - 60.0\,\mathrm{N/C}) \\ &= 3.54 \times 10^{-6}\,\mathrm{C} = 3.54\,\mu\mathrm{C}\,. \end{aligned}$$

15P

The total flux through any surface that completely surrounds the point charge is q/ϵ_0. If you stack identical cubes side by side and directly on top of each other, you will find that 8 cubes meet at any corner. Thus one-eighth of the field lines emanating from the point charge pass through a cube with a corner at the charge and the total flux through the surface of such a cube is $q/8\epsilon_0$. Now the field lines are radial, so at each of the 3 cube faces that meet at the charge the lines are parallel to the face and the flux through the face is zero. The fluxes through each of the other 3 faces are the same so the flux through each of them is one-third the total. That is, the flux through each of these faces is $(1/3)(q/8\epsilon_0) = q/24\epsilon_0$.

16P

Consider a spherical surface of radius r centered at the location of a point mass m. Then by symmetry the gravitational flux through the surface is

$$\oint \mathbf{g} \cdot d\mathbf{A} = 4\pi r^2 g\,.$$

Thus

$$\mathbf{g} = \frac{\mathbf{e}_r}{4\pi r^2} \oint \mathbf{g} \cdot d\mathbf{A} = \frac{\mathbf{e}_r}{4\pi r^2}(-4\pi m G) = -\left(\frac{Gm}{r^2}\right)\mathbf{e}_r \,,$$

where $\mathbf{e}_r$ is the unit vector along $\mathbf{r}$. This is the Newton's law of gravitation. Here the minus sign indicates that the gravitational force is attractive.

17E

The surface charge density is $\sigma = E\epsilon_o = (2.3 \times 10^5\,\mathrm{N/C})(8.85 \times 10^{-12}\,\mathrm{C^2/N{\cdot}m^2}) = 2.0 \times 10^{-6}\mathrm{C/m^2}$.

18E

(a) The charge on the surface of the sphere is the product of the surface charge density σ and the surface area of the sphere ($4\pi r^2$, where r is the radius). Thus

$$q = 4\pi r^2 \sigma = 4\pi \left(\frac{1.2\,\mathrm{m}}{2}\right)^2 (8.1 \times 10^{-6}\,\mathrm{C/m^2}) = 3.7 \times 10^{-5}\,\mathrm{C}\,.$$

(b) Choose a Gaussian surface in the form a sphere, concentric with the conducting sphere and with a slightly larger radius. The flux is given by Gauss' law:

$$\Phi = \frac{q}{\epsilon_0} = \frac{3.66 \times 10^{-5}\,\mathrm{C}}{8.85 \times 10^{-12}\,\mathrm{C^2\,/N{\cdot}m^2}} = 4.1 \times 10^6\,\mathrm{N{\cdot}m^2\,/C}\,.$$

19E

(a)

$$\sigma = \frac{q}{\pi D^2} = \frac{2.4 \times 10^{-6}\,\mathrm{C}}{\pi(1.3\,\mathrm{m})^2} = 4.5 \times 10^{-7}\mathrm{C/m^2}\,.$$

(b)

$$E = \frac{\sigma}{\epsilon_o} = \frac{4.5 \times 10^{-7}\,\mathrm{C/m^2}}{8.85 \times 10^{-12}\,\mathrm{C^2/N{\cdot}m^2}} = 5.1 \times 10^4\,\mathrm{N/C}\,.$$

20E

(a) Draw a Gaussian surface A which is just outside the inner surface of the spherical shell. Then $\mathbf{E}$ is zero everywhere on surface A. Thus

$$\oint_A \mathbf{E} \cdot d\mathbf{A} = \frac{(Q' + Q)}{\epsilon_0} = 0\,,$$

where Q' is the charge on the inner surface of the shell. This gives $Q' = -Q$.

(b) Since $\mathbf{E}$ remains zero on surface A the result is unchanged.
(c) Now

$$\oint_A \mathbf{E}\cdot d\mathbf{A} = \frac{(Q'+q+Q)}{\epsilon_0} = 0\,,$$

so $Q' = -(Q+q)$.
(d) Yes, since $\mathbf{E}$ remains zero on surface A regardless of where you place the sphere inside the shell.

21P

(a) Consider a Gaussian surface that is completely within the conductor and surrounds the cavity. Since the electric field is zero everywhere on the surface, the net charge it encloses is zero. The net charge is the sum of the charge q in the cavity and the charge q_w on the cavity wall, so $q + q_w = 0$ and $q_w = -q = -3.0\times10^{-6}\,\text{C}$.

(b) The net charge Q of the conductor is the sum of the charge on the cavity wall and the charge q_s on the outer surface of the conductor, so $Q = q_w + q_s$ and $q_s = Q - q_w = (10\times10^{-6}\,\text{C}) - (-3.0\times10^{-6}\,\text{C}) = +1.3\times10^{-5}\,\text{C}$.

22E

The magnitude of the electric field produced by a uniformly charged infinite line is $E = \lambda/2\pi\epsilon_0 r$, where λ is the linear charge density and r is the distance from the line to the point where the field is measured. See Eq. 24-12. Thus

$$\lambda = 2\pi\epsilon_0 Er = 2\pi(8.85\times10^{12}\,\text{C}^2\,/\text{N}\cdot\text{m}^2)(4.5\times10^4\,\text{N/C})(2.0\,\text{m}) = 5.0\times10^{-6}\,\text{C/m}\,.$$

23E

(a) The side surface area A for the drum of diameter D and length h is given by $A = \pi Dh$. Thus

$$\begin{aligned} q &= \sigma A = \sigma\pi Dh = \pi\epsilon_0 EDh \\ &= \pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(2.3\times10^5\,\text{N/C})(0.12\,\text{m})(0.42\,\text{m}) \\ &= 3.2\times10^{-7}\,\text{C}\,. \end{aligned}$$

