CHAPTER 25

Answer to Checkpoint Questions

1. (a) negative; (b) increase
2. (a) positive; (b) higher
3. (a) rightward; (b) 1, 2, 3, 5: positive; 4: negative; (c) 3, then 1, 2, and 5 tie, then 4
4. all tie
5. a, c (zero), b
6. (a) 2, then 1 and 3 tie; (b) 3; (c) accelerate leftward
7. closer (half of 9.23 fm)

Answer to Questions

1. (a) higher; (b) positive; (c) negative; (d) all tie
2. (a) left; (b) $-50\,\mathrm{V}$; (c) positive; (d) negative
3. (a) 1 and 2; (b) none; (c) no; (d) 1 and 2 yes, 3 and 4 no
4. $-4q/4\pi\epsilon_0 d$
5. (b), then (a), (c) and (d) tie
6. all tie
7. (a) negative; (b) zero
8. (a)–(c) $Q/4\pi\epsilon_0 R$; (d) a, b, c
9. (a) 1, then 2 and 3 tie; (b) 3
10. E_z, E_y, E_x
11. left
12. (a) 2, 4, and then a tie of 1, 3, and 5 (where $E = 0$); (b) negative x direction; (c) positive x direction
13. (a), (b), (c)
14. (a) 3 and 4 tie, then 1 and 2 tie; (b) 1 and 2, increase; 3 and 4, decrease

15. (a) c, b, a; (b) zero
16. (a) – (d) zero
17. (a) positive; (b) positive; (c) negative; (d) all tie
18. farther
19. (a) no; (b) yes
20. no
21. no (a particle at the intersection would have two different potential energies)
22. (a) – (c) C, B, A; (d) all tie
23. (a) – (b) all tie; (c) C, B, A; (d) all tie
24. situation 2

Solutions to Exercises & Problems

1E

The magnitude is $\Delta U = e\Delta V = 1.2 \times 10^9\,\text{eV} = 1.2\,\text{GeV}$.

2E

(a) An ampere is a coulomb per second, so

$$84\,\text{A}\cdot\text{h} = (84\ \text{C}\cdot\text{h/s})\,(3600\,\text{s/h}) = 3.0 \times 10^5\,\text{C}\,.$$

(b) The change in potential energy is $\Delta U = q\,\Delta V = (3.0 \times 10^5\,\text{C})(12\,\text{V}) = 3.6 \times 10^6\,\text{J}$.

3P

(a) When charge q moves through a potential difference ΔV its potential energy changes by $\Delta U = q\,\Delta V$. In this case, $\Delta U = (30\,\text{C})(1.0 \times 10^9\,\text{V}) = 3.0 \times 10^{10}\,\text{J}$.

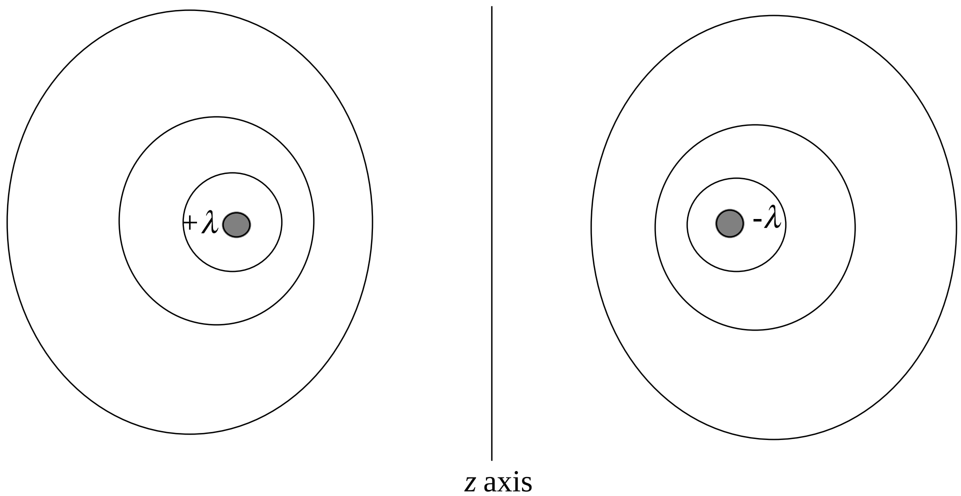
(b) Equate the final kinetic energy of the automobile to the energy released by the lightning: $\Delta U = \frac{1}{2}mv^2$, where m is the mass of the automobile and v is its final speed. Thus

$$v = \sqrt{\frac{2\,\Delta U}{m}} = \sqrt{\frac{2(3.0 \times 10^{10}\,\text{J})}{1000\,\text{kg}}} = 7.7 \times 10^3\,\text{m/s}\,.$$

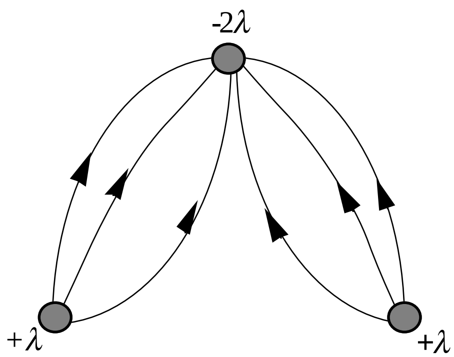
(c) Equate the energy required to melt mass m of ice to the energy released by the lightning: $\Delta U = mL_F$, where L_F is the heat of fusion for water. Thus

$$m = \frac{\Delta U}{L_F} = \frac{3.0 \times 10^{10}\,\text{J}}{3.3 \times 10^5\,\text{J/kg}} = 9.0 \times 10^4\,\text{kg}\,.$$

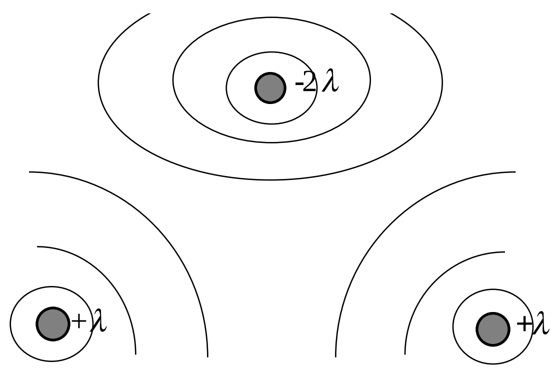
4E



5P



some electric field lines



some equipotential cross sections

6E

(a) $V_B - V_A = \Delta U/(-e) = 3.94 \times 10^{-19}\,\mathrm{J}/(-1.60 \times 10^{-19}\,\mathrm{C}) = -2.46\,\mathrm{V}$.

(b) $V_C - V_A = V_B - V_A = -2.46\,\mathrm{V}$.

(c) $V_C - V_B = 0$ (Since C and B are on the same equipotential line).

7E

$\Delta V = E\Delta s = (1.92 \times 10^5\,\mathrm{N/C})(0.0150\,\mathrm{m}) = 2.90 \times 10^3\,\mathrm{V}$.

8E

(*a*) $E = F/e = 3.9 \times 10^{-15}\,\mathrm{N}/(1.60 \times 10^{-19}\,\mathrm{C}) = 2.4 \times 10^4\,\mathrm{N/C}$.
(*b*) $\Delta V = E\Delta s = (2.4 \times 10^4\,\mathrm{N/C})(0.12\,\mathrm{m}) = 2.9 \times 10^3\,\mathrm{V}$.

9E

The electric field produced by an infinite sheet of charge has magnitude $E = \sigma/2\epsilon_0$, where σ is the surface charge density. The field is normal to the sheet and is uniform. Place the origin of a coordinate system at the sheet and take the x axis to be parallel to the field and positive in the direction of the field. Then the electric potential is

$$V = V_s - \int_0^x E\,dx = V_s - Ex\,,$$

where V_s is the potential at the sheet. The equipotential surfaces are surfaces of constant x; that is, they are planes that are parallel to the plane of charge. If two surfaces are separated by Δx then their potentials differ by $\Delta V = -E\Delta x = (\sigma/2\epsilon_0)\Delta x$. Thus

$$\Delta x = \frac{2\epsilon_0\,\Delta V}{\sigma} = \frac{2(8.85 \times 10^{-12}\,\mathrm{C^2/N{\cdot}m^2})(50\,\mathrm{V})}{0.10 \times 10^{-6}\,\mathrm{C/m^2}} = 8.8 \times 10^{-3}\,\mathrm{m}\,.$$

10P

(*a*)

$$W = \int_i^f q_0\mathbf{E}\cdot d\mathbf{s} = \frac{q_0\sigma}{2\epsilon_0}\int_0^z dz = \frac{q_0\sigma z}{2\epsilon_0}\,.$$

(*b*) Since $V - V_0 = -W/q_0 = -\sigma z/2\epsilon_0$,

$$V = V_0 - \frac{\sigma z}{2\epsilon_0}\,.$$

11P

The potential difference between the wire and cylinder is given, not the linear charge density on the wire. Use Gauss' law to find an expression for the electric field a distance r from the center of the wire, between the wire and the cylinder, in terms of the linear charge density. Then integrate with respect to r to find an expression for the potential difference between the wire and cylinder in terms of the linear charge density. Use this result to obtain an expression for the linear charge density in terms of the potential difference and substitute the result into the equation for the electric field. This will give the electric field in terms of the potential difference and will allow you to compute numerical values for the field at the wire and at the cylinder.

