

CHAPTER 23

Answer to Checkpoint Questions

1. (a) rightward; (b) leftward; (c) leftward; (d) rightward; (p and e have same charge magnitude and p is farther)
2. all tie
3. (a) toward positive y ; (b) toward positive x ; (c) toward negative y
4. (a) leftward; (b) leftward; (c) decrease
5. (a) leftward; (b) 1 and 3 tie, then 2 and 4 tie

Answer to Questions

1. (a) toward positive x ; (b) downward and to the right; (c) A
2. (a) to their left; (b) no
3. two points: one to the left of the particles, the other between the protons
4. $q/4\pi\epsilon_0 d^2$, leftward
5. (a) yes; (b) toward; (c) no (the field vectors are not along the same line; (d) cancel; (e) add; (f) adding components; (g) toward negative y
6. all tie
7. (a) 3, then 1 and 2 tie (zero); (b) all tie; (c) 1 and 2 tie, then 3
8. e, b, then a and c tie, then d (zero)
9. (a) rightward; (b) $+q_1$ and $-q_3$, increase; q_2 , decrease; n , same
10. (a) toward the bottom; (b) 2 and 4 toward the bottom, 3 toward the top
11. a, b, c
12. (a) positive; (b) same
13. (a) 4, 3, 1, 2; (b) 3, then 1 and 4 tie, then 2

14. Accumulated excess charge creates an electric field; proximity of a second body concentrates the excess charge and increases the field, causing electrical breakdown in the air (sparking).

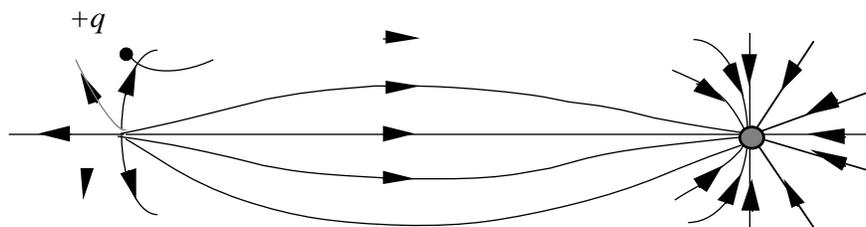
Solutions to Exercises & Problems

1E

(a) $F_A = eE_A = (1.60 \times 10^{-19} \text{ C})(40 \text{ N/C}) = 6.4 \times 10^{-18} \text{ C}$.

(b) $E_B = E_A/2 = 20 \text{ N/C}$.

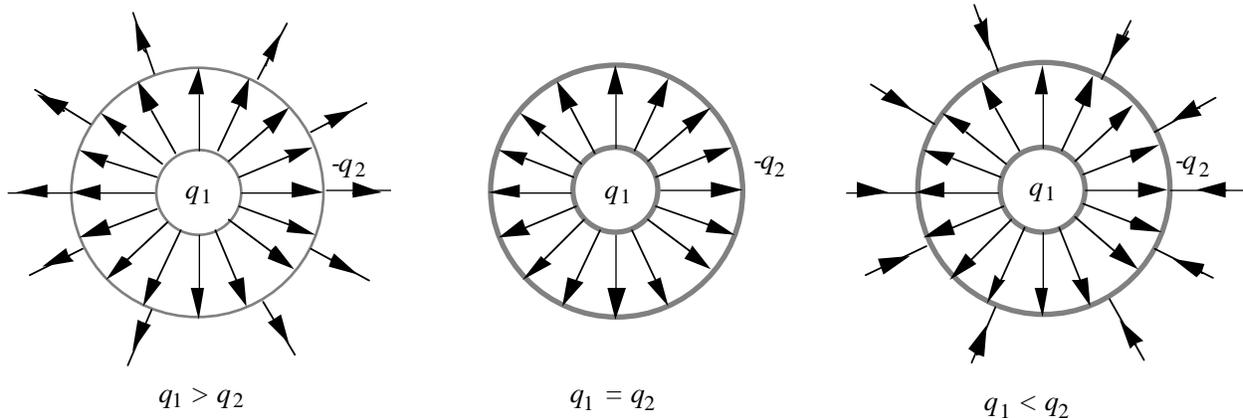
2E



3E

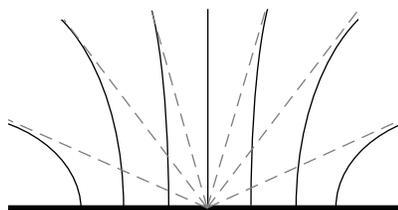
The lines of forces due to $+Q$ and $-Q$ are similar to those in Fig. 23-5. Just turn the book by 90° so that the line joining the two charges in Fig. 23-5 becomes horizontal, with $+Q$ on the left. Obviously the force on $+q$ is to the right.

4E



5E

The diagram to the right is an edge view of the disk and shows the field lines above it. Near the disk the lines are perpendicular to the surface and since the disk is uniformly charged the lines are uniformly distributed over the surface. Far away from the disk the lines are like those of a single point charge (the charge on the disk). Extended back to the disk (along the dotted lines of the diagram) they intersect at the center of the disk.



If the disk is positively charged the lines are directed outward from the disk. If the disk is negatively charged they are directed inward toward the disk. Lines below the disk are exactly like those above.

6E

Solve q from $E = q/4\pi\epsilon_0 r^2$:

$$q = 4\pi\epsilon_0 E r^2 = \frac{(1.00 \text{ N/C})(1.00 \text{ m})^2}{8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2} = 1.11 \times 10^{-10} \text{ C}.$$

7E

Since the magnitude of the electric field produced by a point charge q is given by $E = q/4\pi\epsilon_0 r^2$, where r is the distance from the charge to the point where the field has magnitude E , the charge is

$$q = 4\pi\epsilon_0 r^2 E = \frac{(0.50 \text{ m})^2 (2.0 \text{ N/C})}{8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2} = 5.6 \times 10^{-11} \text{ C}.$$

8E

$$E = \frac{2Q}{4\pi\epsilon_0 (r/2)^2} = \frac{8Q}{4\pi\epsilon_0 r^2} = \frac{(8)(2.0 \times 10^{-7} \text{ C})(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)}{(0.15 \text{ m})^2} = 6.4 \times 10^5 \text{ N/C}.$$

\mathbf{E} points toward the negative charge.

(b) $F = eE = (1.60 \times 10^{-19} \text{ C})(6.4 \times 10^5 \text{ N/C}) = 1.0 \times 10^{-13} \text{ N}$. \mathbf{F} points toward the positive charge.

9E

Since the charge is uniformly distributed throughout a sphere the electric field at the

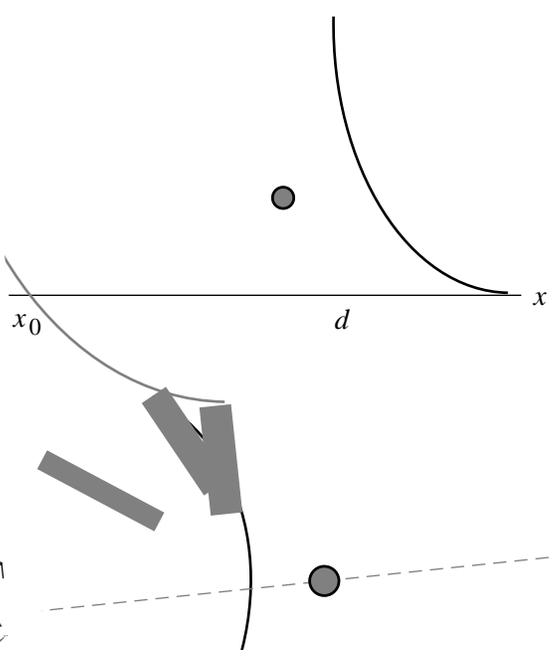
Set the net force to 0

$$F_{\text{net}} = F + F' = \frac{-q_1q_2}{4\pi\epsilon_0 x^2} + \frac{-q_1q_2}{4\pi\epsilon_0 (x' - x)^2}$$

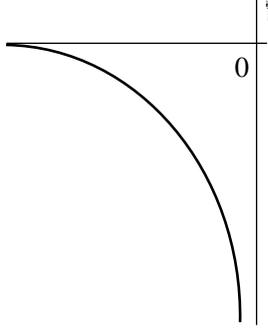
Solve for x' : $x' = 3x = 3(2.0 \text{ mm}) = 6.0 \text{ mm}$.