(b) The new charge is

$$\begin{aligned} q' &= q\left(\frac{A'}{A}\right) = q\left(\frac{\pi D'h'}{\pi Dh}\right) \\ &= (3.2\times10^{-7}\text{C})\left[\frac{(8.0\,\text{cm})(28\,\text{cm})}{(12\,\text{cm})(42\,\text{cm})}\right] = 1.4\times10^{-7}\,\text{C}\,. \end{aligned}$$

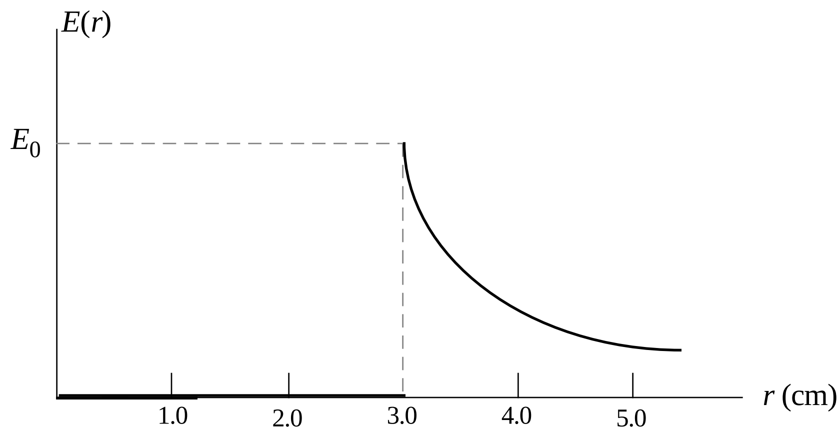
24P

Draw a cylindrical Gaussian surface A of radius r and unit length concentric with the metal tube. Then by symmetry

$$\oint_A \mathbf{E}\cdot d\mathbf{A} = 2\pi r E = \frac{q_{\text{enclosed}}}{\epsilon_0}\,.$$

(a) For $r > R$, $q_{\text{enclosed}} = \lambda$, so $E(r) = \lambda/2\pi r\epsilon_0$.
(b) For $r < R$, $q_{\text{enclosed}} = 0$, so $E = 0$.
The plot of E vs r is shown below. Here

$$E_0 = \frac{\lambda}{2\pi r\epsilon_0} = \frac{(2.0\times10^{-8}\,\text{C/m})}{2\pi(0.030\,\text{m})(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)} = 1.2\times10^4\,\text{N/C}\,.$$



25P

Consider a cylindrical Gaussian surface A of radius r and unit length, concentric with both the cylinders. Then from

$$\oint_A \mathbf{E}\cdot d\mathbf{A} = 2\pi r E = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

we have $E = q_{\text{enclosed}}/2\pi\epsilon_0 r$.
(a) For $r < a$ $\;q_{\text{enclosed}} = 0$, so $E = 0$
(b) For $a < r < b$ $\;q_{\text{enclosed}} = -\lambda$, so $|E| = \lambda/2\pi\epsilon_0 r$.

26P

The net electric field within the cyinder is always given by $E = \lambda/2\pi\epsilon_0 r$, where $\lambda = 3.6\,\text{nC/m}$ and $0 < r < R = 1.5\,\text{cm}$ is the distance measured from the wire. It will not vanish regardless of σ. The $\mathbf{E}$ field outside the cylinder can indeed be zero, provided that $2\pi R\sigma = \lambda$, or

$$\sigma = \frac{\lambda}{2\pi R} = \frac{3.6\times10^{-9}\,\text{C/m}}{(2\pi)(1.5\times10^{-2}\,\text{m})} = 3.8\times10^{-8}\,\text{C/m}^2\,.$$

27P

Use $E(r) = \lambda_{\text{enclosed}}/2\pi\epsilon_0 r$ (see 25P).

(*a*) $E(r) = \dfrac{(q-2q)}{2\pi\epsilon_0 Lr} = -\dfrac{q}{2\pi\epsilon_0 Lr}$.

(*b*) The inner surface of the shell is charged to $-q$ while the out shell is charged to $-2q - (-q) = -q$.

(*c*) Now $\lambda_{\text{enclosed}} = q/L$ so

$$E(r) = \frac{q}{2\pi\epsilon_0 Lr}\,.$$

$\mathbf{E}(r)$ points radially outward by symmetry.

28P

Denote the inner (outer) cylinders with subscripts i and o, respectively.

(*a*) Since $r_i < r = 4.0\,\text{cm} < r_o$,

$$E(r) = \frac{\lambda_i}{2\pi\epsilon_0 r} = \frac{5.0\times10^{-6}\,\text{C/m}}{2\pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(4.0\times10^{-2}\,\text{m})} = 2.3\times10^{6}\,\text{N/C}\,.$$

$\mathbf{E}(r)$ points radially outward.

(*b*) Since $r > r_o$,

$$E(r) = \frac{\lambda_i + \lambda_o}{2\pi\epsilon_0 r} = \frac{5.0\times10^{-6}\,\text{C/m} - 7.0\times10^{-6}\,\text{C/m}}{2\pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(8.0\times10^{-2}\,\text{m})} = -4.5\times10^{5}\,\text{N/C},$$

where the minus sign indicates that $\mathbf{E}(r)$ points radially inward.