For the Gaussian surface use a cylinder of radius r and length ℓ, concentric with the wire and cylinder. The electric field is normal to the rounded portion of the cylinder's surface and is uniform over that surface. This means the electric flux through the Gaussian surface is given by $2\pi r\ell E$, where E is the magnitude of the electric field. The charge enclosed by the Gaussian surface is $q = \lambda\ell$, where λ is the linear charge density on the wire. Gauss' law yields $2\pi\epsilon_0 r\ell E = \lambda\ell$. Thus

$$E = \frac{\lambda}{2\pi\epsilon_0 r}\,.$$

Since the field is radial, the difference in the potential V_c of the cylinder and the potential V_w of the wire is

$$\Delta V = V_w - V_c = -\int_{r_c}^{r_w} E\,dr = \int_{r_w}^{r_c} \frac{\lambda}{2\pi\epsilon_0 r}\,dr = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{r_c}{r_w}\right),$$

where r_w is the radius of the wire and r_c is the radius of the cylinder. This means that

$$\lambda = \frac{2\pi\epsilon_0\,\Delta V}{\ln(r_c/r_w)}$$

and

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{\Delta V}{r\ln(r_c/r_w)}\,.$$

(a) Substitute r_w for r to obtain the field at the surface of the wire:

$$\begin{aligned} E &= \frac{\Delta V}{r_w\ln(r_c/r_w)} = \frac{850\,\mathrm{V}}{(0.65\times10^{-6}\,\mathrm{m})\ln\left[(1.0\times10^{-2}\,\mathrm{m})/(0.65\times10^{-6}\,\mathrm{m})\right]} \\ &= 1.36\times10^{8}\,\mathrm{V/m}\,. \end{aligned}$$

(b) Substitute r_c for r to find the field at the surface of the cylinder:

$$\begin{aligned} E &= \frac{\Delta V}{r_w\ln(r_c/r_w)} = \frac{850\,\mathrm{V}}{(1.0\times10^{-2}\,\mathrm{m})\ln\left[(1.0\times10^{-2}\,\mathrm{m})/(0.65\times10^{-6}\,\mathrm{m})\right]} \\ &= 8.82\times10^{3}\,\mathrm{V/m}\,. \end{aligned}$$

12P

(a)

$$V(r) = V(0) - \int_0^r E(r)dr = 0 - \int_0^r \frac{qr}{4\pi\epsilon_0 R^3}\,dr = -\frac{qr^2}{8\pi\epsilon_0 R^3}\,.$$

(b) $\Delta V = V(0) - V(R) = q/8\pi\epsilon_0 R$.

(c) Since $\Delta V = V(0) - V(R) > 0$, the potential at the center of the sphere is higher.

13P

(a) Use Gauss' law to find expressions for the electric field inside and outside the spherical charge distribution. Since the field is radial the electric potential can be written as an integral of the field along a sphere radius, extended to infinity. Since different expressions for the field apply in different regions the integral must be split into two parts, one from infinity to the surface of the distribution and one from the surface to a point inside.

Outside the charge distribution the magnitude of the field is $E = q/4\pi\epsilon_0 r^2$ and the potential is $V = q/4\pi\epsilon_0 r$, where r is the distance from the center of the distribution.

To find an expression for the magnitude of the field inside the charge distribution use a Gaussian surface in the form of a sphere with radius r, concentric with the distribution. The field is normal to the Gaussian surface and its magnitude is uniform over it, so the electric flux through the surface is $4\pi r^2 E$. The charge enclosed is qr^3/R^3. Gauss' law becomes

$$4\pi\epsilon_0 r^2 E = \frac{qr^3}{R^3},$$

so

$$E = \frac{qr}{4\pi\epsilon_0 R^3}.$$

If V_s is the potential at the surface of the distribution ($r = R$) then the potential at a point inside, a distance r from the center, is

$$V = V_s - \int_R^r E\,dr = V_s - \frac{q}{4\pi\epsilon_0 R^3}\int_R^r r\,dr = V_s - \frac{qr^2}{8\pi\epsilon_0 R^3} + \frac{q}{8\pi\epsilon_0 R}.$$

The potential at the surface can be found by replacing r with R in the expression for the potential at points outside the distribution. It is $V_s = q/4\pi\epsilon_0 R$. Thus

$$V = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{R} - \frac{r^2}{2R^3} + \frac{1}{2R}\right) = \frac{q}{8\pi\epsilon_0 R^3}(3R^2 - r^2).$$

(b) In 12P the electric potential was taken to be zero at the center of the sphere. In this problem it is zero at infinity. According to the expression derived in part (a) the potential at the center of the sphere is $V_c = 3q/8\pi\epsilon_0 R$. Thus $V - V_c = -qr^2/8\pi\epsilon_0 R^3$. This is the result of 12P.

(c) The potential difference is

$$\Delta V = V_s - V_c = \frac{2q}{8\pi\epsilon_0 R} - \frac{3q}{8\pi\epsilon_0 R} = -\frac{q}{8\pi\epsilon_0 R}.$$

The same value as is given by the expression obtained in 12P.

(d) Only potential differences have physical significance, not the value of the potential at any particular point. The same value can be added to the potential at every point without changing the electric field, for example. Changing the reference point from the center of the distribution to infinity changes the value of the potential at every point but it does not change any potential differences.

14P

(a) For $r > r_2$ the field is like that of a point charge and

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r},$$

where the zero of potential was taken to be at infinity.

(b) To find the potential in the region $r_1 < r < r_2$, first use Gauss's law to find an expression for the electric field, then integrate along a radial path from r_2 to r. The Gaussian surface is a sphere of radius r, concentric with the shell. The field is radial and therefore normal to the surface. Its magnitude is uniform over the surface, so the flux through the surface is $\Phi = 4\pi r^2 E$. The volume of the shell is $(4\pi/3)(r_2^3 - r_1^3)$, so the charge density is

$$\rho = \frac{3Q}{4\pi(r_2^3 - r_1^3)}$$

and the charge enclosed by the Gaussian surface is

$$q = \left(\frac{4\pi}{3}\right)(r^3 - r_1^3)\rho = Q\left(\frac{r^3 - r_1^3}{r_2^3 - r_1^3}\right) .$$

Gauss' law yields

$$4\pi\epsilon_0 r^2 E = Q\left(\frac{r^3 - r_1^3}{r_2^3 - r_1^3}\right)$$

and the magnitude of the electric field is

$$E = \frac{Q}{4\pi\epsilon_0}\,\frac{r^3 - r_1^3}{r^2(r_2^3 - r_1^3)} .$$

If V_s is the electric potential at the outer surface of the shell ($r = r_2$) then the potential a distance r from the center is given by

$$\begin{aligned} V = V_s - \int_{r_2}^{r} E\,dr &= V_s - \frac{Q}{4\pi\epsilon_0}\,\frac{1}{r_2^3 - r_1^3}\int_{r_2}^{r}\left(r - \frac{r_1^3}{r^2}\right)dr \\ &= V_s - \frac{Q}{4\pi\epsilon_0}\,\frac{1}{r_2^3 - r_1^3}\left(\frac{r^2}{2} - \frac{r_2^2}{2} + \frac{r_1^3}{r} - \frac{r_1^3}{r_2}\right) . \end{aligned}$$

The potential at the outer surface is found by placing $r = r_2$ in the expression found in part (a). It is $V_s = Q/4\pi\epsilon_0 r_2$. Make this substitution and collect terms to find

$$V = \frac{Q}{4\pi\epsilon_0}\,\frac{1}{r_2^3 - r_1^3}\left(\frac{3r_2^2}{2} - \frac{r^2}{2} - \frac{r_1^3}{r}\right) .$$

Since $\rho = 3Q/4\pi(r_2^3 - r_1^3)$ this can also be written

$$V = \frac{\rho}{3\epsilon_0}\left(\frac{3r_2^2}{2} - \frac{r^2}{2} - \frac{r_1^3}{r}\right) .$$

(c) The electric field vanishes in the cavity, so the potential is everywhere the same inside and has the same value as at a point on the inside surface of the shell. Put $r = r_1$ in the result of part (b). After collecting terms the result is

$$V = \frac{Q}{4\pi\epsilon_0}\,\frac{3(r_2^2 - r_1^2)}{2(r_2^3 - r_1^3)} ,$$

or in terms of the charge density $V = (\rho/2\epsilon_0)(r_2^2 - r_1^2)$.
(d) The solutions agree at $r = r_1$ and at $r = r_2$.

15E

Let $V = 0$ at $r \to \infty$. Then $V(r) = q/4\pi\epsilon_0 r$. Thus
(a)

$$\begin{aligned} V_A - V_B &= \frac{q}{4\pi\epsilon_0 r_A} - \frac{q}{4\pi\epsilon_0 r_B} \\ &= (1.0 \times 10^{-6}\,\mathrm{C})(8.99 \times 10^9\,\mathrm{N{\cdot}m^2/C^2})\left(\frac{1}{2.0\,\mathrm{m}} - \frac{1}{1.0\,\mathrm{m}}\right) = -4500\,\mathrm{V}. \end{aligned}$$

(b) Since $V(r)$ depends only on the magnitude of $\mathbf{r}$, the result is unchanged.

16E

(a) From symmetry consideration, the equipotential surface with $V = 30\,\mathrm{V}$ must be a sphere (of radius R) centered at q, where $V = q/4\pi\epsilon_0 R$. Solve for R:

$$R = \frac{q}{4\pi\epsilon_0 V} = \frac{(1.5 \times 10^{-8}\,\mathrm{C})(8.99 \times 10^9\,\mathrm{N{\cdot}m^2/C^2})}{30\,\mathrm{V}} = 4.5\,\mathrm{m}\,.$$

(b) Since $\Delta R = (q/4\pi\epsilon_0)\Delta(1/V)$, ΔR is not proportional to ΔV (but rather to ΔV^{-1}), so the surfaces are not evenly spaced.

17E

Imagine moving all the charges on the surface of the sphere to the center of the the sphere. Using Gauss' law you can see that this would not change the electric field *outside* the sphere. The magnitude of the electric field E of the uniformly charged sphere as a function of r, the distance from the center of the sphere, is thus given by $E(r) = q/(4\pi\epsilon_0 r^2)$ for $r > R$. Here R is the radius of the sphere. Thus the potential V at the surface of the sphere (where $r = R$) is given by

$$\begin{aligned} V(R) &= V\Big|_{r=\infty} + \int_R^\infty E(r)\,dr = \int_\infty^R \frac{q}{4\pi\epsilon_0 r^2}\,dr = \frac{q}{4\pi\epsilon_0 R} \\ &= \frac{(8.99 \times 10^9\,\mathrm{N{\cdot}m^2/C^2})(1.50 \times 10^8\,\mathrm{C})}{16.0\,\mathrm{cm}} = 8.43 \times 10^2\,\mathrm{V}\,. \end{aligned}$$