(b) The direction of the electric field produced by either of the two charges is in the x direction at $x = 2.0 \text{ mm}$. Therefore the net field is also in the negative x direction.

11P



The $E(x)$ vs. x curve is plotted above. Here $x_0 = d/(\sqrt{3} + 1)$, where $E = 0$.



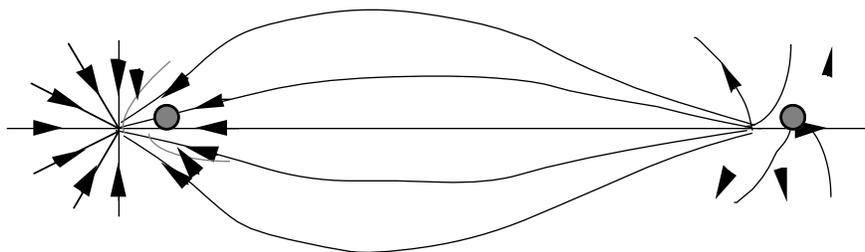
12P

(a) Convince yourself that a point where $E = 0$ can only be located to the right of the positive charge. Denote the separation between the point and the positive charge by x , then

$$E = \frac{1}{4\pi\epsilon_0} \left[\frac{2.0q}{x^2} - \frac{5.0q}{(x+d)^2} \right] = 0.$$

Solve for x : $x = 1.7d$.

(b)

**13P**

(a)

$$E_A = \frac{1}{4\pi\epsilon_0} \left[\frac{1.0q}{d^2} - \frac{2.0q}{(2d)^2} \right] = \frac{q}{8\pi\epsilon_0 d^2}.$$

E_A points to the left.

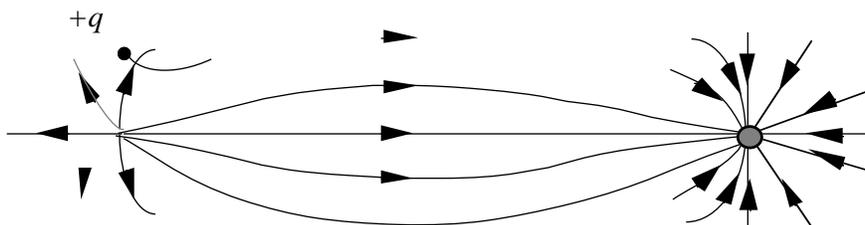
$$E_B = \frac{1}{4\pi\epsilon_0} \left[\frac{1.0q}{(d/2)^2} + \frac{2.0q}{(d/2)^2} \right] = \frac{3q}{\pi\epsilon_0 d^2}.$$

E_B points to the right.

$$E_C = \frac{1}{4\pi\epsilon_0} \left[\frac{2.0q}{d^2} - \frac{1.0q}{(2d)^2} \right] = \frac{7q}{16\pi\epsilon_0 d^2}.$$

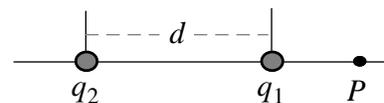
E_C points to the left.

(b)



14P

At points between the charges the individual electric fields are in the same direction and do not cancel. Charge q_2 has a greater magnitude than charge q_1 so a point of zero field must be closer to q_1 than to q_2 . It must be to the right of q_1 on the diagram.



Put the origin at q_2 and let x be the coordinate of P , the point where the field vanishes. Then the total electric field at P is given by

$$E = \frac{1}{4\pi\epsilon_0} \left[\frac{q_2}{x^2} - \frac{q_1}{(x-d)^2} \right],$$

where q_1 and q_2 are the magnitudes of the charges. If the field is to vanish, $q_2/x^2 = q_1/(x-d)^2$. Take the square root of both sides to obtain $\sqrt{q_1}/x = \sqrt{q_2}/(x-d)$. The solution for x is

$$\begin{aligned} x &= \left(\frac{\sqrt{q_2}}{\sqrt{q_2} - \sqrt{q_1}} \right) d = \left(\frac{\sqrt{4.0q_1}}{\sqrt{4.0q_1} - \sqrt{q_1}} \right) d \\ &= \left(\frac{2.0}{2.0 - 1.0} \right) d = 2.0d = (2.0)(50 \text{ cm}) = 100 \text{ cm}. \end{aligned}$$

The point is 50 cm to the right of q_1 .

15P

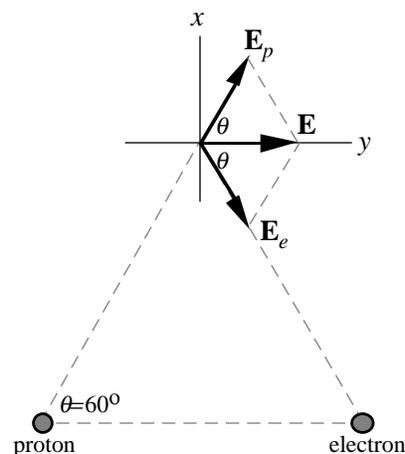
(a) The fields from the two charges of $5.0q$ each cancel out. So

$$E_p = \frac{1}{4\pi\epsilon_0} \left[\frac{3.0q}{d^2} - \frac{12q}{(2d)^2} \right] = 0.$$

16P

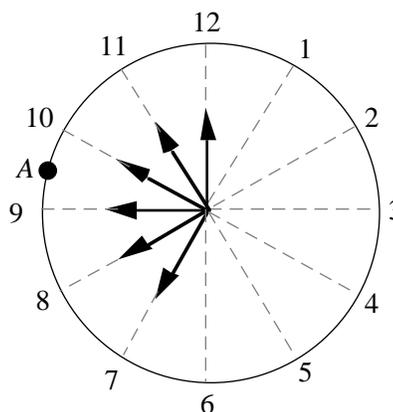
Denote the electron with subscript e and the proton with p . From the figure to the right we see that the $|E_e| = |E_p| = e/(4\pi\epsilon_0 d^2)$, where $d = 2.0 \times 10^{-6}$ m, and the magnitude of the net electric field is

$$\begin{aligned} E &= 2E_e \cos \theta = 2 \left(\frac{e}{4\pi\epsilon_0 d^2} \right) \cos \theta \\ &= \left[\frac{2(1.6 \times 10^{-19} \text{ C})(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)}{2.0 \times 10^{-6} \text{ m}^2} \right] (\cos 60^\circ) \\ &= 3.6 \times 10^2 \text{ N/C}. \end{aligned}$$



17P

In the figure to the right, each of the six arrows represents the net electric field of a pair of diametrically opposite charges (e.g. the one which points vertically upward represents the field of the $-12q$ and the $-6q$ charges). Since each of the six \mathbf{E} -fields has a magnitude of $E = 6q/4\pi\epsilon_0 r^2$, where r is roughly the radius of the clock, by symmetry the net \mathbf{E} -field should point at A , i.e., the 9:30 mark.