29P

The electric field is radially outward from the central wire. You want to find its magnitude in the region between the wire and the cylinder as a function of the distance r from the wire. Use a Gaussian surface in the form of a cylinder with radius r and length ℓ, concentric with the wire. The radius is greater than the radius of the wire and less than in the radius of the inner cylinder wall. Only the charge on the wire is enclosed by the Gaussian surface; denote it by q. The area of the rounded surface of the Gaussian cylinder is $2\pi r\ell$ and the flux through it is $\Phi = 2\pi r\ell E$. If we neglect fringing there is no flux through the ends of the cylinder, so Φ is the total flux. Gauss' law yields $q = 2\pi\epsilon_0 r\ell E$. Since the magnitude of the field at the cylinder wall is known take the Gaussian surface to coincide with the wall. Then r is the radius of the wall and

$$\begin{aligned} q &= 2\pi(8.85\times10^{-12}\,\text{C}^2\,/\text{N}\cdot\text{m}^2)(0.014\,\text{m})(0.16\,\text{m})(2.9\times10^{4}\,\text{N/C}) \\ &= 3.6\times10^{-9}\,\text{C}\,. \end{aligned}$$

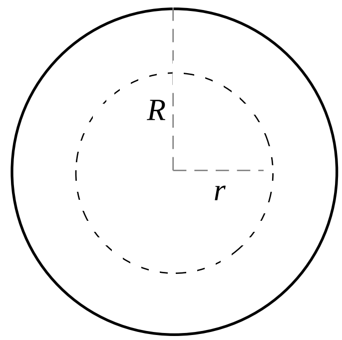
30P

Use $mv^2/r = F_c = |e|E$. Thus

$$K = \frac{1}{2}mv^2 = \frac{|e|Er}{2} = \frac{|e|\lambda r}{4\pi\epsilon_0 r}$$
$$= \frac{|e|(30 \times 10^{-9}\mathrm{C/m})}{4\pi(8.85 \times 10^{-12}\,\mathrm{C^2/N{\cdot}m^2})} = 270\,|e|\,\mathrm{V} = 270\,\mathrm{eV}\,.$$

31P

(a) The diagram to the right shows a cross section of the charged cylinder (solid circle). Consider a Gaussian surface in the form of a cylinder with radius r and length ℓ, concentric with the cylinder of charge The cross section is shown as a dotted circle. Use Gauss' law to find an expression for the magnitude of the electric field at the Gaussian surface.



The charge within the Gaussian cylinder is $q = \rho V = \pi r^2 \ell \rho$, where $V\ (= \pi r^2 \ell)$ is the volume of the cylinder.

If ρ is positive the electric field lines are radially outward and are therefore normal to the rounded portion of the Gaussian cylinder and are distributed uniformly over it. None pass through the ends of the cylinder. Thus the total flux through the Gaussian cylinder is $\Phi = EA = 2\pi r\ell E$, where $A\ (= 2\pi r\ell)$ is the area of rounded portion of the cylinder.

Gauss' law ($\epsilon_0\Phi = q$) yields $2\pi\epsilon_0 r\ell E = \pi r^2\ell\rho$, so

$$E = \frac{\rho r}{2\epsilon_0}\,.$$

(b) Take the Gaussian surface to be a cylinder with length ℓ and radius r (greater than R). The flux is again $\Phi = 2\pi r\ell E$. The charge enclosed is the total charge in a section of the charged cylinder with length ℓ. That is, $q = \pi R^2\ell\rho$. Gauss' law yields $2\pi\epsilon_0 r\ell E = \pi R^2\ell\rho$, so

$$E = \frac{R^2\rho}{2\epsilon_0 r}\,.$$

32E

According to Eq. 24-13 the electric field due to either sheet of charge with surface charge density σ is perpendicular to the plane of the sheet and has magnitude $E = \sigma/2\epsilon_0$. Thus by the superpostion principle:

(a) $E = \sigma/\epsilon_0$, pointing to the left;
(b) $E = 0$;
(c) $E = \sigma/\epsilon_0$, pointing to the right.

33E

(a) To calculate the electric field at a point very close to the center of a large, uniformly charged, conducting plate, we may replace the finite plate with an infinite plate with the same area charge density and take the magnitude of the field to be $E = \sigma/\epsilon_0$, where σ is the area charge density for the surface just under the point. The charge is distributed uniformly over both sides of the original plate, with half being on the side near the field point. Thus

$$\sigma = \frac{q}{2A} = \frac{6.0 \times 10^{-6}\,\mathrm{C}}{2(0.080\,\mathrm{m})^2} = 4.69 \times 10^{-4}\,\mathrm{C/m^2}\,.$$

The magnitude of the field is

$$E = \frac{4.69 \times 10^{-4}\,\mathrm{C/m^2}}{8.85 \times 10^{-12}\,\mathrm{C^2/N{\cdot}m^2}} = 5.3 \times 10^{7}\,\mathrm{N/C}\,.$$

The field is normal to the plate and since the charge on the plate is positive it points away from the plate.

(b) At a point far away from the plate the electric field is nearly that of a point particle with charge equal to the total charge on the plate. The magnitude of the field is $E = q/4\pi\epsilon_0 r^2$, where r is the distance from the plate. Thus

$$E = \frac{(8.99 \times 10^{9}\,\mathrm{N{\cdot}m^2/C^2})(6.0 \times 1o^{-6}\,\mathrm{C})}{(30\,\mathrm{m})^2} = 60\,\mathrm{N/C}\,.$$

34E

The charge distribution in this problem is equivalent to that of an infinite sheet of charge with charge density σ plus a small circular pad of radius R located at the middle of the sheet with charge density $-\sigma$. Denote the electric fields produced by the sheet and the pad with subscripts 1 and 2, respectively. The net electric field $\mathbf{E}$ is then

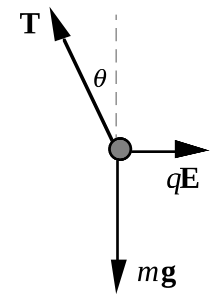
$$\begin{aligned}\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 &= \left(\frac{\sigma}{2\epsilon_0}\right)\mathbf{k} + \frac{(-\sigma)}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)\mathbf{k} \\ &= \frac{\sigma z}{2\epsilon_0\sqrt{z^2+R^2}}\,\mathbf{k}\,,\end{aligned}$$

where Eq. 23-24 was used for $\mathbf{E}_2$.

35P

The forces acting on the ball are shown in the diagram next page. The gravitational force has magnitude mg, where m is the mass of the ball; the electrical force has magnitude qE, where q is the charge on the ball and E is the electric field at the position of the ball; and the tension in the thread is denoted by T. The electric field produced by the plate is normal to the plate and points to the right. Since the ball is positively charged the electric force on it also points to the right. The tension in the thread makes the angle θ (= 30°) with the vertical.