18E

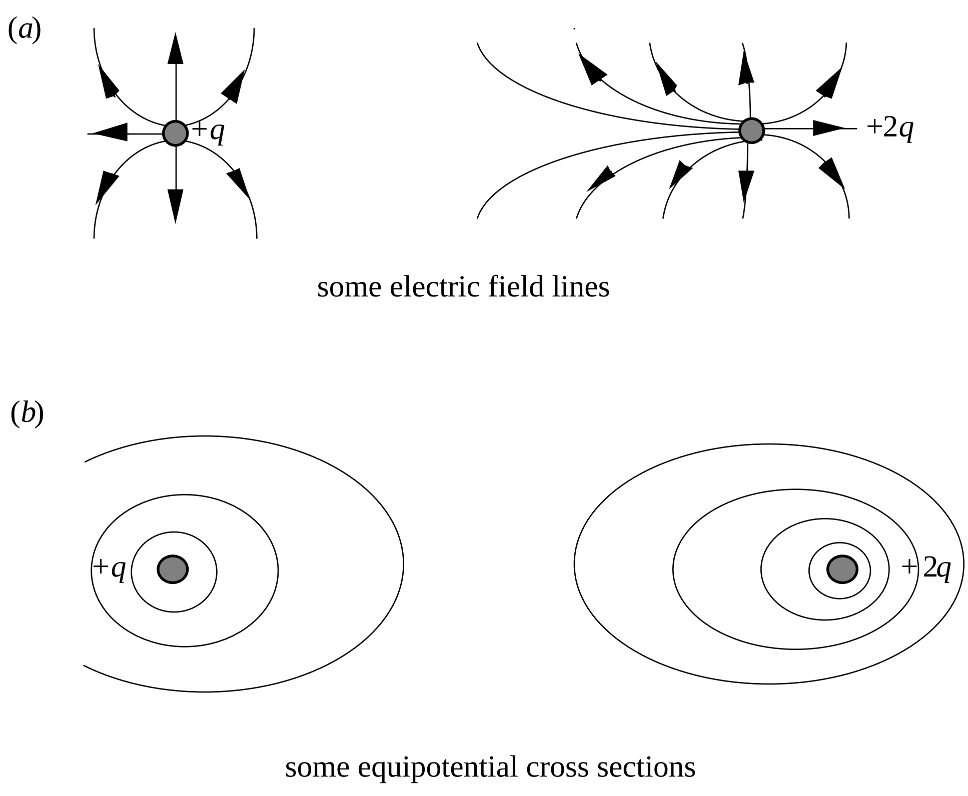
$$q = 4\pi\epsilon_0 RV = \frac{(10\,\mathrm{m})(-1.0\,\mathrm{V})}{8.99 \times 10^9\,\mathrm{N{\cdot}m^2/C^2}} = -1.1 \times 10^{-9}\,\mathrm{C}\,.$$

19E

If the electric potential is zero at infinity then at the surface of a uniformly charged sphere it is $V = q/4\pi\epsilon_0 R$, where q is the charge on the sphere and R is the sphere radius. Thus $q = 4\pi\epsilon_0 RV$ and the number of electrons is

$$N = \frac{|q|}{e} = \frac{4\pi\epsilon_0 R|V|}{e} = \frac{(1.0 \times 10^{-6}\,\mathrm{m})(400\,\mathrm{V})}{(8.99 \times 10^9\,\mathrm{N{\cdot}m^2/C^2})(1.60 \times 10^{-19}\,\mathrm{C})} = 2.8 \times 10^5\,.$$

20E



21E

First, convince yourself that $V(x)$ cannot be equal to zero for $x > d$. In fact $V(x)$ is always negative for $x > d$. Now consider the two remaining regions on the x axis: $x < 0$ and

$0 < x < d$. For $x < 0$ the separation between q_1 and a point on the x axis whose coordinate is x is given by $d_1 = -x$; while the corresponding separation for q_2 is $d_2 = d - x$. Now set

$$V(x) = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{d_1} + \frac{q_2}{d_2}\right) = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{-x} + \frac{-3}{d-x}\right) = 0$$

to obtain $x = -d/2$. Similarly, for $0 < x < d$ we have $d_1 = x$ and $d_2 = d - x$. Let

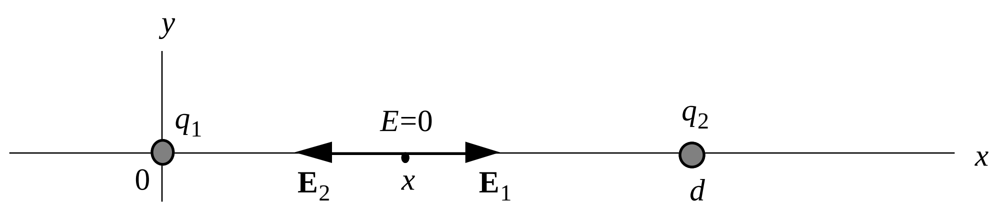
$$V(x) = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{d_1} + \frac{q_2}{d_2}\right) = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{x} + \frac{-3}{d-x}\right) = 0$$

and solve for x: $x = d/4$.

22E

(a) Since both charges are positive, so is the electric potential created by each of them. Thus the net potential cannot possibly be zero anywhere except at infinity.

(b)



In the figure above, let $E(x) = E_1 - E_2 = 0$, we obtain

$$E_1(x) = \frac{1}{4\pi\epsilon_0}\frac{q_1}{x^2} = E_2(x) = \frac{1}{4\pi\epsilon_0}\frac{q_2}{(d-x)^2}\,.$$

Combine this equation with $q_1 = +q$ and $q_2 = +2q$ and solve for x:

$$x = \frac{d}{1+\sqrt{2}} = \frac{1.0\,\mathrm{m}}{1+\sqrt{2}} = 0.41\,\mathrm{m}\,.$$

23E

Since according to the problem statement there is a point in between the two charges on the x axis where the net electric field is zero, the fields at that point due to q_1 and q_2 must be directed opposite to each other. This means that q_1 and q_2 must have the same sign (i.e., either both are positive or both negative). Thus the potentials due to either of them must be of the same sign. Therefore the net electric potential cannot possibly be zero anywhere except at infinity.

24E

(*a*)

$$V = \frac{q}{4\pi\epsilon_0 R} = \frac{(4.0 \times 10^{-6}\,\mathrm{C})(8.99 \times 10^9\,\mathrm{N\cdot m^2/C^2})}{0.10\,\mathrm{m}} = 3.6 \times 10^5\,\mathrm{V}\,.$$

(*b*) The field just outside the sphere would be

$$E = \frac{q}{4\pi\epsilon_0 R^2} = \frac{V}{R} = \frac{3.6 \times 10^5\,\mathrm{V}}{0.10\,\mathrm{m}} = 3.6 \times 10^6\,\mathrm{V/m}\,,$$

which would have exceeded 3.0 MV/m. So this situation cannot occur.

25E

(*a*)

$$q = 4\pi\epsilon_0 VR = \frac{(200\,\mathrm{V})(0.15\,\mathrm{m})}{8.99 \times 10^9\,\mathrm{N\cdot m^2/C^2}} = 3.3 \times 10^{-9}\,\mathrm{C}\,.$$

(*b*)

$$\sigma = \frac{q}{4\pi R^2} = \frac{3.3 \times 10^{-9}\,\mathrm{C}}{4\pi(0.15\,\mathrm{m})^2} = 1.2 \times 10^{-8}\,\mathrm{C/m^2}\,.$$

26P

(*a*) The electric potential V at the surface of the drop, the charge q on the drop, and the radius R of the drop are related by $V = q/4\pi\epsilon_0 R$. Thus

$$R = \frac{q}{4\pi\epsilon_0 V} = \frac{(8.99 \times 10^9\,\mathrm{N\cdot m^2/C^2})(30 \times 10^{-12}\,\mathrm{C})}{500\,\mathrm{V}} = 5.4 \times 10^{-4}\,\mathrm{m}\,.$$

(*b*) After the drops combine the total volume is twice the volume of an original drop, so the radius R' of the combined drop is given by $(R')^3 = 2R^3$ and $R' = 2^{1/3}R$. The charge is twice the charge of original drop: $q' = 2q$. Thus

$$V' = \frac{1}{4\pi\epsilon_0}\frac{q'}{R'} = \frac{1}{4\pi\epsilon_0}\frac{2q}{2^{1/3}R} = \frac{2V}{2^{1/3}} = \frac{2(500\,\mathrm{V})}{2^{1/3}} = 790\,\mathrm{V}\,.$$

27P

Assume the charge on the Earth is distributed with spherical symmetry. If the electric potential is zero at infinity then at the surface of the Earth it is $V = q/4\pi\epsilon_0 R$, where q is the charge on the Earth and R $(= 6.37\times 10^6\,\mathrm{m})$ is the radius of the Earth. The magnitude of the electric field at the surface is $E = q/4\pi\epsilon_0 R^2$, so $V = ER = (100\,\mathrm{V/m})(6.37 \times 10^6\,\mathrm{m}) = 6.4 \times 10^8\,\mathrm{V}$.

28P

A charge $-5q$ is a distance $2d$ from point P, a charge $-5q$ is a distance d from P, and two charges $+5q$ are each a distance d from P, so the electric potential at P is

$$V = \frac{q}{4\pi\epsilon_0}\left(-\frac{5}{2d} - \frac{5}{d} + \frac{5}{d} + \frac{5}{d}\right) = -\frac{5q}{8\pi\epsilon_0}\,.$$

The zero of the electric potential was taken to be at infinity.

29P

Use $q = 137,000\,\mathrm{C}$ from Sample Problem 22-4 to find V:

$$V = \frac{q}{4\pi\epsilon_0 R_e} = \frac{(137,000\,\mathrm{C})(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})}{6.37\times10^6\,\mathrm{m}} = 1.93\times10^8\,\mathrm{V}\,.$$

30P

The net electric potential at point P is the sum of those due to the six charges:

$$\begin{aligned}
V_P &= \sum_{i=1}^{6} V_{Pi} = \sum_{i=1}^{6}\frac{q_i}{4\pi\epsilon_0 r_i} \\
&= \frac{1}{4\pi\epsilon_0}\left[\frac{5.0q}{\sqrt{d^2+(d/2)^2}} + \frac{-2.0q}{d/2} + \frac{-3.0q}{\sqrt{d^2+(d/2)^2}}\right. \\
&\qquad \left. + \frac{3.0q}{\sqrt{d^2+(d/2)^2}} + \frac{-2.0q}{d/2} + \frac{-5.0q}{\sqrt{d^2+(d/2)^2}}\right] \\
&= \frac{-0.94q}{4\pi\epsilon_0 d}\,.
\end{aligned}$$

31P

(a) and (b). Consider the two points A and B, with $(A_x, A_y, A_z) = (R + x_c,\ 0,\ 0)$ and $(B_x, B_y, B_z) = (R - x_c,\ 0,\ 0)$, where the circle intersects the x axis. We have

$$\begin{cases}
4\pi\epsilon_0 V_A = \dfrac{q_1}{R+x_c} + \dfrac{q_2}{x_2-(R+x_c)} = 0\,, \\
4\pi\epsilon_0 V_B = \dfrac{q_1}{R-x_c} + \dfrac{q_2}{x_2+(R-x_c)} = 0\,.
\end{cases}$$

Solve for R and x_c:

$$\begin{cases}
x_c = \dfrac{q_1^2 x_2}{q_1^2-q_2^2} = \dfrac{(6.0e)^2(8.6\,\mathrm{nm})}{(6.0e)^2-(-10e)^2} = -4.8\,\mathrm{nm}\,, \\
R = \dfrac{q_1 q_2 x_2}{q_1^2-q_2^2} = \dfrac{(6.0e)(-10e)(8.6\,\mathrm{nm})}{(6.0e)^2-(-10e)^2} = 8.1\,\mathrm{nm}\,.
\end{cases}$$

(c) No. In fact the $V = 0$ one is the only circular one.