**18P**

By symmetry the fields due to the two electrons placed in line with the midpoint of a side cancel out. So

$$\begin{aligned} E &= \frac{1}{4\pi\epsilon_0} \frac{e}{(\sqrt{3}a/2)^2} = \frac{e}{3\pi\epsilon_0 a^2} \\ &= \frac{4(1.60 \times 10^{-19} \text{ C})(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)}{3(0.20 \text{ m})^2} = 4.8 \times 10^{-8} \text{ N/C}. \end{aligned}$$

\mathbf{E} is perpendicular to the side line and points toward the corner of the triangle opposite to the side.

19P

By symmetry the fields due to the two charges of magnitude q each cancel out. So

$$\mathbf{E}_p = \frac{1}{4\pi\epsilon_0} \frac{2.0q \mathbf{i}}{(a/\sqrt{2})^2} = \frac{q \mathbf{i}}{\pi\epsilon_0 a^2},$$

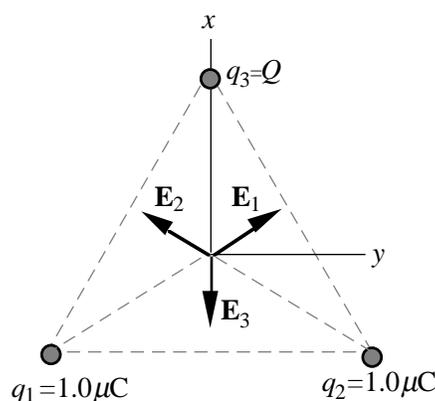
where \mathbf{i} is a unit vector from the $+2.0q$ charge to point P .

20P

Put the origin of a coordinate system at the center of the triangle, where the net electric field is given by

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3.$$

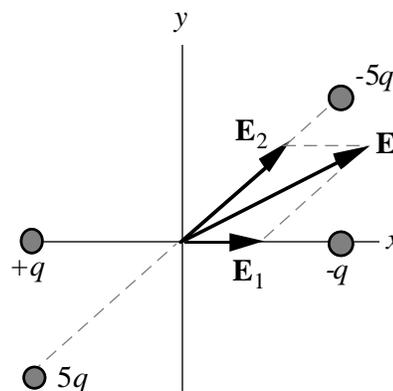
From symmetry consideration it is obvious that $E_1 = E_2 = E_3$. This gives $Q = q_1 = q_2 = +1.0 \mu\text{C}$.



21P

From symmetry, the only two pairs of charges which produce a non-vanishing field \mathbf{E} are: pair 1, which is the one in the middle of the two vertical sides of the square (the $+q, -2q$ pair); and pair 2, the $+5q, -5q$ pair. Denote the electric fields produced by each pair as \mathbf{E}_1 and \mathbf{E}_2 , respectively.

Set up a coordinate system as shown to the right, with the origin at the center of the square. Now



$$E_1 = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{d^2} + \frac{2q}{d^2} \right) = \frac{3q}{4\pi\epsilon_0 d^2}$$

and

$$E_2 = \frac{1}{4\pi\epsilon_0} \left[\frac{5q}{(\sqrt{2}d)^2} + \frac{5q}{(\sqrt{2}d)^2} \right] = \frac{5q}{4\pi\epsilon_0 d^2},$$

so the components of the net \mathbf{E} field are given by

$$\begin{aligned} E_x &= E_{1x} + E_{2x} = E_1 + E_2 \cos 45^\circ \\ &= \frac{3q}{4\pi\epsilon_0 d^2} + \left(\frac{5q}{4\pi\epsilon_0 d^2} \right) \cos 45^\circ = 6.536 \left(\frac{q}{4\pi\epsilon_0 d^2} \right) \end{aligned}$$

and

$$E_y = E_{1y} + E_{2y} = E_2 \sin 45^\circ = \left(\frac{5q}{4\pi\epsilon_0 d^2} \right) \sin 45^\circ = 3.536 \left(\frac{q}{4\pi\epsilon_0 d^2} \right).$$

Thus the magnitude of \mathbf{E} is

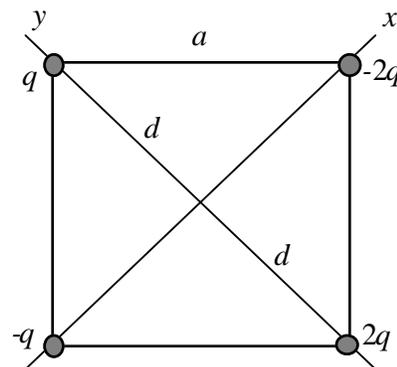
$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(6.536)^2 + (3.536)^2} \left(\frac{q}{4\pi\epsilon_0 d^2} \right) = \frac{7.43q}{4\pi\epsilon_0 d^2},$$

and \mathbf{E} makes an angle θ with the positive x axis, where

$$\theta = \tan^{-1} \left(\frac{E_y}{E_x} \right) = \tan^{-1} \left(\frac{3.536}{6.536} \right) = 28.4^\circ.$$

22P

Choose the coordinate axes as shown on the diagram to the right. At the center of the square the electric fields produced by the charges at the lower left and upper right corners are both along the x axis and each points away from the center and toward the charge that produces it. Since each charge is a distance $d = \sqrt{2}a/2 = a/\sqrt{2}$ away from the center the net field due to these two charges is



$$\begin{aligned} E_x &= \frac{1}{4\pi\epsilon_0} \left(\frac{2q}{a^2/2} - \frac{q}{a^2/2} \right) \\ &= \frac{1}{4\pi\epsilon_0} \frac{q}{a^2/2} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.0 \times 10^{-8} \text{ C})}{(0.050 \text{ m})^2/2} = 7.19 \times 10^4 \text{ N/C}. \end{aligned}$$

At the center of the square the field produced by the charges at the upper left and lower right corners are both along the y axis and each points toward the center, away from the charge that produces it. The net field produced at the center by these charges is

$$E_y = \frac{1}{4\pi\epsilon_0} \left(\frac{2q}{a^2/2} - \frac{q}{a^2/2} \right) = \frac{1}{4\pi\epsilon_0} \frac{q}{a^2/2} = 7.19 \times 10^4 \text{ N/C}.$$

The magnitude of the field is

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{2(7.19 \times 10^4 \text{ N/C})^2} = 1.02 \times 10^5 \text{ N/C}$$

and the angle it makes with the x axis is

$$\theta = \tan^{-1} \left(\frac{E_y}{E_x} \right) = \tan^{-1}(1) = 45^\circ.$$

It is upward in the diagram, from the center of the square toward the center of the upper side.

23E

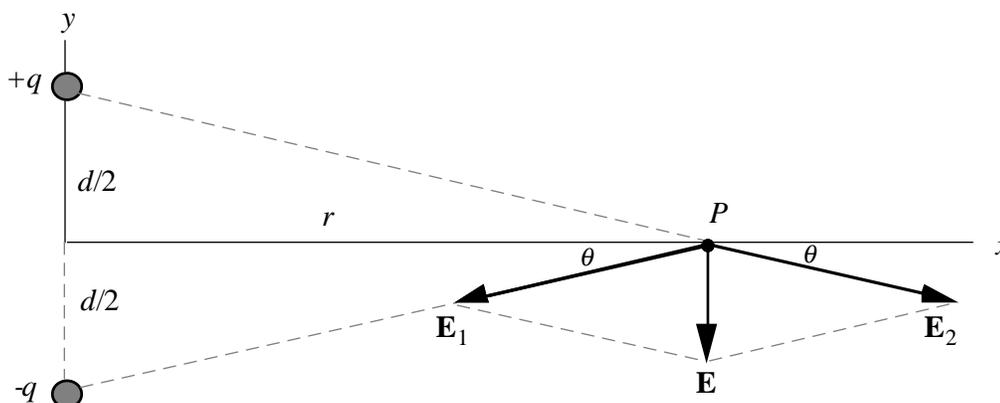
The magnitude of the dipole moment is given by $p = qd$, where q is the positive charge in the dipole and d is the separation of the charges. For the dipole described in the problem $p = (1.60 \times 10^{-19} \text{ C})(4.30 \times 10^{-9} \text{ m}) = 6.88 \times 10^{-28} \text{ C}\cdot\text{m}$. The dipole moment is a vector that points from the negative toward the positive charge.