Since the ball is in equilibrium the net force on it vanishes. The sum of the horizontal components yields $qE - T\sin\theta = 0$ and the sum of the vertical components yields $T\cos\theta - mg = 0$. The expression $T = qE/\sin\theta$, from the first equation, is substituted into the second to obtain $qE = mg\tan\theta$.



The electric field produced by a large uniform plane of charge is given by $E = \sigma/2\epsilon_0$, where σ is the surface charge density. Thus

$$\frac{q\sigma}{2\epsilon_0} = mg\tan\theta$$

and

$$\begin{aligned}\sigma &= \frac{2\epsilon_0 mg\tan\theta}{q}\\ &= \frac{2(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})(1.0\times10^{-6}\,\mathrm{kg})(9.8\,\mathrm{m/s^2})\tan 30^\circ}{2.0\times10^{-8}\,\mathrm{C}}\\ &= 5.0\times10^{-9}\,\mathrm{C/m^2}\,.\end{aligned}$$

36P

Let $\mathbf{i}$ be a unit vector pointing to the left.

(a) To the left of the plates: $\mathbf{E} = (\sigma/2\epsilon_0)\,\mathbf{i}$ (from the right plate)$+(-\sigma/2\epsilon_0)\,\mathbf{i}$ (from the left one)$= 0$.

(b) To the right of the plates: $\mathbf{E} = (\sigma/2\epsilon_0)(-\mathbf{i})$ (from the right plate)$+(-\sigma/2\epsilon_0)(-\mathbf{i})$ (from the left one)$= 0$.

(c) Between the plates:

$$\begin{aligned}\mathbf{E} &= \left(\frac{\sigma}{2\epsilon_0}\right)\mathbf{i} + \left(\frac{-\sigma}{2\epsilon_0}\right)(-\mathbf{i}) = \left(\frac{\sigma}{\epsilon_0}\right)\mathbf{i}\\ &= \left(\frac{7.0\times10^{-22}\,\mathrm{C/m^2}}{8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}}\right)\mathbf{i} = (7.9\times10^{-11}\,\mathrm{N/C})\,\mathbf{i}\,.\end{aligned}$$

37P

The charge on the metal plate, which is negative, exerts a force of repulsion on the electron and stops it. First find an expression for the acceleration of the electron, then use kinematics to find the stopping distance. Take the initial direction of motion of the electron to be positive. Then the electric field is given by $E = \sigma/\epsilon_0$, where σ is the surface charge density on the plate. The force on the electron is $F = -eE = -e\sigma/\epsilon_0$ and the acceleration is

$$a = \frac{F}{m_e} = -\frac{e\sigma}{\epsilon_0 m_e}\,,$$

where m_e is the mass of the electron.

The force is constant, so we use constant acceleration kinematics. If v_0 is the initial velocity of the electron, v is the final velocity, and x is the distance traveled between the initial and final positions, then $v^2 - v_0^2 = 2ax$. Set $v = 0$ and replace a with $-e\sigma/\epsilon_0 m_e$, then solve for x. You should get

$$x = -\frac{v_0^2}{2a} = \frac{\epsilon_0 m_e v_0^2}{2e\sigma}.$$

Now $\frac{1}{2}m_e v_0^2$ is the initial kinetic energy K_0, so

$$x = \frac{\epsilon_0 K_0}{e\sigma}.$$

You must convert the given value of K_0 to joules. Since $1.00\,\text{eV} = 1.60 \times 10^{-19}\,\text{J}$, $100\,\text{eV} = 1.60 \times 10^{-17}\,\text{J}$. Thus

$$x = \frac{(8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(1.60 \times 10^{-17}\,\text{J})}{(1.60 \times 10^{-19}\,\text{C})(2.0 \times 10^{-6}\,\text{C/m}^2)} = 4.4 \times 10^{-4}\,\text{m}.$$

38P

Use the result of part (c) of 36P to obtain σ: $E = \sigma/\epsilon_0$, or

$$\sigma = \epsilon_0 E = (8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(55\,\text{N/C}) = 4.9 \times 10^{-10}\,\text{C/m}^2.$$

39P

(a) Use $m_e g = eE = e\sigma/\epsilon_0$ to obtain σ:

$$\sigma = \frac{m_e g \epsilon_0}{e} = \frac{(9.11 \times 10^{-31}\,\text{kg})(9.80\,\text{m/s}^2)(8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}{1.60 \times 10^{-19}\,\text{C}} = 4.9 \times 10^{-22}\,\text{C/m}^2.$$

(b) Downward (since the electric force exerted on the electron must be upward).

40

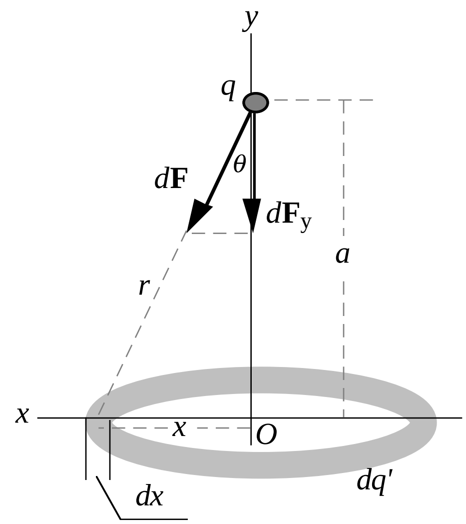
(a)

$$E = \frac{\sigma}{\epsilon_0} = \frac{-qa}{2\pi\epsilon_0 r^3}.$$

(b) Convince yourself, by considering an infinitely large Gaussian surface which encloses all of the conducting plane and the point charge $+q$, that Gauss' law should predict the total charge induced to be $-q$. Alternatively you may also perform the following integration:

$$q_{\text{induced}} = \int_{\text{plane}} \sigma\, dA = \int_0^\infty \frac{-qa}{2\pi(x^2 + a^2)^{3/2}} 2\pi x\, dx = -q.$$