32P

(a) The net charge carried by the sphere after a time t is $q = \frac{1}{2}aet$, where a is the activity of the nickel coating. Thus from $V = q/4\pi\epsilon_0 R = aet/8\pi\epsilon_0 R$ we solve for t:

$$t = \frac{8\pi\epsilon_0 VR}{ae} = \frac{2(1000\,\mathrm{V})(0.010\,\mathrm{m})}{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(3.70\times10^8/\mathrm{s})(1.60\times10^{-19}\,\mathrm{C})} = 38\,\mathrm{s}\,.$$

(b) The time t' required satisfies $\frac{1}{2}a\epsilon t' = c\Delta T$, where $\epsilon = 100\,\mathrm{keV}$, $c = 14.3\,\mathrm{J/K}$, and $\Delta T = 5.0\,\mathrm{C}^\circ$. Thus

$$\begin{aligned} t' &= \frac{2c\Delta T}{a\epsilon} = \frac{2(14.3\,\mathrm{J/K})(5.0\,\mathrm{C}^\circ)}{(3.70\times10^8/\mathrm{s})(100\times10^3\times1.6\times10^{-19}\,\mathrm{J})} \\ &= 2.4\times10^9\,\mathrm{s} = 2.8\times10^2\,\mathrm{da}\,. \end{aligned}$$

33E

Use Eq. 25-20:

$$V = \frac{1}{4\pi\epsilon_0}\frac{p}{r^2} = \frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(1.47\times3.34\times10^{-30}\,\mathrm{C\cdot m})}{(52.0\times10^{-9}\,\mathrm{m})^2} = 1.63\times10^{-5}\,\mathrm{V}\,.$$

34E

A positive charge q is a distance $r-d$ from point P, another positive charge q is a distance r from P, and a negative charge $-q$ is a distance $r+d$ from P. Sum the individual electric potentials created at P to find the total:

$$V = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r-d} + \frac{1}{r} - \frac{1}{r+d}\right).$$

Use the binomial theorem to approximate $1/(r-d)$ for r much larger than d:

$$\frac{1}{r-d} = (r-d)^{-1} \approx (r)^{-1} - (r)^{-2}(-d) = \frac{1}{r} + \frac{d}{r^2}\,.$$

Similarly,

$$\frac{1}{r+d} \approx \frac{1}{r} - \frac{d}{r^2}\,.$$

Only the first two terms of each expansion were retained. Thus

$$V \approx \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r} + \frac{d}{r^2} + \frac{1}{r} - \frac{1}{r} + \frac{d}{r^2}\right) = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r} + \frac{2d}{r^2}\right) = \frac{q}{4\pi\epsilon_0 r}\left(1 + \frac{2d}{r}\right).$$

35E

(a) From Eq. 25-35

$$V = 2\frac{\lambda}{4\pi\epsilon_0}\ln\left[\frac{L/2 + (L^2/4 + d^2)^{1/2}}{d}\right].$$

(b) $V = 0$ due to superposition.

36E

$$V_P = \frac{1}{4\pi\epsilon_0}\int_{\text{rod}}\frac{dq}{R} = \frac{1}{4\pi\epsilon_0 R}\int_{\text{rod}} dq = \frac{-Q}{4\pi\epsilon_0 R}.$$

37E

(a) All the charge is the same distance R from C, so the electric potential at C is

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{R} - \frac{6Q}{R}\right) = -\frac{5Q}{4\pi\epsilon_0 R},$$

where the zero was taken to be at infinity.

(b) All the charge is the same distance from P. That distance is $\sqrt{R^2 + z^2}$, so the electric potential at P is

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{\sqrt{R^2+z^2}} - \frac{6Q}{\sqrt{R^2+z^2}}\right) = -\frac{5Q}{4\pi\epsilon_0\sqrt{R^2+z^2}}.$$

38E

The disk is uniformly charged. This means that when the full disk is present each quadrant contributes equally to the electric potential at P, so the potential at P due to a single quadrant is one-fourth the potential due to the entire disk. First find an expression for the potential at P due to the entire disk.

Consider a ring of charge with radius r and width dr. Its area is $2\pi r\,dr$ and it contains charge $dq = 2\pi\sigma r\,dr$. All the charge in it is a distance $\sqrt{r^2 + z^2}$ from P, so the potential it produces at P is

$$dV = \frac{1}{4\pi\epsilon_0}\frac{2\pi\sigma r\,dr}{\sqrt{r^2+z^2}} = \frac{\sigma r\,dr}{2\epsilon_0\sqrt{r^2+z^2}}.$$

The total potential at P is

$$V = \frac{\sigma}{2\epsilon_0}\int_0^R \frac{r\,dr}{\sqrt{r^2+z^2}} = \frac{\sigma}{2\epsilon_0}\sqrt{r^2+z^2}\Big|_0^R = \frac{\sigma}{2\epsilon_0}\left(\sqrt{R^2+z^2} - z\right).$$

The potential V_{sq} at P due to a single quadrant is

$$V_{sq} = \frac{V}{4} = \frac{\sigma}{8\epsilon_0}\left(\sqrt{R^2+z^2} - z\right).$$

39P

Consider an infinitesimal segment of the ring, located between r' and $r + dr'$ from the center of the ring. The area of this segment is $dA = 2\pi r' dr'$, and the charge it carries is $dq = \sigma dA = 2\pi\sigma r' dr'$. The separation between any point on this segment of the ring and point P is $a = \sqrt{r'^2 + z^2}$. Thus the contribution to V_P from this segment is given by $dV_P = dq/(4\pi\epsilon_0 a)$. Integrate over the entire ring to obtain V_P:

$$V_P = \int_{\text{ring}} dV_P = \frac{1}{4\pi\epsilon_0}\int_{\text{ring}} \frac{dq}{a} = \frac{1}{4\pi\epsilon_0}\int_r^R \frac{2\pi\sigma r'}{\sqrt{r'^2 + z^2}}\, dr' \,.$$

Upon making a change of variable $u = r'^2$ the integral above can be readily evaluated to yield the result

$$V_P = \frac{\sigma}{2\epsilon_0}[\sqrt{r'^2 + z^2}]\bigg|_r^R = \left(\frac{\sigma}{2\epsilon_0}\right)(\sqrt{R^2 + z^2} - \sqrt{r^2 + z^2})\,.$$

For $z = 2.00R$ and $r = 0.200R$ this gives $V_P = 0.113\sigma R/\epsilon_0$.

40P

(a) Denote the surface charge density of the disk as σ_1 for $0 < r < R/2$, and as σ_2 for $R/2 < r < R$. Thus the total charge on the disk is given by

$$\begin{aligned} q &= \int_{\text{disk}} dq = \int_0^{R/2} 2\pi\sigma_1 r dr + \int_{R/2}^R 2\pi\sigma_2 r dr = \frac{\pi}{4}R^2(\sigma_1 + 3\sigma_2) \\ &= \frac{\pi}{4}(2.20 \times 10^{-2}\,\text{m})^2[1.50 \times 10^{-6}\,\text{C/m}^2 + 3(8.00 \times 10^{-7}\,\text{C/m}^2)] \\ &= 1.48 \times 10^{-9}\,\text{C}\,. \end{aligned}$$

(b) Use Eq. 25-36:

$$\begin{aligned} V(z) &= \int_{\text{disk}} dV = \frac{1}{4\pi\epsilon_0}\left[\int_0^{R/2} \frac{\sigma_1(2\pi R')dR'}{\sqrt{z^2 + R'^2}} + \int_{R/2}^R \frac{\sigma_2(2\pi R')dR'}{\sqrt{z^2 + R'^2}}\right] \\ &= \frac{\sigma_1}{2\epsilon_0}\left(\sqrt{z^2 + \frac{R^2}{4}} - z\right) + \frac{\sigma_2}{2\epsilon_0}\left(\sqrt{z^2 + R^2} - \sqrt{z^2 + \frac{R^2}{4}}\right). \end{aligned}$$

Plug in thenumerical values of σ_1, σ_2, R and z to obtain $V(z) = 7.95 \times 10^2\,\text{V}$.

41P

(a) Consider an infinitesimal segment of the rod, located between x and $x + dx$. It has length dx and contains charge $dq = \lambda\, dx$, where $\lambda = Q/L$ is the linear charge density of the rod. Its distance from P_1 is $d + x$ and the potential it creates at P_1 is

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{d + x} = \frac{1}{4\pi\epsilon_0}\frac{\lambda\, dx}{d + x}\,.$$

To find the total potential at P_1, integrate over the rod:

$$V = \frac{\lambda}{4\pi\epsilon_0}\int_0^L \frac{dx}{d+x} = \frac{\lambda}{4\pi\epsilon_0}\ln(d+x)\Big|_0^L = \frac{Q}{4\pi\epsilon_0 L}\ln\left(1+\frac{L}{d}\right).$$

42P

(a) Similar to the last problem, let us consider an infinitesimal segment of the rod, located between x and $x+dx$. It has length dx and contains charge $dq = \lambda\, dx = cxdx$. Its distance from P_1 is $d+x$ and the potential it creates at P_1 is

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{d+x} = \frac{1}{4\pi\epsilon_0}\frac{cx\,dx}{d+x}.$$

To find the total potential at P_1, integrate over the rod:

$$V = \frac{c}{4\pi\epsilon_0}\int_0^L \frac{x\,dx}{d+x} = \frac{c}{4\pi\epsilon_0}[x - d\ln(x+d)]\Big|_0^L = \frac{c}{4\pi\epsilon_0}\left[L - d\ln\left(1+\frac{L}{d}\right)\right].$$