24E

$$E = \frac{1}{4\pi\epsilon_0} \left[\frac{q}{(z - d/2)^2} + \frac{q}{(z + d/2)^2} \right].$$

For $z \gg d$ we have $(z \pm d/2)^{-2} \approx z^{-2}$, so

$$E \approx \frac{1}{4\pi\epsilon_0} \left(\frac{q}{z^2} + \frac{q}{z^2} \right) = \frac{2q}{4\pi\epsilon_0 z^2}.$$

25E

From the figure above it is obvious that the net electric field at point P is in the negative y direction. Its magnitude is

$$\begin{aligned} E &= 2E_1 \sin \theta = 2 \left[\frac{1}{4\pi\epsilon_0} \frac{q}{(d/2)^2 + r^2} \right] \frac{d/2}{\sqrt{(d/2)^2 + r^2}} \\ &= \frac{1}{4\pi\epsilon_0} \frac{qd}{[(d/2)^2 + r^2]^{3/2}}. \end{aligned}$$

For $r \gg d$ $[(d/2)^2 + r^2]^{3/2} \approx r^3$ so the expression above reduces to

$$E \approx \frac{1}{4\pi\epsilon_0} \frac{qd}{r^3}.$$

Since $\mathbf{p} = (qd)(-\mathbf{j})$,

$$\mathbf{E} \approx -\frac{1}{4\pi\epsilon_0} \frac{\mathbf{p}}{r^3}.$$

26P

Think of the quadrupole as composed of two dipoles, each with a dipole moment of magnitude $p = qd$. The two dipoles point in opposite directions and produce fields in opposite

directions at points on the dipole axis. Consider a point on the axis a distance z above the center of the quadrupole and take an upward pointing field to be positive. Then the field produced by the upper dipole of the pair is $qd/2\pi\epsilon_0(z-d/2)^3$ and the field produced by the lower is $-qd/2\pi\epsilon_0(z+d/2)^3$. Use the binomial expansions $(z-d/2)^{-3} \approx z^{-3} - 3z^{-4}(-d/2)$ and $(z+d/2)^{-3} \approx z^{-3} - 3z^{-4}(d/2)$ to obtain

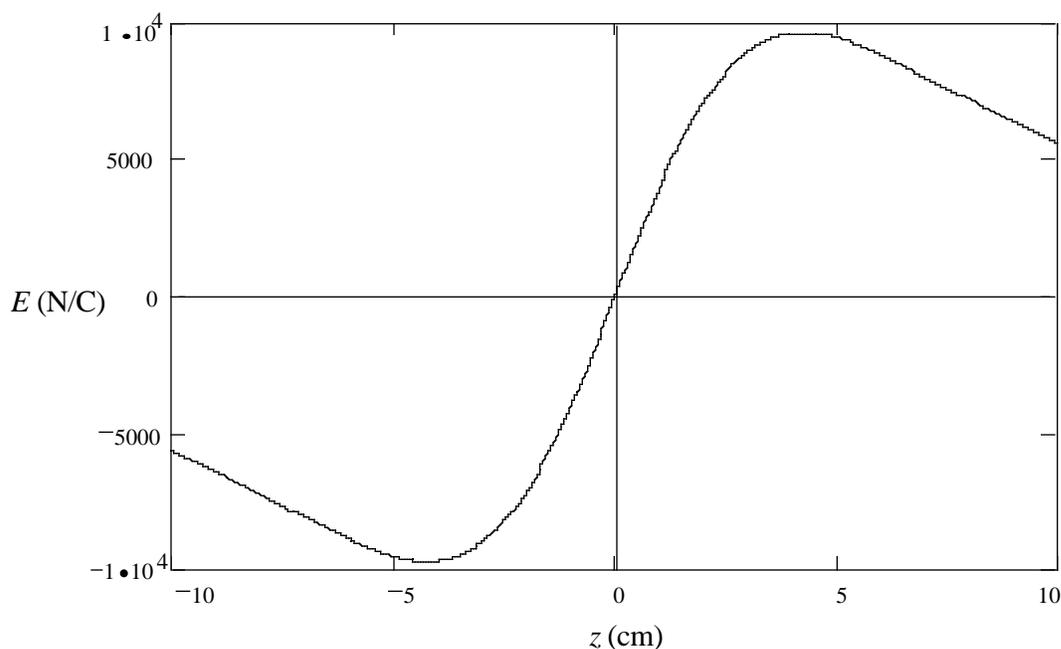
$$E = \frac{qd}{2\pi\epsilon_0} \left(\frac{1}{z^3} + \frac{3d}{2z^4} - \frac{1}{z^3} + \frac{3d}{2z^4} \right) = \frac{6qd^2}{4\pi\epsilon_0 z^4}.$$

Let $Q = 2qd^2$, then

$$E = \frac{3Q}{4\pi\epsilon_0 z^4}.$$

27E

Use Eq. 23-16. Take $E > 0$ for \mathbf{E} pointing in the positive z direction. The plot is shown below.



28E

From Eq. 23-16 the magnitude of the field at point P produced by ring 1 is given by

$$E_1 = \frac{q_1 R}{4\pi\epsilon_0 (R^2 + R^2)^{3/2}},$$

while that by ring 2 is

$$E_2 = \frac{q_2(2R)}{4\pi\epsilon_0[(2R)^2 + R^2]^{3/2}}.$$

Since \mathbf{E}_1 and \mathbf{E}_2 have opposite directions the condition for $\mathbf{E}_1 + \mathbf{E}_2 = 0$ is $E_1 = E_2$, or

$$\frac{q_1 R}{(R^2 + R^2)^{3/2}} = \frac{q_2(2R)}{[(2R)^2 + R^2]^{3/2}}.$$

Solve for q_1/q_2 to obtain $q_1/q_2 = 2(2/5)^{3/2} \approx 0.51$.

29P

Set $dE/dz = 0$ in Eq. 23-16:

$$\frac{dE}{dz} = \frac{d}{dz} \left[\frac{qz}{4\pi\epsilon_0(z^2 + R^2)^{3/2}} \right] = \frac{q(z^2 + R^2)^{3/2} - 3qz(z^2 + R^2)^{1/2}}{4\pi\epsilon_0(z^2 + R^2)^3} = 0,$$

i.e., $(z^2 + R^2)^{1/2} - 3z = 0$. Solve for z : $z = \pm R/\sqrt{2}$.

30P

The electric field at a point on the axis of a uniformly charged ring, a distance z from the ring center, is given by

$$E = \frac{qz}{4\pi\epsilon_0(z^2 + R^2)^{3/2}},$$

where q is the charge on the ring and R is the radius of the ring (see Eq. 23-16). For q positive the field points upward at points above the ring and downward at points below the ring. Take the positive direction to be upward. Then the force acting on an electron on the axis is

$$F = -\frac{eqz}{4\pi\epsilon_0(z^2 + R^2)^{3/2}}.$$

For small amplitude oscillations $z \ll R$ and z can be neglected in the denominator. Thus

$$F = -\frac{eqz}{4\pi\epsilon_0 R^3}.$$

The force is a restoring force: it pulls the electron toward the equilibrium point $z = 0$. Furthermore, the magnitude of the force is proportional to z , just as if the electron were attached to a spring with spring constant $k = eq/4\pi\epsilon_0 R^3$. The electron moves in simple harmonic motion with an angular frequency given by

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{eq}{4\pi\epsilon_0 m R^3}},$$

where m is the mass of the electron.