(c) Set up a coordinate system as shown. Consider the force exerted on the charge $+q$ from an infinitesimal ring of induced charge dq', located between x and $x+dx$ from the origin. The area of this ring is $dA = 2\pi x dx$, so



$$dq' = \sigma dA = -\left(\frac{qa}{2\pi r^3}\right)(2\pi x dx)\,,$$

where $r = (x^2 + a^2)^{1/2}$. By symmetry only the y-component of the force dF_y is non-zero. Here $dF_y = (qdq'/r^2)\cos\theta$, where $\cos\theta = a/r$. So

$$F = \int_{\text{plane}} dF_y = \frac{q}{4\pi\epsilon_0}\int_{\text{plane}} \frac{\cos\theta dq'}{r^2} = -\frac{q^2a^2}{4\pi\epsilon_0}\int_0^\infty \frac{2\pi x dx}{2\pi(x^2+a^2)^3} = -\frac{q^2}{16\pi\epsilon_0 a^2}\,.$$

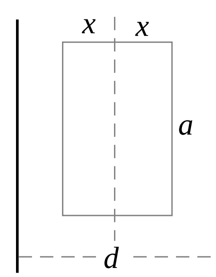
Here the minus sign indicates that the force is attractive.
(d) Let the magnitude of the other charge be q'. Then

$$F = \frac{1}{4\pi\epsilon_0}\frac{qq'}{(2a)^2} = -\frac{q^2}{16\pi\epsilon_0 a^2}\,,$$

or $q' = -q$.

41P

(a) Use a Gaussian surface in the form of a box with rectangular sides. The cross section is shown with dotted lines in the diagram to the right. It is centered at the central plane of the slab so the left and right faces are each a distance x from the central plane. Take the thickness of the rectangular solid to be a, the same as its length, so the left and right faces are squares.



The electric field is normal to the left and right faces and is uniform over them. If ρ is positive it points outward at both faces: toward the left at the left face and toward the right at the right face. Furthermore, the magnitude is the same at both faces. The electric flux through each of these faces is Ea^2. The field is parallel to the other faces of the Gaussian surface and the flux through them is zero. The total flux through the Gaussian surface is $\Phi = 2Ea^2$.

The volume enclosed by the Gaussian surface is $2a^2x$ and the charge contained within it is $q = 2a^2x\rho$. Gauss' law yields $2\epsilon_0 Ea^2 = 2a^2x\rho$. Solve for E:

$$E = \frac{\rho x}{\epsilon_0}\,.$$

(b) Take a Gaussian surface of the same shape and orientation, but with $x > d/2$, so the left and right faces are outside the slab. The total flux through the surface is again $\Phi = 2Ea^2$ but the charge enclosed is now $q = a^2 d\rho$. Gauss's law yields $2\epsilon_0 E a^2 = a^2 d\rho$, so

$$E = \frac{\rho d}{2\epsilon_0}\,.$$

42E

Charge is distributed uniformly over the surface of the sphere and the electric field it produces at points outside the sphere is like the field of a point particle with charge equal to the net charge on the sphere. That is, the magnitude of the field is given by $E = q/4\pi\epsilon_0 r^2$, where q is the magnitude of the charge on the sphere and r is the distance from the center of the sphere to the point where the field is measured. Thus

$$q = 4\pi\epsilon_0 r^2 E = \frac{(0.15\,\text{m})^2(3.0\times10^3\,\text{N/C})}{8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2} = 7.5\times10^{-9}\,\text{C}\,.$$

The field points inward, toward the sphere center, so the charge is negative: -7.5×10^{-9} C.

43E

(a) The flux is still $-750\,\text{N}\cdot\text{m}^2/\text{C}$, since it depends only on the amount of charge enclosed.
(b) Use $\Phi = q/\epsilon_0$ to obtain q:

$$q = \epsilon_0\Phi = (8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(-750\,\text{N}\cdot\text{m}^2/\text{C}) = -6.64\times10^{-10}\text{C}\,.$$

44E

(a) For $r < R$, $E = 0$ (see Eq. 24-16).
(b) For r slightly greater than R,

$$\begin{aligned} E &= \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \approx \frac{q}{4\pi\epsilon_0 R^2} = \frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(2.0\times10^{-7}\text{C})}{(0.25\,\text{m})^2} \\ &= 2.9\times10^4\,\text{N/C}\,. \end{aligned}$$

(c) For $r > R$,

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = (2.9\times10^4\,\text{N/C})\left(\frac{0.25\,\text{m}}{3.0\,\text{m}}\right)^2 = 200\,\text{N/C}\,.$$

45E

(a)

$$E = \frac{q}{4\pi\epsilon_0 r_1^2} = \frac{(8.99\times10^9\text{N}\cdot\text{m}^2/\text{C}^2)(1.0\times10^{-7}\text{C})}{(0.015\,\text{m})^2} = 4.0\times10^6\,\text{N/C}\,.$$

E points from q_1 to P.
(b) $\mathbf{E} = 0$, since P_2 is inside the metal.

46E
(a) Since $r_1 = 10.0\,\text{cm} < r = 12.0\,\text{cm} < r_2 = 15.0\,\text{cm}$,

$$E(r) = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2} = \frac{(8.99\times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(4.00\times 10^{-8}\,\text{C})}{(0.120\,\text{m})^2} = 2.50\times 10^4\,\text{N/C}\,.$$

(b) Since $r_1 < r_2 < r = 20.0\,\text{cm}$,

$$\begin{aligned} E(r) &= \frac{1}{4\pi\epsilon_0}\frac{q_1+q_2}{r^2} = \frac{(8.99\times 10^9\text{N}\cdot\text{m}^2/\text{C}^2)(4.00+2.00)(10^{-8}\text{C})}{(0.200\,\text{m})^2} \\ &= 1.35\times 10^4\,\text{N/C}\,. \end{aligned}$$

47E
Use Eqs. 24-15, 24-16 and the superposition principle.
(a) $E = 0$.
(b) $E = (1/4\pi\epsilon_0)(q_a/r^2)$.
(c) $E = (1/4\pi\epsilon_0)(q_a+q_b)/r^2$.
(d) Since $E = 0$ for $r < a$ the charge on the inner surface of the inner shell is always zero. The charge on the outer surface of the inner shell is therefore q_a. Since $E = 0$ inside the metallic outer shell the net charge enclosed in a Gaussian surface that lies in between the inner and outer surfaces of the outer shell is zero. Thus the inner surface of the outer shell must carry a charge $-q_a$, leaving the charge on the outer surface of the outer shell to be $q_b + q_a$.