43E

The magnitede of the electric field is given by

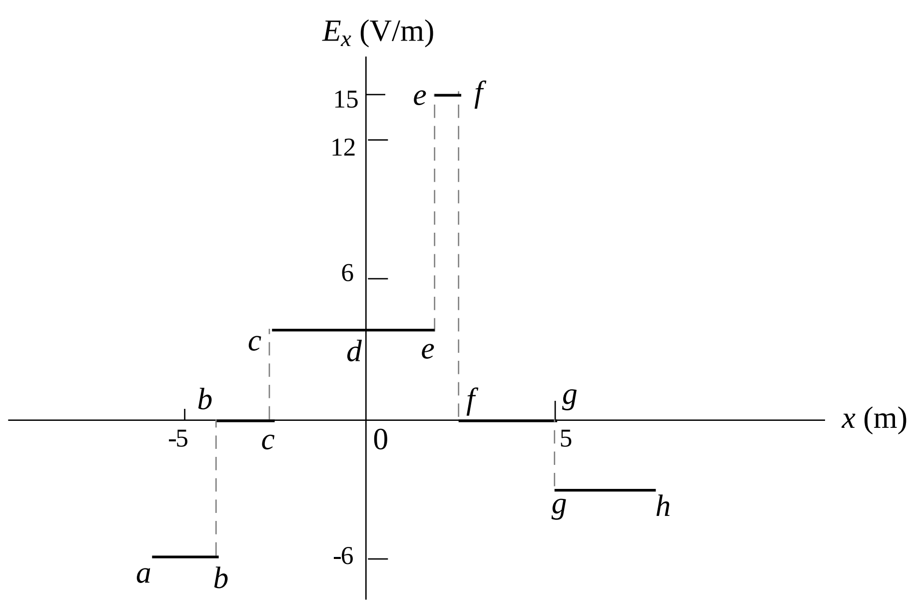
$$|E| = \left|-\frac{\Delta V}{\Delta s}\right| = \frac{2(5.0\,\mathrm{V})}{0.015\,\mathrm{m}} = 6.7\times10^2\,\mathrm{V/m}.$$

E points from the positively charged plate to the negatively charged one.

44E

Use $E_x = -dV/dx$, where dV/dx is the local slope of the V vs. x curve depicted in Fig. 25-48. The results are:

$E_x(ab) = -6.0\,\mathrm{V/m}$, $E_x(bc) = 0$, $E_x(cd) = E_x(de) = 3.0\,\mathrm{V/m}$,
$E_x(ef) = 15\,\mathrm{V/m}$, $E_x(fg) = 0$, $E_x(gh) = -3.0\,\mathrm{V/m}$.



45E

On the dipole axis $\theta = 0$ or π so $|\cos\theta| = 1$. The magnitude of the electric field is thus given by

$$|E(r)| = \left| -\frac{\partial V}{\partial r} \right| = \frac{p}{4\pi\epsilon_0}\left|\frac{d}{dr}\left(\frac{1}{r^2}\right)\right| = \frac{p}{2\pi\epsilon_0 r^3}\,.$$

46E

Use Eq. 25-41:

$$\begin{aligned} E_x(x,y) &= -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}[(2.0\,\mathrm{V/m^2})x^2 - (3.0\,\mathrm{V/m^2})y^2] = -2(2.0\,\mathrm{V/m^2})x\,; \\ E_y(x,y) &= -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}[(2.0\,\mathrm{V/m^2})x^2 - (3.0\,\mathrm{V/m^2})y^2] = 2(3.0\,\mathrm{V/m^2})y\,. \end{aligned}$$

Now plug in $x = 3.0\,\mathrm{m}$ and $y = 2.0\,\mathrm{m}$ to obtain the magnitude of **E**: $E = \sqrt{E_x^2 + E_y^2} = 17\,\mathrm{V/m}$. **E** makes an angle θ with the positive x axis, where

$$\theta = \tan^{-1}\left(\frac{E_y}{E_x}\right) = 135^\circ\,.$$

47E

$$\begin{aligned} \mathbf{E} &= -\left(\frac{dV}{dx}\right)\mathbf{i} = -\frac{d}{dx}(1500x^2)\,\mathbf{i} = (-3000x)\,\mathbf{i} \\ &= (-3000\,\mathrm{V/m^2})(0.0130\,\mathrm{m})\,\mathbf{i} = (-39\,\mathrm{V/m})\,\mathbf{i}\,. \end{aligned}$$

48E

(*a*) The E-field corresponding to V is

$$\begin{aligned} E &= -\frac{dV}{dr} = -\frac{d}{dr}\left[\frac{Ze}{4\pi\epsilon_0}\left(\frac{1}{r} - \frac{3}{2R} + \frac{r^2}{2R^3}\right)\right] \\ &= \frac{Ze}{4\pi\epsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right). \end{aligned}$$

(*b*) The expression for $V(r)$ is valid inside the atom only and cannot be extended to $r \to \infty$.

49P

(a) The charge on every part of the ring is the same distance from any point P on the axis. This distance is $r = \sqrt{z^2 + R^2}$, where R is the radius of the ring and z is the distance from the center of the ring to P. The electric potential at P is

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r} = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{\sqrt{z^2+R^2}} = \frac{1}{4\pi\epsilon_0}\frac{1}{\sqrt{z^2+R^2}}\int dq = \frac{1}{4\pi\epsilon_0}\frac{q}{\sqrt{z^2+R^2}}\,.$$

(b) The electric field is along the axis and its component is given by

$$\begin{aligned} E &= -\frac{\partial V}{\partial z} = -\frac{q}{4\pi\epsilon_0}\frac{d}{dz}(z^2+R^2)^{-1/2} \\ &= -\frac{q}{4\pi\epsilon_0}\left(\frac{1}{2}\right)(z^2+R^2)^{-3/2}(2z) = \frac{q}{4\pi\epsilon_0}\frac{z}{(z^2+R^2)^{3/2}}\,. \end{aligned}$$

This agrees with the result of Section 23-6.

50P

From 41P the electric potential for a point on the x axis whose x coordinate is given by $x = -d < 0$ is

$$V(x=-d) = \frac{Q}{4\pi\epsilon_0 L}\ln\left(1+\frac{L}{d}\right).$$

Replace $x = -d$ with a general value x to obtain

$$V(x) = \frac{Q}{4\pi\epsilon_0 L}\ln\left(1-\frac{L}{x}\right).$$

Thus

$$E_x(x=-d) = -\frac{dV}{dx}\bigg|_{x=-d} = -\frac{d}{dx}\left[\frac{Q}{4\pi\epsilon_0 L}\ln\left(1-\frac{L}{x}\right)\right]\bigg|_{x=-d} = -\frac{1}{4\pi\epsilon_0}\frac{Q}{d(L+d)}\,.$$

Note that here $E_x < 0$, indicating that $\mathbf{E}$ is in the negative x direction, as expected.
(b) From symmetry it is obvious that the electric field does not have any y component, i.e., $E_y = 0$.

51P

(a) Consider an infinitesimal segment of the rod from x to $x + dx$. Its contribution to the potential at point P is

$$dV_P = \frac{1}{4\pi\epsilon_0}\frac{\lambda(x)dx}{\sqrt{x^2+y^2}} = \frac{1}{4\pi\epsilon_0}\frac{cx}{\sqrt{x^2+y^2}}\,dx.$$

Thus

$$V_P = \int_{\text{rod}} dV_P = \frac{c}{4\pi\epsilon_0}\int_0^L \frac{x}{\sqrt{x^2+y^2}}dx = \frac{c}{4\pi\epsilon_0}(\sqrt{L^2+y^2}-y)\,.$$

(b)

$$E_{P,y} = -\frac{\partial V_P}{\partial y} = -\frac{c}{4\pi\epsilon_0}\frac{d}{dy}(\sqrt{L^2+y^2}-y) = \frac{c}{4\pi\epsilon_0}\left(1-\frac{y}{\sqrt{L^2+y^2}}\right).$$

(c) All we obtained above for the potential is its value at any point P on the y-axis. In order to obtain $E_x(x,y)$ we need to first calculate $V(x,y)$, i.e., the potential for an arbitrary point located at $(x,\,y)$. Then $E_x(x,\,y)$ can be obtained from $E_x(x,\,y) = -\partial V(x,\,y)/\partial x$.

52E

(a) Use Eq. 25-43 with $q_1 = q_2 = -e$ and $r = 2.00\,\text{nm}$:

$$U = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r} = \frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\,\text{C})^2}{2.00\times10^{-9}\,\text{m}} = 1.15\times10^{-19}\,\text{J}\,.$$

(b) Since $U>0$ and $U \propto r^{-1}$ the potential energy U decreases as r increases.

53E

(a) The charges are equal and are the same distance from C. Use the Pythagorean theorem to find the distance $r = \sqrt{(d/2)^2+(d/2)^2} = d/\sqrt{2}$. The electric potential at C is the sum of the potential due to the individual charges but since they produce the same potential, it is twice that of either one:

$$\begin{aligned} V &= \frac{2q}{4\pi\epsilon_0}\frac{\sqrt{2}}{d} = \frac{2\sqrt{2}q}{4\pi\epsilon_0 d} \\ &= \frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(2)\sqrt{2}(2.0\times10^{-6}\,\text{C})}{0.020\,\text{m}} = 2.5\times10^6\,\text{V}\,. \end{aligned}$$

(b) As you move the charge into position from far away the potential energy changes from zero to qV, where V is the electric potential at the final location of the charge. The change in the potential energy equals the work you must do to bring the charge in: $W = qV = (2.0\times10^{-6}\,\text{C})(2.5\times10^6\,\text{V}) = 5.1\,\text{J}$.

(c) The work calculated in part (b) represents the potential energy of the interactions between the charge brought in from infinity and the other two charges. To find the total potential energy of the three-charge system you must add the potential energy of the interaction between the fixed charges. Their separation is d so this potential energy is $q^2/4\pi\epsilon_0 d$. The total potential energy is

$$\begin{aligned} U &= W + \frac{q^2}{4\pi\epsilon_0 d} \\ &= 5.1\,\text{J} + \frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(2.0\times10^{-6}\,\text{C})^2}{0.020\,\text{m}} = 6.9\,\text{J}\,. \end{aligned}$$

54E

The potential energy of the two-charge system is

$$\begin{aligned}U &= \frac{1}{4\pi\epsilon_0}\left[\frac{q_1q_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}}\right]\\ &= \frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(3.0\times10^{-6}\,\mathrm{C})(-4.0\times10^{-6}\,\mathrm{C})}{\sqrt{(3.5+2.0)^2+(0.50-1.5)^2}\,\mathrm{cm}} = -1.9\,\mathrm{J}\,.\end{aligned}$$

Thus $-1.9\,\mathrm{J}$ of work is needed.