31P

By symmetry, each of the two rods produces the same electric field \mathbf{E}_0 pointing in the $+y$ axis at the center of the circle. So the net field is

$$\begin{aligned}\mathbf{E} &= 2E_0\mathbf{j} = 2\mathbf{j} \int \frac{\cos\theta dq}{4\pi\epsilon_0 R^2} = 2\mathbf{j} \int_{-\pi/2}^{+\pi/2} \frac{\cos\theta \cdot qR d\theta}{4\pi\epsilon_0 R^2 \cdot \pi R} \\ &= \left(\frac{1}{4\pi\epsilon_0} \frac{4q}{\pi R^2} \right) \mathbf{j}.\end{aligned}$$

32P

By symmetry, both $+q$ and $-q$ produce an equal value of electric field, \mathbf{E}_0 , pointing vertically downward. The magnitude of \mathbf{E}_P is thus

$$E_P = 2E_0 = 2 \int_{\text{rod}} \frac{\sin\theta dq}{4\pi\epsilon_0 r^2} = 2 \int_0^{\pi/2} \frac{qr \sin\theta d\theta}{4\pi\epsilon_0 r^2 (\pi r/2)} = \frac{q}{\pi^2 \epsilon_0 r^2},$$

and \mathbf{E}_P points vertically downward.

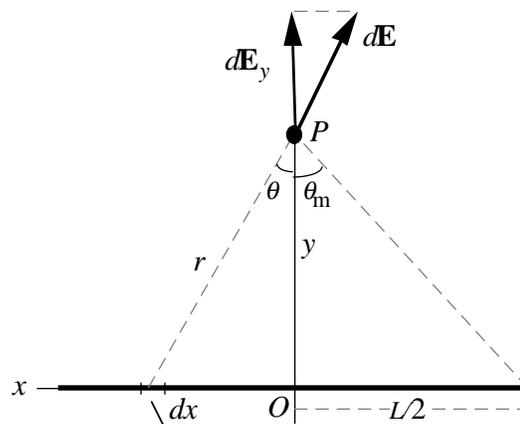
33P

By symmetry we only need to consider the y -component of \mathbf{E}_P . Consider an infinitesimal segment of length dx in the rod. We have

$$dE_y = \cos\theta dE = \frac{y}{r} \cdot \frac{q dx/L}{4\pi\epsilon_0 r^2} = \frac{q \cos\theta d\theta}{4\pi\epsilon_0 Ly},$$

where we used $x = y \tan\theta$ and $r = y/\cos\theta$. Thus

$$\begin{aligned}E &= 2 \int_0^{\theta_m} \frac{q \cos\theta d\theta}{4\pi\epsilon_0 Ly} = \frac{2q}{4\pi\epsilon_0 Ly} \sin\theta_m \\ &= \frac{qL/2}{2\pi\epsilon_0 Ly \sqrt{y^2 + (L/2)^2}} \\ &= \frac{q}{2\pi\epsilon_0 y \sqrt{4y^2 + L^2}}.\end{aligned}$$



Note that the factor of 2 in the first step above is due to the fact that each half of the rod contributes equally to E .

34P

(a) $\lambda = -q/L$.

(b)

$$\mathbf{E}_P = \frac{\mathbf{i}}{4\pi\epsilon_0} \int_0^L \frac{\lambda dx}{(a+L-x)^2} = -\frac{q\mathbf{i}}{4\pi\epsilon_0 L} \int_0^L \frac{dx}{(a+L-x)^2} = \frac{q(-\mathbf{i})}{4\pi\epsilon_0 a(L+a)}.$$

(c) If $a \gg L$ then $a(L+a) \approx a^2$, and

$$\mathbf{E}_P \approx \frac{q(-\mathbf{i})}{4\pi\epsilon_0 a^2},$$

indeed the electric field of a point charge.

35P

Consider an infinitesimal section of the rod of length dx , a distance x from the left end, as shown in the diagram to the right. It contains charge $dq = \lambda dx$ and is a distance r from P . The magnitude of the field it produces at P is given by

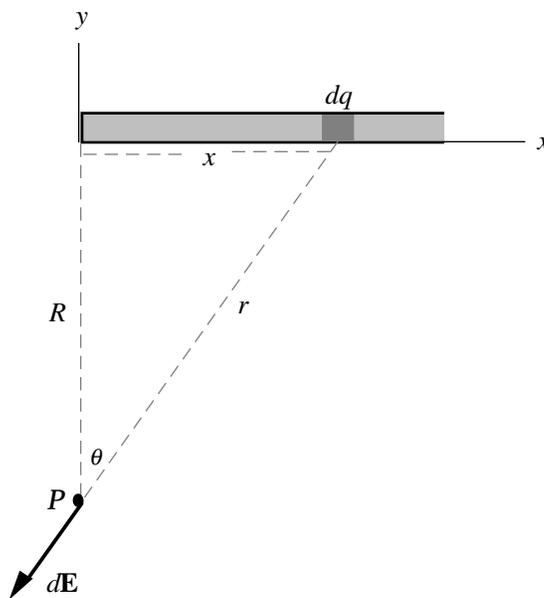
$$dE = \frac{1}{4\pi\epsilon_0} \frac{\lambda dx}{r^2}.$$

The x component is

$$dE_x = -\frac{1}{4\pi\epsilon_0} \frac{\lambda dx}{r^2} \sin \theta$$

and the y component is

$$dE_y = -\frac{1}{4\pi\epsilon_0} \frac{\lambda dx}{r^2} \cos \theta.$$



Use θ as the variable of integration. Substitute $r = R/\cos \theta$, $x = R \tan \theta$, and $dx = (R/\cos^2 \theta) d\theta$. The limits of integration are 0 and $\pi/2$ rad. Thus

$$E_x = -\frac{\lambda}{4\pi\epsilon_0} \int_0^{\pi/2} \sin \theta d\theta = \frac{\lambda}{4\pi\epsilon_0} \cos \theta \Big|_0^{\pi/2} = -\frac{\lambda}{4\pi\epsilon_0 R}.$$

and

$$E_y = -\frac{\lambda}{4\pi\epsilon_0} \int_0^{\pi/2} \cos \theta d\theta = -\frac{\lambda}{4\pi\epsilon_0} \sin \theta \Big|_0^{\pi/2} = -\frac{\lambda}{4\pi\epsilon_0 R}.$$

Notice that $E_x = E_y$ no matter what the value of R . Thus \mathbf{E} makes an angle of 45° with the negative x axis for all values of R .

36E

From Eq. 23-24

$$E = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right) \\ = \frac{5.3 \mu\text{C}/\text{m}^2}{2(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)} \left[1 - \frac{12 \text{ cm}}{\sqrt{(12 \text{ cm})^2 + (2.5 \text{ cm})^2}} \right] = 6.3 \times 10^3 \text{ N/C}.$$

37P

(a) Let $E = \sigma/2\epsilon_0 = E_0 = 3 \times 10^6 \text{ N/C}$. Thus

$$q = \pi R^2 \sigma = 2\pi\epsilon_0 R^2 E_0 = \frac{(2.5 \times 10^{-2} \text{ m})^2 (3.0 \times 10^6 \text{ N/C})}{2(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)} = 1.0 \times 10^{-7} \text{ C}.$$

(b)

$$N = \frac{\pi(2.5 \times 10^{-2} \text{ m})^2}{0.015 \times 10^{-18} \text{ m}^2} = 1.3 \times 10^{17}.$$

(c) The fraction is

$$\text{frac} = \frac{q}{Ne} = \frac{1.0 \times 10^{-7} \text{ C}}{(1.3 \times 10^{17})(1.6 \times 10^{-19} \text{ C})} = 5.0 \times 10^{-6}.$$

38P

At a point on the axis of a uniformly charged disk a distance z above the center of the disk the magnitude of the electric field is

$$E = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right),$$

where R is the radius of the disk and σ is the surface charge density on the disk. The magnitude of the field at the center of the disk ($z = 0$) is $E_c = \sigma/2\epsilon_0$. You want to solve for the value of z such that $E/E_c = 1/2$. This means

$$1 - \frac{z}{\sqrt{z^2 + R^2}} = \frac{1}{2}$$

or

$$\frac{z}{\sqrt{z^2 + R^2}} = \frac{1}{2}.$$

Square both sides, then multiply them by $z^2 + R^2$ to obtain $z^2 = (z^2/4) + (R^2/4)$. Thus $z^2 = R^2/3$ and $z = R/\sqrt{3}$.