48E
Use Gauss' law to find an expression for the magnitude of the electric field a distance r from the center of the atom. The field is radially outward and is uniform over any sphere centered at the atom's center. Take the Gaussian surface to be a sphere of radius r with its center at the center of the atom. If E is the magnitude of the field then the total flux through the Gaussian sphere is $\Phi = 4\pi r^2 E$. The charge enclosed by the Gaussian surface is the positive charge at the center of the atom and that portion of the negative charge within the surface. Since the negative charge is uniformly distributed throughout a sphere of radius R we can compute the charge inside the Gaussian sphere using a ratio of volumes. That is, the negative charge inside is $-Zer^3/R^3$. Thus the total charge enclosed is $Ze - Zer^3/R^3$. Gauss' law yields

$$4\pi\epsilon_0 r^2 E = Ze\left(1 - \frac{r^3}{R^3}\right)\,.$$

Solve for E:

$$E = \frac{Ze}{4\pi\epsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right) .$$

49E

Since $\sigma = q/4\pi r^2$,

$$E = \frac{\sigma}{\epsilon_0} = \frac{1}{\epsilon_0}\left(\frac{q}{4\pi r^2}\right) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} .$$

50P

The proton is in uniform circular motion, with the electrical force of the charge on the sphere providing the centripetal force. According to Newton's second law $F = m_p v^2/r$, where F is the magnitude of the force, v is the speed of the proton, and r is the radius of its orbit, essentially the same as the radius of the sphere.

The magnitude of the force on the proton is $F = eq/4\pi\epsilon_0 r^2$, where q is the magnitude of the charge on the sphere. Thus

$$\frac{1}{4\pi\epsilon_0}\frac{eq}{r^2} = \frac{m_p v^2}{r} ,$$

so

$$\begin{aligned} q &= \frac{4\pi\epsilon_0 m_p v^2 r}{e} = \frac{(1.67\times10^{-27}\,\text{kg})(3.00\times10^{5}\,\text{m/s})^2(0.0100\,\text{m})}{(8.99\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\,\text{C})} \\ &= 1.04\times10^{-9}\,\text{C} . \end{aligned}$$

The force must be inward, toward the center of the sphere, and since the proton is positively charged the electric field must also be inward. The charge on the sphere is negative: $q = -1.04\times10^{-9}$ C.

51P

Draw a spherical Gaussian surface of radius r centered at the point charge $+q$. From symmetry consideration E is the same throughout the surface, so

$$\oint \mathbf{E}\cdot d\mathbf{A} = 4\pi r^2 E = \frac{q_{\text{encl}}}{\epsilon_0} ,$$

which gives

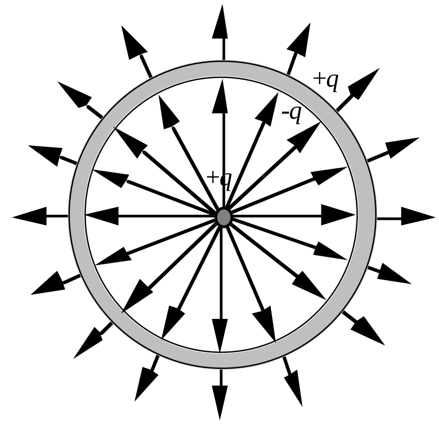
$$E(r) = \frac{q_{\text{encl}}}{4\pi\epsilon_0 r^2} ,$$

where q_{encl} is the net charge enclosed by the Gaussian surface.

(a) Now $a < r < b$, where $E = 0$. Thus $q_{\text{encl}} = 0$, so the charge on the inner surface of the shell is $q_i = -q$.

(b) The shell as a whole is electrically neutral so the outer shell must carry a charge of $q_o = +q$.
(c) For $r < a$ $q_{\rm encl} = +q$, so

$$E\Big|_{r<a} = \frac{q}{4\pi\epsilon_0 r^2}\,.$$



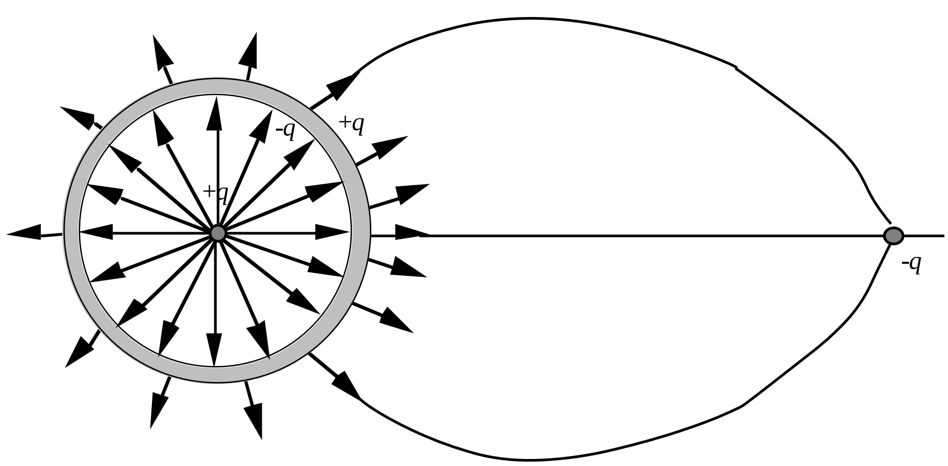
(d) For $b > r > a$ $E = 0$, since this region is inside the metallic part of the shell.
(e) For $r > b$ $q_{\rm encl} = +q$, so

$$E\Big|_{r<a} = \frac{q}{4\pi\epsilon_0 r^2}\,.$$

The field lines are sketched to the right.