55E

(a) Denote the side length of the triangle as a. Then the electric potential energy is

$$\begin{aligned}U &= \frac{3(e/3)^2}{4\pi\epsilon_0 a} = \frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(1.60\times10^{-19}\,\mathrm{C})^2}{3(2.82\times10^{-31})}\\ &= 2.72\times10^{-14}\,\mathrm{J}\,.\end{aligned}$$

(b) $U/c^2 = 2.72\times10^{-14}\,\mathrm{J}/(3.00\times10^8\,\mathrm{m/s})^2 = 3.02\times10^{-31}\,\mathrm{kg}$. This is about a third of the accepted value of the electron mass.

56E

Choose the zero of electric potential to be at infinity. The initial electric potential energy U_i of the system before the particles are brought together is therefore zero. After the system is set up the final potential energy is

$$\begin{aligned}U_f &= \frac{q^2}{4\pi\epsilon_0}\left(-\frac{1}{a}-\frac{1}{a}+\frac{1}{\sqrt{2}a}-\frac{1}{a}-\frac{1}{a}+\frac{1}{\sqrt{2}a}\right)\\ &= \frac{2q^2}{4\pi\epsilon_0 a}\left(\frac{1}{\sqrt{2}}-2\right) = -\frac{0.21q^2}{\epsilon_0 a}\,.\end{aligned}$$

Thus the amount of work required to set up the system is given by $W = \Delta U = U_f - U_i = -0.21q^2/(\epsilon_0 a)$.

57E

Let the quark-quark separation be a. Then

(a)

$$\begin{aligned}U_{\text{up-up}} &= \frac{1}{4\pi\epsilon_0}\frac{(2e/3)(2e/3)}{a} = \frac{4e^2}{4\pi\epsilon_0 a}\\ &= \frac{4(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(1.60\times10^{-19}\,\mathrm{C})e}{9(1.32\times10^{-15}\,\mathrm{m})}\\ &= 4.84\times10^5\,\mathrm{eV} = 0.484\,\mathrm{MeV}.\end{aligned}$$

(*b*)

$$U = \frac{1}{4\pi\epsilon_0}\left[\frac{(2e/3)(2e/3)}{a} - \frac{2(2e/3)(2e/3)}{a}\right] = 0\,.$$

58E

$$\begin{aligned}U &= \frac{1}{4\pi\epsilon_0}\sum_{i\neq j}\frac{q_iq_j}{r_{ij}} = \frac{1}{4\pi\epsilon_0 d}\left(q_1q_2 + q_1q_3 + q_2q_4 + q_3q_4 + \frac{q_1q_4}{\sqrt{2}} + \frac{q_2q_3}{\sqrt{2}}\right)\\ &= \frac{(8.99\times 10^9\,\mathrm{N{\cdot}m^2/C^2})}{1.3\,\mathrm{m}}\left[(12)(-24) + (12)(31) + (-24)(17) + (31)(17)\right.\\ &\qquad\left. + \frac{(12)(17)}{\sqrt{2}} + \frac{(-24)(31}{\sqrt{2}}\right](10^{-19}\,\mathrm{C})^2\\ &= -1.2\times 10^{-6}\,\mathrm{J}\,.\end{aligned}$$

59P

Let $q = 0.12\,\mathrm{C}$ and $a = 1.7\,\mathrm{m}$. The change in electric potential energy of the three-charge system as one of the charges is moved as described in the problem is

$$\begin{aligned}\Delta U &= \frac{q^2}{4\pi\epsilon_0}\left(\frac{2}{a/2} - \frac{2}{a}\right) = \frac{2q^2}{4\pi\epsilon_0 a}\\ &= \frac{2(0.12\,\mathrm{C})^2(8.99\times 10^9\,\mathrm{N{\cdot}m^2/C^2})}{1.7\,\mathrm{m}} = 1.5\times 10^8\,\mathrm{J}.\end{aligned}$$

Thus the number of days required would be

$$n = \frac{(1.5\times 10^8\,\mathrm{J})}{(0.83\times 10^3\,\mathrm{W})(86400\,\mathrm{s/d})} = 2.1\,\mathrm{d}\,.$$

60P

(*a*) Let ℓ $(= 0.15\,\mathrm{m})$ be the length of the rectangle and w $(= 0.050\,\mathrm{m})$ be its width. Charge q_1 is a distance ℓ from point A and charge q_2 is a distance w, so the electric potential at A is

$$\begin{aligned}V_A &= \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{\ell} + \frac{q_2}{w}\right)\\ &= (8.99\times 10^9\,\mathrm{N{\cdot}m^2/C^2})\left(\frac{-5.0\times 10^{-6}\,\mathrm{C}}{0.15\,\mathrm{m}} + \frac{2.0\times 10^{-6}\,\mathrm{C}}{0.050\,\mathrm{m}}\right)\\ &= 6.0\times 10^4\,\mathrm{V}\,.\end{aligned}$$

(b) Charge q_1 is a distance w from point b and charge q_2 is a distance ℓ, so the electric potential at B is

$$\begin{aligned} V_B &= \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{w}+\frac{q_2}{\ell}\right) \\ &= (8.99\times 10^9\,\mathrm{N{\cdot}m^2/C^2})\left(\frac{-5.0\times 10^{-6}\,\mathrm{C}}{0.050\,\mathrm{m}}+\frac{2.0\times 10^{-6}\,\mathrm{C}}{0.15\,\mathrm{m}}\right) \\ &= -7.8\times 10^5\,\mathrm{V}\,. \end{aligned}$$

(c) Since the kinetic energy is zero at the beginning and end of the trip, the work done by an external agent equals the change in the potential energy of the system. The potential energy is the product of the charge q_3 and the electric potential. If U_A is the potential energy when q_3 is at A and U_B is the potential energy when q_3 is at B, then the work done in moving the charge from B to A is $W = U_A - U_B = q_3(V_A - V_B) = (3.0\times 10^{-6}\,\mathrm{C})(6.0\times 10^4\,\mathrm{V} + 7.8\times 10^5\,\mathrm{V}) = 2.5\,\mathrm{J}$.

(d) The work done by the external agent is positive, so the energy of the three-charge system increases.

(e) and (f) The electrostatic force is conservative, so the work is the same no matter what the path.

61P

The work required is

$$W = \Delta U = \frac{1}{4\pi\epsilon_0}\left[\frac{(4q)(5q)}{2d}+\frac{(5q)(-2q)}{d}\right] = 0\,.$$

62P

The particle with charge $-q$ has both potential and kinetic energy and both these change when the radius of the orbit is changed. Find an expression for the total energy in terms of the orbit radius. Q provides the centripetal force required for $-q$ to move in uniform circular motion. The magnitude of the force is $F = Qq/4\pi\epsilon_0 r^2$, where r is the orbit radius. The acceleration of $-q$ is v^2/r, where v is its speed. Newton's second law yields $Qq/4\pi\epsilon_0 r^2 = mv^2/r$, so $mv^2 = Qq/4\pi\epsilon_0 r$ and the kinetic energy is $K = \frac{1}{2}mv^2 = Qq/8\pi\epsilon_0 r$. The potential energy is $U = -Qq/4\pi\epsilon_0 r$ and the total energy is

$$E = K + U = \frac{Qq}{8\pi\epsilon_0 r} - \frac{Qq}{4\pi\epsilon_0 r} = -\frac{Qq}{8\pi\epsilon_0 r}\,.$$

When the orbit radius is r_1 the energy is $E_1 = -Qq/8\pi\epsilon_0 r_1$ and when it is r_2 the energy is $E_2 = -Qq/8\pi\epsilon_0 r_2$. The difference $E_2 - E_1$ is the work W done by an external agent to change the radius:

$$W = E_2 - E_1 = -\frac{Qq}{8\pi\epsilon_0}\left(\frac{1}{r_2}-\frac{1}{r_1}\right) = \frac{Qq}{8\pi\epsilon_0}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\,.$$

63P

(a)

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{e}{r} = \frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(1.60\times10^{-19}\,\mathrm{C})}{5.29\times10^{-11}\,\mathrm{m}} = 27.2\,\mathrm{V}.$$

(b) $U = -eV(r) = -27.2\,\mathrm{eV}$.

(c) Since $m_e v^2/r = -e^2/4\pi\epsilon_0 r^2$,

$$K = \frac{1}{2}mv^2 = -\frac{1}{2}\left(\frac{e^2}{4\pi\epsilon_0 r}\right) = -\frac{1}{2}V(r) = \frac{27.2\,\mathrm{eV}}{2} = 13.6\,\mathrm{eV}.$$

(d) The energy required is

$$\Delta E = 0 - [V(r) + K] = 0 - (-27.2\,\mathrm{eV} + 13.6\,\mathrm{eV}) = 13.6\,\mathrm{eV}.$$

64P

Use the conservation of energy principle. The initial potential energy is $U_i = q^2/4\pi\epsilon_0 r_1$, the initial kinetic energy is $K_i = 0$, the final potential energy is $U_f = q^2/4\pi\epsilon_0 r_2$, and the final kinetic energy is $K_f = \frac{1}{2}mv^2$, where v is the final speed of the particle. Conservation of energy yields

$$\frac{q^2}{4\pi\epsilon_0 r_1} = \frac{q^2}{4\pi\epsilon_0 r_2} + \frac{1}{2}mv^2\,.$$

The solution for v is

$$\begin{aligned} v &= \sqrt{\frac{2q^2}{4\pi\epsilon_0 m}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)} \\ &= \sqrt{\frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(2)(3.1\times10^{-6}\,\mathrm{C})^2}{20\times10^{-6}\,\mathrm{kg}}\left(\frac{1}{0.90\times10^{-3}\,\mathrm{m}}-\frac{1}{2.5\times10^{-3}\,\mathrm{m}}\right)} \\ &= 2.5\times10^3\,\mathrm{m/s}\,. \end{aligned}$$

65P

Let $r = 1.5\,\mathrm{m}$, $x = 3.0\,\mathrm{m}$, $q_1 = -9.0\,\mathrm{nC}$, and $q_2 = -6.0\,\mathrm{pC}$. The work done is given by

$$\begin{aligned} W = \Delta U &= \frac{q_1 q_2}{4\pi\epsilon_0}\left(\frac{1}{r}-\frac{1}{\sqrt{r^2+x^2}}\right) \\ &= (-9.0\times10^{-9}\,\mathrm{C})(-6.0\times10^{-12}\,\mathrm{C})(8.99\times10^9\,\mathrm{N\cdot m^2/C^2}) \\ &\quad\cdot\left[\frac{1}{1.5\,\mathrm{m}}-\frac{1}{\sqrt{(1.5\,\mathrm{m})^2+(3.0\,\mathrm{m})^2}}\right] \\ &= 1.8\times10^{-10}\,\mathrm{J}\,. \end{aligned}$$

66P

(a) The potential energy is

$$U = \frac{q^2}{4\pi\epsilon_0 d} = \frac{(8.99 \times 10^9\,\mathrm{N{\cdot}m^2/C^2})(5.0 \times 10^{-6}\,\mathrm{C})^2}{1.00\,\mathrm{m}} = 0.225\,\mathrm{J}\,,$$

relative to the potential energy at infinite separation.