39E

The magnitude of the force acting on the electron is $F = eE$, where E is the magnitude of the electric field at its location. The acceleration of the electron is given by Newton's second law:

$$a = \frac{F}{m_e} = \frac{eE}{m_e} = \frac{(1.60 \times 10^{-19} \text{ C})(2.00 \times 10^4 \text{ N/C})}{9.11 \times 10^{-31} \text{ kg}} = 3.51 \times 10^{15} \text{ m/s}^2.$$

40E

$$E = \frac{F}{e} = \frac{m_e a}{e} = \frac{(9.11 \times 10^{-31} \text{ kg})(1.80 \times 10^9 \text{ m/s}^2)}{1.60 \times 10^{-19} \text{ C}} = 1.02 \times 10^{-2} \text{ N/C},$$

\mathbf{E} points westward.

41E

Use Eq. 23-9:

$$\begin{aligned} F = qE &= \frac{ep}{2\pi\epsilon_0 z^3} = \frac{2(1.60 \times 10^{-19} \text{ C})(3.6 \times 10^{-29} \text{ C}\cdot\text{m})(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)}{(25 \times 10^{-9} \text{ m})^3} \\ &= 6.6 \times 10^{-15} \text{ N}. \end{aligned}$$

42E

$$(a) F_e = Ee = (3.0 \times 10^6 \text{ N/C})(1.6 \times 10^{-19} \text{ C}) = 4.8 \times 10^{-13} \text{ N}.$$

$$(b) F_i = Eq_{\text{net}} = Ee = 4.8 \times 10^{-13} \text{ N}.$$

43E

Let $mg = qE = 2eE$. Thus

$$E = \frac{mg}{2e} = \frac{(6.64 \times 10^{-27} \text{ kg})(9.80 \text{ m/s}^2)}{2(1.6 \times 10^{-19} \text{ C})} = 2.03 \times 10^{-7} \text{ N/C}.$$

\mathbf{E} should point upward.

44E

(a) The magnitude of the force on the particle is given by $F = qE$, where q is the magnitude of the charge carried by the particle and E is the magnitude of the electric field at the location of the particle. Thus

$$E = \frac{F}{q} = \frac{3.0 \times 10^{-6} \text{ N}}{2.0 \times 10^{-9} \text{ C}} = 1.5 \times 10^3 \text{ N/C}.$$

The force points downward and the charge is negative, so the field points upward.

(b) The magnitude of the electrostatic force on a proton is

$$F_e = eE = (1.60 \times 10^{-19} \text{ C})(1.5 \times 10^3 \text{ N/C}) = 2.4 \times 10^{-16} \text{ N}.$$

A proton is positively charged, so the force is in the same direction as the field, upward.

(c) The magnitude of the gravitational force on the proton is

$$F_g = mg = (1.67 \times 10^{-27} \text{ kg})(9.8 \text{ m/s}^2) = 1.6 \times 10^{-26} \text{ N}.$$

The force is downward.

(d) The ratio of the forces is

$$\frac{F_e}{F_g} = \frac{2.4 \times 10^{-16} \text{ N}}{1.64 \times 10^{-26} \text{ N}} = 1.5 \times 10^{10}.$$

45E

Let $mg = |q|E$ and solve for $|q|$:

$$|q| = \frac{mg}{E} = \frac{4.4 \text{ N}}{150 \text{ N/C}} = 2.9 \times 10^{-2} \text{ C}.$$

Since $q\mathbf{E}$ must be upward while \mathbf{E} is downward, q is negative.

46E

(a)

$$a = \frac{F}{m_e} = \frac{eE}{m_e} = \frac{(1.60 \times 10^{-19} \text{ C})(1.40 \times 10^6 \text{ N/C})}{9.11 \times 10^{-31} \text{ kg}} = 2.46 \times 10^{17} \text{ m/s}^2.$$

(b)

$$t = \frac{v}{a} = \frac{3.00 \times 10^7 \text{ m/s}}{2.46 \times 10^{17} \text{ m/s}^2} = 1.22 \times 10^{-10} \text{ s}.$$

(c)

$$s = \frac{1}{2}at^2 = \frac{1}{2}(2.46 \times 10^{17} \text{ m/s}^2)(1.22 \times 10^{-10} \text{ s})^2 = 1.83 \times 10^{-3} \text{ m}.$$

47E

(a) The magnitude of the force acting on the proton is $F = eE$, where E is the magnitude of the electric field. According to Newton's second law the acceleration of the proton is $a = F/m_p = eE/m_p$, where m_p is the mass of the proton. Thus

$$a = \frac{(1.60 \times 10^{-19} \text{ C})(2.00 \times 10^4 \text{ N/C})}{1.67 \times 10^{-27} \text{ kg}} = 1.92 \times 10^{12} \text{ m/s}^2.$$

(b) Assume the proton starts from rest and use the kinematic equations $x = \frac{1}{2}at^2$ and $v = at$ to show that $v = \sqrt{2ax} = \sqrt{2(1.92 \times 10^{12} \text{ m/s}^2)(0.0100 \text{ m})} = 1.96 \times 10^5 \text{ m/s}$.

48E

(a) Use $v_f^2 - v_i^2 = -v_i^2 = 2as$ and $a = F/m = -eE/m$ to solve for s :

$$s = \frac{-v_i^2}{2a} = \frac{-m_e v_i^2}{-2eE} = \frac{-(9.11 \times 10^{-31} \text{ kg})(5.00 \times 10^6 \text{ m/s})^2}{-2(1.60 \times 10^{-19} \text{ C})(1.00 \times 10^3 \text{ N/C})} = 7.12 \times 10^{-2} \text{ m}.$$

(b)

$$t = \frac{s}{v} = \frac{2s}{v_i} = \frac{2(7.12 \times 10^{-2} \text{ m})}{5.00 \times 10^6 \text{ m/s}} = 2.85 \times 10^{-8} \text{ s}.$$

(c) From $\Delta v^2 = 2a\Delta s$

$$\begin{aligned} \frac{\Delta K}{K_i} &= \frac{\Delta(\frac{1}{2}m_e v^2)}{\frac{1}{2}m_e v_i^2} = \frac{\Delta v^2}{v_i^2} = \frac{2a\Delta s}{v_i^2} = \frac{-2eE\Delta s}{m_e v_i^2} \\ &= \frac{-2(1.60 \times 10^{-19} \text{ C})(1.00 \times 10^3 \text{ N/C})(8.00 \times 10^{-3} \text{ m})}{(9.11 \times 10^{-31} \text{ kg})(5.00 \times 10^6 \text{ m/s})^2} = -11.2\%. \end{aligned}$$