(f) The net charge of the central point charge-inner surface combination is zero. Thus the eletric field it produces is also zero.
(g) The outer shell has a spherically symmetric charge distribution with a net charge $+q$. Thus the field it produces for $r > b$ is $E = q/(4\pi\epsilon_0 r^2)$.
(h) Yes. In fact there will be a distribution of induced charges on the outer shell, as a result of a flow of positive charges toward the side of the surface that is closer to the negative point charge outside the shell.
(i) No. The change in the charge distribution on the outer shell cancels the effect of the negative point charge.

The field lines are sketched below.



(j) The shell plus the positive point charge, viewed as a single system, is positively charged and will exert an attractive force on the negative point charge.
(k) No. Again the change in the charge distribution on the outer shell cancels the effect of the negative point charge.
(l) No, because the net force on each point charge is the combined force given by the other point charge *and* the shell, not just that of the other charge. In fact the force exerted on the positive charge on the negative one alone is indeed equal in magnitude and opposite in direction with that exerted by the negative charge on the positive one.

52P

(a)

$$\begin{aligned} Q &= \int \rho dV = 4\pi \int_0^R \rho(r) r^2 dr = 4\pi \int_0^R \left(\frac{\rho_s r}{R}\right) r^2 dr \\ &= \pi \rho_s R^3. \end{aligned}$$

(b) Since the distribution of charge is spherically symmetrical so must be the electric field. Thus

$$\begin{aligned} E(r) &= \frac{1}{4\pi\epsilon_0}\frac{q_{\text{encl}}}{r^2} = \frac{1}{4\pi\epsilon_0 r^2}\int_0^r \frac{4\pi\rho_s r^3}{R} dr \\ &= \frac{\rho_s r^2}{4\epsilon_0 R} = \frac{(\pi\rho_s R^3) r^2}{4\pi\epsilon_0 R^4} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^4} r^2. \end{aligned}$$

53P

At all points where there is an electric field it is radially outward. For each part of the problem use a Gaussian surface in the form of a sphere that is concentric with the sphere of charge and passes through the point where the electric field is to be found. The field is uniform on the surface, so

$$\oint \mathbf{E} \cdot d\mathbf{A} = 4\pi r^2 E\,,$$

where r is the radius of the Gaussian surface.
(a) Here r is less than a and the charge enclosed by the Gaussian surface is $q(r/a)^3$. Gauss' law yields

$$4\pi r^2 E = \left(\frac{q}{\epsilon_0}\right)\left(\frac{r}{a}\right)^3,$$

so

$$E = \frac{qr}{4\pi\epsilon_0 a^3}\,.$$

(b) Here r is greater than a but less than b. The charge enclosed by the Gaussian surface is q, so Gauss' law becomes

$$4\pi r^2 E = \frac{q}{\epsilon_0}$$

and

$$E = \frac{q}{4\pi\epsilon_0 r^2}\,.$$

(c) The shell is conducting so the electric field inside it is zero.

(d) For $r > c$ the charge enclosed by the Gaussian surface is zero (charge q is inside the shell cavity and charge $-q$ is on the shell). Gauss' law yields

$$4\pi r^2 E = 0\,,$$

so $E = 0$.

(e) Consider a Gaussian surface that lies completely within the conducting shell. Since the electric field is everywhere zero on the surface $\oint \mathbf{E}\cdot d\mathbf{A} = 0$ and, according to Gauss' law, the net charge enclosed by the surface is zero. If Q_i is the charge on the inner surface of the shell, then $q + Q_i = 0$ and $Q_i = -q$. Let Q_0 be the charge on the outer surface of the shell. Since the net charge on the shell is $-q$, $Q_i + Q_0 = -q$. This means $Q_0 = -q - Q_i = -q - (-q) = 0$.

54P

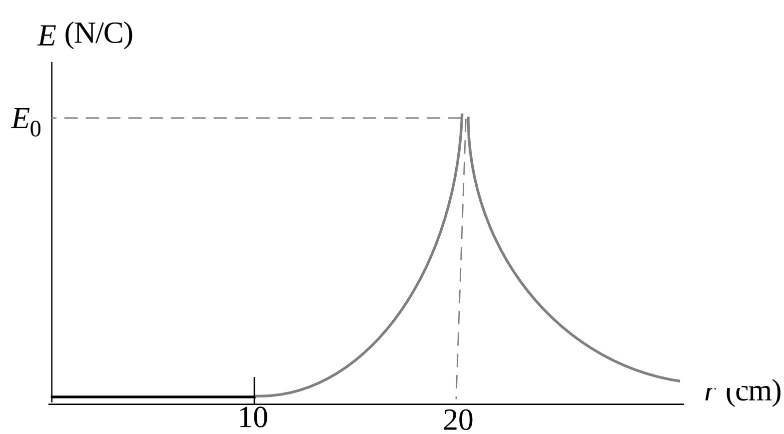
For $r < a$, $E = 0$; for $a < r < b$,

$$E = \frac{1}{4\pi\epsilon_0}\frac{q_{\rm encl}}{r^2} = \frac{\rho}{4\pi\epsilon_0 r^2}\cdot\frac{4\pi}{3}(r^3 - a^3)\,;$$

and for $r > b$,

$$E = \frac{1}{4\pi\epsilon_0}\frac{q_{\rm encl}}{r^2} = \frac{\rho}{4\pi\epsilon_0 r^2}\cdot\frac{4\pi}{3}(b^3 - a^3)\,.$$

The E vs r curve is plotted below. Here $E_0 = 6.6\times10^3$ N/C.