(b) Each sphere repels the other with a force that has magnitude

$$F = \frac{q^2}{4\pi\epsilon_0 d^2} = \frac{(8.99 \times 10^9\,\mathrm{N{\cdot}m^2/C^2})(5.0 \times 10^{-6}\,\mathrm{C})^2}{(1.00\,\mathrm{m})^2} = 0.225\,\mathrm{N}\,.$$

According to Newton's second law the acceleration of each sphere is the force divided by the mass of the sphere. Let m_A and m_B be the masses of the spheres. The acceleration of sphere A is

$$a_A = \frac{F}{m_A} = \frac{0.225\,\mathrm{N}}{5.0 \times 10^{-3}\,\mathrm{kg}} = 45.0\,\mathrm{m/s^2}$$

and the acceleration of sphere B is

$$a_B = \frac{F}{m_B} = \frac{0.225\,\mathrm{N})}{10 \times 10^{-3}\,\mathrm{kg}} = 22.5\,\mathrm{m/s^2}\,.$$

(c) Energy is conserved. The initial potential energy is $U = 0.225\,\mathrm{J}$, as calculated in part (a). The initial kinetic energy is zero since the spheres start from rest. The final potential energy is zero since the spheres are then far apart. The final kinetic energy is $\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$, where v_A and v_B are the final velocities. Thus

$$U = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2\,.$$

Momentum is also conserved, so

$$0 = m_A v_A + m_B v_B\,.$$

Solve these equations simultaneously for v_A and v_B.

Substitute $v_B = -(m_A/m_B)v_A$, from the momentum equation, into the energy equation and collect terms. You should obtain $U = \frac{1}{2}(m_A/m_B)(m_A + m_B)v_A^2$. Thus

$$\begin{aligned} v_A &= \sqrt{\frac{2Um_B}{m_A(m_A+m_B)}} \\ &= \sqrt{\frac{2(0.225\,\mathrm{J})(10 \times 10^{-3}\,\mathrm{kg})}{(5.0 \times 10^{-3}\,\mathrm{kg})(5.0 \times 10^{-3}\,\mathrm{kg} + 10 \times 10^{-3}\,\mathrm{kg})}} = 7.75\,\mathrm{m/s}\,. \end{aligned}$$

Now calculate v_B:

$$v_B = -\frac{m_A}{m_B}v_A = -\left(\frac{5.0 \times 10^{-3}\,\mathrm{kg}}{10 \times 10^{-3}\,\mathrm{kg}}\right)(7.75\,\mathrm{m/s}) = -3.87\,\mathrm{m/s}\,.$$

67P

The initial speed v_i of the electron satisfies $K_i = \frac{1}{2}m_e v_i^2 = e\Delta V$, which gives

$$v_i = \sqrt{\frac{2e\Delta V}{m_e}} = \sqrt{\frac{2(1.60 \times 10^{-19}\,\mathrm{J})(625\,\mathrm{V})}{9.11 \times 10^{-31}\,\mathrm{kg}}} = 1.48 \times 10^7\,\mathrm{m/s}\,.$$

68P

(a) At the smallest center-to-center separation $r_{\min}$ the initial kinetic energy K_i of the proton is entirely converted to the electric potential energy between the proton and the neucleus. Thus

$$K_i = \frac{1}{4\pi\epsilon_0}\frac{eq_{\text{lead}}}{r_{\min}} = \frac{82e^2}{4\pi\epsilon_0 r_{\min}}\,.$$

Thus

$$\begin{aligned} r_{\min} &= \frac{82e^2}{4\pi\epsilon_0 K_i} = \frac{82e^2}{4\pi\epsilon_0(4.80 \times 10^6 e)\,\mathrm{V}}\frac{82(1.6 \times 10^{-19}\,\mathrm{C})(8.99 \times 10^9\,\mathrm{N{\cdot}m^2/C^2})}{4.80 \times 10^6\,\mathrm{V}} \\ &= 2.5 \times 10^{-14}\,\mathrm{m} = 25\,\mathrm{fm}\,. \end{aligned}$$

Note that $1\,\mathrm{eV} = 1e{\cdot}1\,\mathrm{V}$.

(b) In this case

$$K_i = \frac{1}{4\pi\epsilon_0}\frac{q_\alpha q_{\text{lead}}}{r'_{\min}} = 2\left(\frac{82e^2}{4\pi\epsilon_0 r'_{\min}}\right) = \frac{82e^2}{4\pi\epsilon_0 r_{\min}}\,,$$

so the new minimum separation is $r'_{\min} = 2r_{\min} = 50\,\mathrm{fm}$.

69P

The idea for solving this problem is the same as that for the last one. In this case

$$K = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r_{\min}}\,,$$

which gives $r_{\min} = qQ/4\pi\epsilon_0 K$.

70P

The change in electric potential energy of the electron-shell system as the electron starts from its initial position and just reaches the shell is $\Delta U = (-e)(-V) = eV$. Thus from $\Delta U = K = \frac{1}{2}m_e v_i^2$ we find the initial electron speed to be

$$v_i = \sqrt{\frac{2\Delta U}{m_e}} = \sqrt{\frac{2eV}{m_e}}\,.$$

71P

Use the conservation of energy principle. Take the potential energy to be zero when the moving electron is far away from the fixed electrons. The final potential energy is then $U_f = 2e^2/4\pi\epsilon_0 d$, where d is the half the distance between the fixed electrons. The initial kinetic energy is $K_i = \frac{1}{2}m_e v^2$, where m_e is the mass of an electron and v is the initial speed of the moving electron. The final kinetic energy is zero. Thus $K_i = U_f$ or $\frac{1}{2}m_e v^2 = 2e^2/4\pi\epsilon_0 d$. Hence

$$v = \sqrt{\frac{4e^2}{4\pi\epsilon_0 d m_e}} = \sqrt{\frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(4)(1.60\times10^{-19}\,\mathrm{C})^2}{(0.010\,\mathrm{m})(9.11\times10^{-31}\,\mathrm{kg})}} = 3.2\times10^2\,\mathrm{m/s}\,.$$

72P

The potential difference between the surface of the sphere of charge Q and radius r and a point infinitely far from it is $\Delta V = Q/4\pi\epsilon_0 r$. Thus the escape speed v_{esc} of the electron of mass m_e satisfies $K = \frac{1}{2}m_e v_{\mathrm{esc}}^2 = e\Delta V$, or

$$\begin{aligned} v_{\mathrm{esc}} &= \sqrt{\frac{2eQ}{4\pi\epsilon_0 m_e r}} = \sqrt{\frac{2(1.60\times10^{-19}\,\mathrm{C})(1.60\times10^{-15}\,\mathrm{C})(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})}{(9.11\times10^{-31}\,\mathrm{kg})(1.0\times10^{-2}\,\mathrm{m})}} \\ &= 2.2\times10^4\,\mathrm{m/s}\,. \end{aligned}$$

73P

Let the distance in question be r. The initial kinetic energy of the electron is $K_i = \frac{1}{2}m_e v_i^2$, where $v_i = 3.2\times10^5\,\mathrm{m/s}$. As the speed doubles, K becomes $4K_i$. Thus

$$\Delta U = \frac{-e^2}{4\pi\epsilon_0 r} = -\Delta K = -(4K_i - K_i) = -3K_i = -\frac{3}{2}m_e v_i^2\,,$$

or

$$\begin{aligned} r &= \frac{2e^2}{3(4\pi\epsilon_0)m_e v_i^2} = \frac{2(1.6\times10^{-19}\,\mathrm{C})^2(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})}{3(9.11\times10^{-19}\,\mathrm{kg})(3.2\times10^5\,\mathrm{m/s})^2} \\ &= 1.6\times10^{-9}\,\mathrm{m}\,. \end{aligned}$$

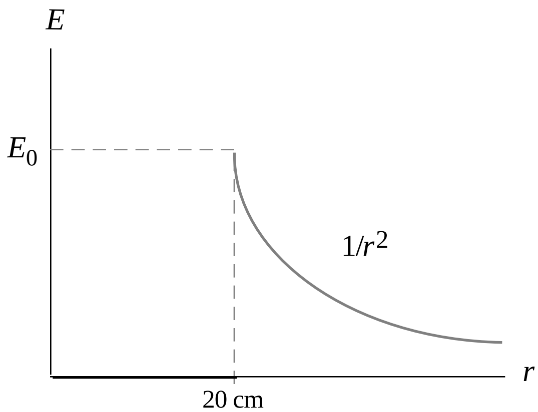
74E

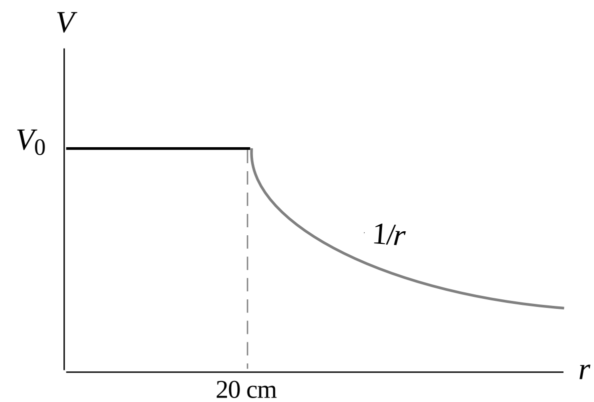
Since the electric potential throughout the entire conductor is a constant, the electric potential at its center is also $+400\,\mathrm{V}$.

75E

(a) and (b) For $r < R = 20\,\mathrm{cm}$ the electric field is zero since this region is inside the conductor. For $r > R$ we have $E(r) = q/(4\pi\epsilon_0 r^2)$, where $q = +3.0\,\mu\mathrm{C}$. In particular, just outside the sphere $E = q/(4\pi\epsilon_0 R^2) = E_0 = 6.8\times10^5\,\mathrm{N/C}$.