49E

(a)

$$W = \rho V = \frac{\pi D^3 \rho}{6} = \frac{\pi(1.2 \times 10^{-6} \text{ m})^3(1.00 \times 10^3 \text{ kg/m}^3)}{6} = 8.87 \times 10^{-15} \text{ N}.$$

(b) From $W = F = qE = NeE$ we solve for N , the number of excess electrons:

$$N = \frac{W}{eE} = \frac{8.87 \times 10^{-15} \text{ N}}{(1.60 \times 10^{-19} \text{ C})(462 \text{ N/C})} = 120.$$

50E

When the drop is in equilibrium the force of gravity is balanced by the force of the electric field: $mg = qE$, where m is the mass of the drop, q is the charge on the drop, and E is the magnitude of the electric field. The mass of the drop is given by $m = (4\pi/3)r^3\rho$, where r is its radius and ρ is its mass density. Thus

$$\begin{aligned} q &= \frac{mg}{E} = \frac{4\pi r^3 \rho g}{3E} \\ &= \frac{4\pi(1.64 \times 10^{-6} \text{ m})^3(851 \text{ kg/m}^3)(9.8 \text{ m/s}^2)}{3(1.92 \times 10^5 \text{ N/C})} = 8.0 \times 10^{-19} \text{ C} \end{aligned}$$

and $q/e = (8.0 \times 10^{-19} \text{ C}) / (1.60 \times 10^{-19} \text{ C}) = 5$.

51P

You need to find the largest common denominator of the group of numbers listed in the problem. One way to start this is to find the closest separation between any pair of the numbers. For example, take $13.13 \times 10^{-19} \text{ C} - 11.50 \times 10^{-19} \text{ C} = 1.63 \times 10^{-19} \text{ C}$. You can easily verify that this value is a common denominator. So this is the value of e that can be deduced.

52P

(a) Use $s = \bar{v}t = vt/2$:

$$v = \frac{2s}{t} = \frac{2(2.0 \times 10^{-2} \text{ m})}{1.5 \times 10^{-8} \text{ s}} = 2.7 \times 10^6 \text{ m/s}.$$

(b) Use $s = \frac{1}{2}at^2$ and $E = F/e = m_e a/e$:

$$E = \frac{m_e a}{e} = \frac{2sm_e}{et^2} = \frac{2(2.0 \times 10^{-2} \text{ m})(9.11 \times 10^{-31} \text{ kg})}{(1.60 \times 10^{-19} \text{ C})(1.5 \times 10^{-8} \text{ s})^2} = 1.0 \times 10^3 \text{ N/C}.$$

53P

(a)

$$\begin{aligned} \mathbf{F} &= q\mathbf{E} = q(E_x \mathbf{i} + E_y \mathbf{j}) \\ &= (8.00 \times 10^{-5} \text{ C})(3.00 \times 10^3 \text{ N/C}) \mathbf{i} + (8.00 \times 10^{-5} \text{ C})(-600 \text{ N/C}) \mathbf{j} \\ &= (0.240 \text{ N}) \mathbf{i} - (0.0480 \text{ N}) \mathbf{j}. \end{aligned}$$

So the magnitude of \mathbf{F} is $F = \sqrt{(0.240 \text{ N})^2 + (0.0480 \text{ N})^2} = 0.245 \text{ N}$, and \mathbf{F} makes an angle θ with the $+x$ direction, where

$$\theta = \tan^{-1} \left(\frac{F_y}{F_x} \right) = \tan^{-1} \left(\frac{-0.0480 \text{ N}}{0.240 \text{ N}} \right) = -11.3^\circ.$$

(b) The coordinates (x, y) at $t = 3.00 \text{ s}$ are

$$\begin{cases} x = \frac{1}{2} a_x t^2 = \frac{F_x t^2}{2m} = \frac{(0.240 \text{ N})(3.00 \text{ s})^2}{2(1.0 \times 10^{-2} \text{ kg})} = 108 \text{ m}, \\ y = \frac{1}{2} a_y t^2 = \frac{x a_y}{a_x} = \frac{x F_y}{F_x} = \frac{(108 \text{ m})(-0.0480 \text{ N})}{0.240 \text{ N}} = -21.6 \text{ m}. \end{cases}$$

54P

(a)

$$\mathbf{a} = \frac{\mathbf{F}}{m_e} = \frac{-e\mathbf{E}}{m_e} = \frac{(-1.60 \times 10^{-19} \text{ C})(120 \text{ N/C})\mathbf{j}}{9.11 \times 10^{-31} \text{ kg}} = -2.11 \times 10^{13} \mathbf{j} \text{ (m/s}^2\text{)}.$$

(b) Since $a_x = 0$, the time t it takes for the x coordinate of the electron to change by 2.0 cm is $t = 2.0 \text{ cm}/(1.5 \times 10^5 \text{ m/s}) = 1.3 \times 10^{-7} \text{ s}$. So v_x remains at $1.5 \times 10^5 \text{ m/s}$, while v_y becomes

$$v_y = 3.0 \times 10^3 \text{ m/s} - (2.11 \times 10^{13} \text{ m/s}^2)(1.3 \times 10^{-7} \text{ s}) = -2.7 \times 10^6 \text{ m/s}.$$

Thus \mathbf{v} becomes $(1.5 \times 10^5 \mathbf{i} - 2.7 \times 10^6 \mathbf{j}) \text{ m/s}$.

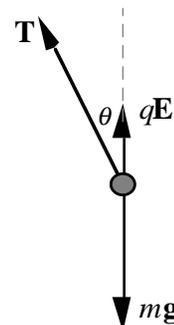
55P

Take the positive direction to be to the right in the diagram. The acceleration of the proton is $a_p = eE/m_p$ and the acceleration of the electron is $a_e = -eE/m_e$, where E is the magnitude of the electric field, m_p is the mass of the proton, and m_e is the mass of the electron. Take the origin to be at the initial position of the proton. Then the coordinate of the proton at time t is $x = \frac{1}{2}a_p t^2$ and the coordinate of the electron is $x = L + \frac{1}{2}a_e t^2$. They pass each other when their coordinates are the same, or $\frac{1}{2}a_p t^2 = L + \frac{1}{2}a_e t^2$. This means $t^2 = 2L/(a_p - a_e)$ and

$$\begin{aligned} x &= \frac{a_p}{a_p - a_e} L = \frac{eE/m_p}{(eE/m_p) + (eE/m_e)} L = \frac{m_e}{m_e + m_p} L \\ &= \frac{9.11 \times 10^{-31} \text{ kg}}{9.11 \times 10^{-31} \text{ kg} + 1.67 \times 10^{-27} \text{ kg}} (0.050 \text{ m}) = 2.7 \times 10^{-5} \text{ m}. \end{aligned}$$

56P

(a) Suppose the pendulum is at the angle θ with the vertical. The force diagram is shown to the right. T is the tension in the thread, mg is the force of gravity, and qE is the force of the electric field. The field points upward and the charge is positive, so the force is upward. Take the angle shown to be positive. Then the torque on the sphere about the point where the thread is attached to the upper plate is $\tau = -(mg - qE)\ell \sin \theta$. If $mg > qE$ then the torque is a restoring torque; it tends to pull the pendulum back to its equilibrium position.



If the amplitude of the oscillation is small, $\sin \theta$ can be replaced by θ in radians and the torque is $\tau = -(mg - qE)\ell\theta$. The torque is proportional to the angular displacement and the pendulum moves in simple harmonic motion. Its angular frequency is $\omega = \sqrt{(mg - qE)\ell/I}$, where I is the rotational inertia of the pendulum. Since $I = m\ell^2$ for a simple pendulum,

$$\omega = \sqrt{\frac{(mg - qE)\ell}{m\ell^2}} = \sqrt{\frac{g - qE/m}{\ell}}$$

and the period is

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{\ell}{g - qE/m}}.$$

If $qE > mg$ the torque is not a restoring torque and the pendulum does not oscillate.