55P

You wish to find an expression for the electric field inside the shell in terms of A and the distance from the center of the shell, then select A so the field does not depend on the distance.

Use a Gaussian surface in the form of a sphere with radius r_g, concentric with the spherical shell and within it ($a < r_g < b$). Gauss' law will be used to find the magnitude of the electric field a distance r_g from the shell center.

The charge that is both in the shell and within the Gaussian sphere is given by the integral $q_s = \int \rho\, dV$ over the portion of the shell within the Gaussian surface. Since the charge distribution has spherical symmetry, we may take dV to be the volume of a spherical shell with radius r and infinitesimal thickness dr: $dV = 4\pi r^2\, dr$. Thus

$$q_s = 4\pi \int_a^{r_g} \rho r^2\, dr = 4\pi \int_a^{r_g} \frac{A}{r}\, r^2\, dr = 4\pi A \int_a^{r_g} r\, dr = 2\pi A(r_g^2 - a^2)\,.$$

The total charge inside the Gaussian surface is $q + q_s = q + 2\pi A(r_g^2 - a^2)$.

The electric field is radial, so the flux through the Gaussian surface is $\Phi = 4\pi r_g^2 E$, where E is the magnitude of the field. Gauss' law yields

$$4\pi\epsilon_0 E r_g^2 = q + 2\pi A(r_g^2 - a^2)\,.$$

Solve for E:

$$E = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r_g^2} + 2\pi A - \frac{2\pi A a^2}{r_g^2}\right).$$

For the field to be uniform the first and last terms in the parenthesis must cancel. They do if $q - 2\pi A a^2 = 0$ or $A = q/2\pi a^2$.

56P

(a) From

$$-e = \int_0^\infty \rho(r) 4\pi r^2 dr = \int_0^\infty A e^{-2r/a_0} 4\pi r^2 dr = \pi a_0^3 A$$

we get $A = -e/\pi a_0^3$.

(b)

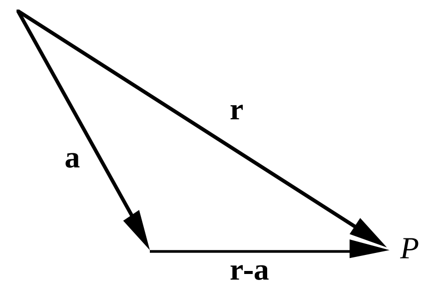
$$\begin{aligned} E &= \frac{q_{\text{encl}}}{4\pi\epsilon_0 a_0^2} = \frac{1}{4\pi\epsilon_0 a_0^2}\left(e + \int_0^{a_0} \rho(r) 4\pi r^2 dr\right) \\ &= \frac{e}{4\pi\epsilon_0 a_0^2}\left(1 - \frac{4}{a_0^3}\int_0^{a_0} e^{-2r/a_0} r^2\, dr\right) \\ &= \frac{5e \cdot \exp(-2)}{4\pi\epsilon_0 a_0^2}\,. \end{aligned}$$

E points radially outward.

57P*

(a) From Gauss' law

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{q_{\text{encl}}\mathbf{r}}{r^3} = \frac{1}{4\pi\epsilon_0}\frac{(4\pi\rho r^3/3)\mathbf{r}}{r^3} = \frac{\rho\mathbf{r}}{3\epsilon_0}.$$



(b) The charge distribution in this case is equivalent to that of a whole sphere of charge density ρ plus a smaller sphere of charge density $-\rho$ which fills the void. By superposition

$$\mathbf{E}(\mathbf{r}) = \frac{\rho\mathbf{r}}{3\epsilon_0} + \frac{(-\rho)(\mathbf{r}-\mathbf{a})}{3\epsilon_0} = \frac{\rho\mathbf{a}}{3\epsilon_0}.$$

58P*

Use

$$E(r) = \frac{q_{\text{encl}}}{4\pi\epsilon_0 r^2} = \frac{1}{4\pi\epsilon_0 r^2}\int_0^r \rho(r)4\pi r^2\,dr$$

to solve for $\rho(r)$:

$$\rho(r) = \frac{\epsilon_0}{r^2}\frac{d}{dr}[r^2E(r)] = \frac{\epsilon_0}{r^2}\frac{d}{dr}(Kr^6) = 6K\epsilon_0 r^3.$$

59

$\alpha = 0.80$.

60

(a) each of the six faces: $1.88\times10^4\,\text{N}\cdot\text{m}^2/\text{C}$; total: $1.13\times10^5\,\text{N}\cdot\text{m}^2/\text{C}$;
(b) each of the four equivalent faces: $1.77\times10^4\,\text{N}\cdot\text{m}^2/\text{C}$; the face at $y=-0.5\,\text{m}$: $1.24\times10^4\,\text{N}\cdot\text{m}^2/\text{C}$; the face at $y=+0.5\,\text{m}$: $2.97\times10^4\,\text{N}\cdot\text{m}^2/\text{C}$; total: $1.13\times10^5\,\text{N}\cdot\text{m}^2/\text{C}$;
(c) each of the four equivalent faces: $1.45\times10^4\,\text{N}\cdot\text{m}^2/\text{C}$; the face at $y=-0.5\,\text{m}$: $8.56\times10^3\,\text{N}\cdot\text{m}^2/\text{C}$; the face at $y=+0.5\,\text{m}$: $4.65\times10^4\,\text{N}\cdot\text{m}^2/\text{C}$; total: $1.13\times10^5\,\text{N}\cdot\text{m}^2/\text{C}$;
(d) each of the four equivalent faces: $1.01\times10^4\,\text{N}\cdot\text{m}^2/\text{C}$; the face at $y=-0.5\,\text{m}$: $6.19\times10^3\,\text{N}\cdot\text{m}^2/\text{C}$; the face at $y=+0.5\,\text{m}$: $-4.65\times10^4\,\text{N}\cdot\text{m}^2/\text{C}$; total: essentially zero