Use $V(r) = \int_0^\infty E(r)dr$ to calculate $V(r)$. For $r < R = 20\,\text{cm}$ the electric potential is a constant since this region is inside the conductor, an equipotential body. For $r \geq R$ we have $V(r) = q/(4\pi\epsilon_0 r)$. In particular, at $r = R$ $V = q/(4\pi\epsilon_0 R) = V_0 = 1.4 \times 10^5\,\text{V}$. This is also the value of $V(r)$ for $r < R$.
Both E and V as functions of r are plotted below.





V0=1.4x105 V.

76E

If the electric potential is zero at infinity, then the potential at the surface of the sphere is given by $V = q/4\pi\epsilon_0 r$, where q is the charge on the sphere and r is its radius. Thus

$$q = 4\pi\epsilon_0 rV = \frac{(0.15\,\text{m})(1500\,\text{V})}{8.99 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2} = 2.5 \times 10^{-8}\,\text{C}\,.$$

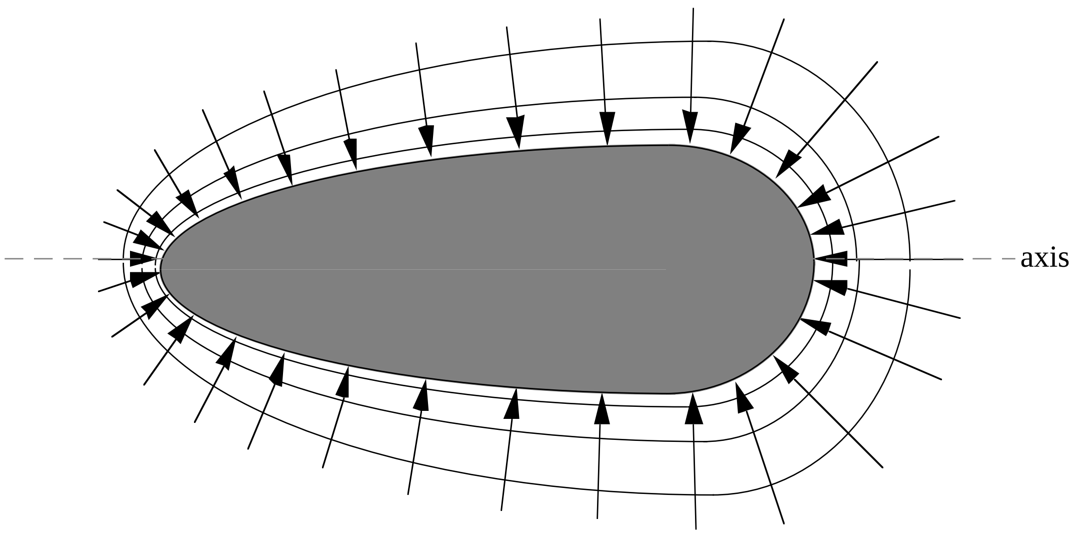
77E

(a) Since the two conductors are connected V_1 and V_2 must be the same.
(b) Let $V_1 = q_1/4\pi\epsilon_0 R_1 = V_2 = q_2/4\pi\epsilon_0 R_2$ and note that $q_1 + q_2 = q$ and $R_2 = 2R_1$. Solve for q_1 and q_2: $q_1 = q/3$, $q_2 = 2q/3$.
(c)

$$\frac{\sigma_1}{\sigma_2} = \frac{q_1/4\pi R_1^2}{q_2/4\pi R_2^2} = \left(\frac{q_1}{q_2}\right)\left(\frac{R_2}{R_1}\right)^2 = 2\,.$$

78P

In sketching the electric field lines and the equipotential lines pay attention to the following: (1) The electric field lines and the equipotential lines and always perpendicular to each other wherever they intersect. (2) The filed lines are perpenduluar to the surface of the conductor (which is an equipotential surface); while the equipotential lines just outside the conductor follow its surface countour. (3) The greater the curvature of the surface of the conductor, the higher the concentration of the surface charge and hence the greater the electric field outside the surface. This means that the electric field just outside the conductor is the strongest near its narrower end; and the weakest in between the two ends, where the surface is flat. The field is of course zero inside the conductor, where the potential is a constant.



some equipotential lines and electric field lines (denoted with arrows)

79P

(a) The potential would be

$$\begin{aligned} V_e &= \frac{Q_e}{4\pi\epsilon_0 R_e} = \frac{4\pi R_e^2\sigma_e}{4\pi\epsilon_0 R_e} \\ &= 4\pi(6.37\times 10^6\,\mathrm{m})(1.0\,\mathrm{electron/\,m^2})(-1.6\times 10^{-19}\,\mathrm{C/\,electron})(8.99\times 10^9\,\mathrm{N\cdot m^2/C^2}) \\ &= -0.12\,\mathrm{V}\,. \end{aligned}$$

(b)

$$E = \frac{\sigma_e}{\epsilon_0} = \frac{V_e}{R_e} = -\frac{0.12\,\mathrm{V}}{6.37\times 10^6\,\mathrm{m}} = -1.8\times 10^{-8}\,\mathrm{N/C}\,,$$

where the minus sign indicates that **E** is radially inward.

80P

(a) The electric potential is the sum of the contributions of the individual spheres. Let q_1 be the charge on one and q_2 be the charge on the other. The point halfway between them is the same distance d ($= 1.0\,\text{m}$) from the center of each sphere, so the potential at the halfway point is

$$V = \frac{q_1 + q_2}{4\pi\epsilon_0 d} = \frac{(8.99 \times 10^9\,\text{N·m}^2/\text{C}^2)(1.0 \times 10^{-8}\,\text{C} - 3.0 \times 10^{-8}\,\text{C})}{1.0\,\text{m}} = -1.80 \times 10^2\,\text{V}\,.$$

(b) The distance from the center of one sphere to the surface of the other is $d - R$, where R is the radius of either sphere. The potential of either one of the spheres is due to the charge on that sphere and the charge on the other sphere. The potential at the surface of sphere 1 is

$$\begin{aligned} V_1 &= \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{R} + \frac{q_2}{d-R}\right) \\ &= (8.99 \times 10^9\,\text{N·m}^2/\text{C}^2)\left(\frac{1.0 \times 10^{-8}\,\text{C}}{0.030\,\text{m}} - \frac{3.0 \times 10^{-8}\,\text{C}}{1.0\,\text{m} - 0.030\,\text{m}}\right) \\ &= 2.7 \times 10^3\,\text{V}\,. \end{aligned}$$

The potential at the surface of sphere 2 is

$$\begin{aligned} V_2 &= \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{d-R} + \frac{q_2}{R}\right) \\ &= (8.99 \times 10^9\,\text{N·m}^2/\text{C}^2)\left(\frac{1.0 \times 10^{-8}\,\text{C}}{1.0\,\text{m} - 0.030\,\text{m}} - \frac{3.0 \times 10^{-8}\,\text{C}}{0.030\,\text{m}}\right) \\ &= -8.9 \times 10^3\,\text{V}\,. \end{aligned}$$

81P

(a)

$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{4\pi\epsilon_0 R^2} = \frac{(3.0 \times 10^{-8}\,\text{C})(8.99 \times 10^9\,\text{N·m}^2/\text{C}^2)}{(0.15\,\text{m})^2} = 1.2 \times 10^4\,\text{N/C}\,.$$

(b) $V = RE = (0.15\,\text{m})(1.2 \times 10^4\,\text{N/C}) = 1.8 \times 10^3\,\text{V}$.

(c) Let the distance be x. Then

$$\Delta V = V(x) - V = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{R+x} - \frac{1}{R}\right) = -500\,\text{V}\,,$$

which gives

$$x = \frac{R\Delta V}{-V - \Delta V} = \frac{(0.15\,\text{m})(-500\,\text{V})}{-1800\,\text{V} + 500\,\text{V}} = 5.8 \times 10^{-2}\,\text{m}\,.$$

82P

Since the charge distribution is spherically symmetric we may write

$$E(r) = \frac{1}{4\pi\epsilon_0}\frac{q_{\text{encl}}}{r},$$

where q_{encl} is the charge enclosed in a sphere of radius r centered at the origin. Also $V(r) = \int_r^\infty E(r)\,dr$. The results are as follows:

For $r > R_2 > R_1$

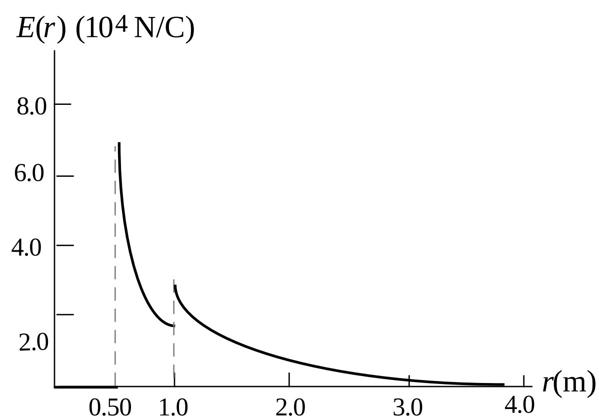
$$\begin{cases} V(r) = \dfrac{q_1 + q_2}{4\pi\epsilon_0 r}, \\ E(r) = \dfrac{q_1 + q_2}{4\pi\epsilon_0 r^2}; \end{cases}$$

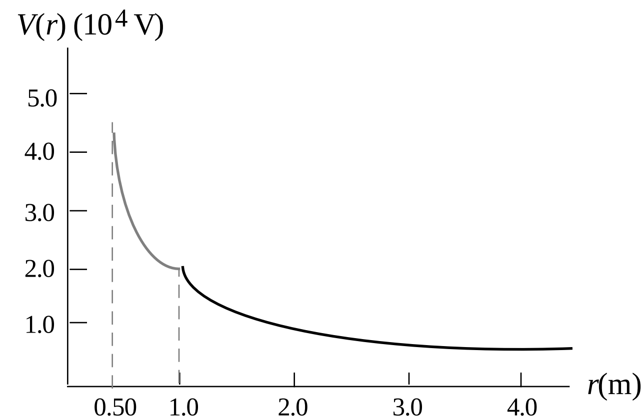
for $R_2 > r > R_1$

$$\begin{cases} V(r) = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q_1}{r} + \dfrac{q_2}{R_2}\right), \\ E(r) = \dfrac{q_1}{4\pi\epsilon_0 r^2}; \end{cases}$$

and for $R_2 > R_1 > r$

$$\begin{cases} E = 0, \\ V(r) = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q_1}{R_1} + \dfrac{q_2}{R_2}\right). \end{cases}$$





83

(c) 4.24 V

84

(a) for $N < 12$ configuration 1 is less energetic, for $N \geq 12$ configuration 2 is less energetic;
(b) 12;
(c) 2