(b) The force of the electric field is now downward and the torque on the pendulum is $\tau = -(mg + qE)\ell\theta$ if the angular displacement is small. The period of oscillation is

$$T = 2\pi \sqrt{\frac{\ell}{g + qE/m}}.$$

57P

The electric field is upward in the diagram and the charge is negative, so the force of the field on it is downward. The magnitude of the acceleration is $a = eE/m_e$, where E is the magnitude of the field and m_e is the mass of the electron. Its numerical value is

$$a = \frac{(1.60 \times 10^{-19} \text{ C})(2.00 \times 10^3 \text{ N/C})}{9.11 \times 10^{-31} \text{ kg}} = 3.51 \times 10^{14} \text{ m/s}^2.$$

Put the origin of a coordinate system at the initial position of the electron. Take the x axis to be horizontal and positive to the right; take the y axis to be vertical and positive toward the top of the page. The kinematic equations are $x = v_0 t \cos \theta$, $y = v_0 t \sin \theta - \frac{1}{2}at^2$, and $v_y = v_0 \sin \theta - at$.

First find the greatest y coordinate attained by the electron. If it is less than d the electron does not hit the upper plate. If it is greater than d it will hit the upper plate if the corresponding x coordinate is less than L . The greatest y coordinate occurs when $v_y = 0$. This means $v_0 \sin \theta - at = 0$ or $t = (v_0/a) \sin \theta$ and

$$\begin{aligned} y_{\max} &= \frac{v_0^2 \sin^2 \theta}{a} - \frac{1}{2}a \frac{v_0^2 \sin^2 \theta}{a^2} = \frac{1}{2} \frac{v_0^2 \sin^2 \theta}{a} \\ &= \frac{(6.00 \times 10^6 \text{ m/s})^2 \sin^2 45^\circ}{2(3.51 \times 10^{14} \text{ m/s}^2)} = 2.56 \times 10^{-2} \text{ m}. \end{aligned}$$

Since this is greater than d ($= 2.00 \text{ cm}$) the electron might hit the upper plate.

Now find the x coordinate of the position of the electron when $y = d$. Since

$$v_0 \sin \theta = (6.00 \times 10^6 \text{ m/s}) \sin 45^\circ = 4.24 \times 10^6 \text{ m/s}$$

and

$$2ad = 2(3.51 \times 10^{14} \text{ m/s}^2)(0.0200 \text{ m}) = 1.40 \times 10^{13} \text{ m}^2/\text{s}^2,$$

the solution to $d = v_0 t \sin \theta - \frac{1}{2} a t^2$ is

$$t = \frac{v_0 \sin \theta - \sqrt{v_0^2 \sin^2 \theta - 2ad}}{a}$$

$$= \frac{4.24 \times 10^6 \text{ m/s} - \sqrt{(4.24 \times 10^6 \text{ m/s})^2 - 1.40 \times 10^{13} \text{ m}^2/\text{s}^2}}{3.51 \times 10^{14} \text{ m/s}^2} = 6.43 \times 10^{-9} \text{ s}.$$

The negative root was used because we want the *earliest* time for which $y = d$.

The x coordinate is

$$x = v_0 t \cos \theta = (6.00 \times 10^6 \text{ m/s})(6.43 \times 10^{-9} \text{ s}) \cos 45^\circ = 2.72 \times 10^{-2} \text{ m}.$$

This is less than L so the electron hits the upper plate at $x = 2.72 \text{ cm}$.

58E

(a) $p = (1.5 \times 10^{-9} \text{ C})(6.20 \times 10^{-6} \text{ m}) = 9.30 \times 10^{-15} \text{ C}\cdot\text{m}.$

(b) $\Delta U = pE - (-pE) = 2pE = 2(9.30 \times 10^{-15} \text{ C}\cdot\text{m})(1100 \text{ N/C}) = 2.05 \times 10^{-11} \text{ J}.$

59E

Use $\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$.

(a) Now $\boldsymbol{\tau} = 0$ as $\mathbf{p} \parallel \mathbf{E}$.

(b) Now $\tau = pE \sin 90^\circ = 2(1.6 \times 10^{-19} \text{ C})(0.78 \times 10^{-9} \text{ m})(3.4 \times 10^6 \text{ N/C}) = 8.5 \times 10^{-22} \text{ N}\cdot\text{m}.$

(c) Now $\boldsymbol{\tau} = 0$ again, since $\mathbf{p} \parallel (-\mathbf{E})$.

60P

$$W = \Delta U = -\mathbf{p}_f \cdot \mathbf{E} - (-\mathbf{p}_i \cdot \mathbf{E}) = (\mathbf{p}_i - \mathbf{p}_f) \cdot \mathbf{E} = pE \cos \theta_0 - pE \cos(\theta_0 + \pi) = 2pE \cos \theta_0.$$

61P

The magnitude of the torque acting on the dipole is given by $\tau = pE \sin \theta$, where p is the magnitude of the dipole moment, E is the magnitude of the electric field, and θ is the angle between the dipole moment and the field. It is a restoring torque: it always tends to rotate the dipole moment toward the direction of the electric field. If θ is positive the torque is negative and vice-versa. Write $\tau = -pE \sin \theta$. If the amplitude of the motion is small we may replace $\sin \theta$ with θ in radians. Thus $\tau = -pE\theta$. Since the magnitude of the torque is proportional to the angle of rotation, the dipole oscillates in simple harmonic motion, just like a torsional pendulum with torsion constant $\kappa = pE$. The angular frequency ω is given by

$$\omega^2 = \frac{\kappa}{I} = \frac{pE}{I},$$

where I is the rotational inertia of the dipole. The frequency of oscillation is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{pE}{I}}.$$

62P

(a) From Eq. 23-38

$$\begin{aligned} U &= -\mathbf{p} \cdot \mathbf{E} = -[(3.00 \mathbf{i} + 4.00 \mathbf{j})(1.24 \times 10^{-30} \text{ C}\cdot\text{m})] \cdot [(4000 \text{ N/C}) \mathbf{i}] \\ &= -1.49 \times 10^{-26} \text{ J}. \end{aligned}$$

(b) From Eq. 23-34

$$\begin{aligned} \boldsymbol{\tau} &= \mathbf{p} \times \mathbf{E} = [(3.00 \mathbf{i} + 4.00 \mathbf{j})(1.24 \times 10^{-30} \text{ C}\cdot\text{m})] \times [(4000 \text{ N/C}) \mathbf{i}] \\ &= (-1.98 \times 10^{-26} \text{ N}\cdot\text{m}) \mathbf{k}. \end{aligned}$$

(c) The work done is

$$\begin{aligned} W &= \Delta U = \Delta(-\mathbf{p} \cdot \mathbf{E}) = (\mathbf{p}_i - \mathbf{p}_f) \cdot \mathbf{E} \\ &= [(3.00 \mathbf{i} + 4.00 \mathbf{j}) - (-4.00 \mathbf{i} + 3.00 \mathbf{j})](1.24 \times 10^{-30} \text{ C}\cdot\text{m}) \cdot [(4000 \text{ N/C}) \mathbf{i}] \\ &= 3.47 \times 10^{-26} \text{ J}. \end{aligned}$$

63

(a)

$$E \Big|_{x=\alpha d} = \frac{2q}{4\pi\epsilon_0 d^2} \left[\frac{\alpha}{(1 + \alpha^2)^{3/2}} \right].$$

(c) 0.707

(d) 0.21 and 1.9

64

(a) first row: 4, 8, 12; second row: 5, 10, 14; thier row: 7, 11, 16;

(b) $1.63 \times 10^{-19} \text{ C}$