CHAPTER 26

Answer to Checkpoint Questions

1. (a) same; (b) same

2. (a) decreases; (b) increases; (c) decreases

3. (a) V, $q/2$; (b) $V/2$, q

4. (a) $q_0 = q_1 + q_{34}$; (b) equal (C_3 and C_4 are in series)

5. (a) same; (b) – (d) increase; (e) same (same potential difference across same plate separation)

6. (a) same; (b) decrease; (c) increase

Answer to Questions

1. a, 2; b, 1; c, 3

2. B and C tie, then A

3. (a) increase; (b) same

4. (a) series; (b) parallel; (c) parallel

5. (a) parallel; (b) series

6. (a) no; (b) yes; (c) all tie

7. (a) $C/3$; (b) $3C$; (c) parallel

8. parallel, C_1 alone, C_2 alone, series

9. (a) equal; (b) less

10. (a) same; (b) same; (c) more; (d) more

11. (a) – (d) less

12. (a) decreases; (b) decreases; (c) increases

13. (a) 2; (b) 3; (c) 1

14. (a) less; (b) more; (c) equal; (d) more

15. Increase plate separation d, but also plate area A, keeping A/d constant

16. (*a*) increases; (*b*) increases; (*c*) decreases; (*d*) decreases; (*e*) same, increases, increases, increases

Solutions to Exercises & Problems

1E

The minimum charge measurable is

$$q_{\text{min}} = CV_{\text{min}} = (50\,\text{pF})(0.15\,\text{V}) = 7.5\,\text{pC}\,.$$

2E

(*a*)

$$C = \frac{q}{\Delta V} = \frac{70\,\text{pC}}{20\,\text{V}} = 3.5\,\text{pF}\,.$$

(*b*) The capacitance is independent of q, i.e. it is still $3.5\,\text{pF}$.

(*c*)

$$\Delta V = \frac{q}{C} = \frac{200\,\text{pC}}{3.5\,\text{pF}} = 57\,\text{V}\,.$$

3E

Charge flows until the potential difference across the capacitor is the same as the emf of the battery. The charge on the capacitor is then $q = C\mathcal{E}$ and this is the same as the total charge that has passed through the battery. Thus $q = (25 \times 10^{-6}\,\text{F})(120\,\text{V}) = 3.0 \times 10^{-3}\,\text{C}$.

4E

$$[\epsilon_0] = \frac{\text{F}}{\text{m}} = \frac{\text{C}}{\text{V}\cdot\text{m}} = \frac{\text{C}}{(\,\text{N}\cdot\text{m}/\text{C})\,\text{m}} = \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\,.$$

5E

(*a*) The capacitance of a parallel plate capacitor is given by $C = \epsilon_0 A/d$, where A is the area of each plate and d is the plate separation. Since the plates are circular the plate area is $A = \pi R^2$, where R is the radius of a plate. Thus

$$C = \frac{\epsilon_0 \pi R^2}{d} = \frac{(8.85 \times 10^{-12}\ \text{F/m})\pi(8.2 \times 10^{-2}\,\text{m})^2}{1.3 \times 10^{-3}\,\text{m}} = 1.4 \times 10^{-10}\,\text{F} = 140\ \text{pF}\,.$$

(b) The charge on the positive plate is given by $q = CV$, where V is the potential difference across the plates. Thus $q = (1.4 \times 10^{-10}\,\mathrm{F})(120\,\mathrm{V}) = 1.7 \times 10^{-8}\,\mathrm{C} = 17$ nC.

6E

Use $C = A\epsilon_0/d$. Thus

$$d = \frac{A\epsilon_0}{C} = \frac{(1.00\,\mathrm{m}^2)(8.85 \times 10^{-12}\,\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)}{1.00\,\mathrm{F}} = 8.85 \times 10^{-12}\,\mathrm{m}\,.$$

Since d is much less than the size of an atom ($\sim 10^{-10}$ m), this capacitor cannot be constructed.

7E

(a) Use Eq. 26-17:

$$C = 4\pi\epsilon_0 \frac{ab}{b-a} = \frac{(40.0\,\mathrm{mm})(38.0\,\mathrm{mm})}{(8.99 \times 10^9\,\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2)(40.0\,\mathrm{mm} - 38.0\,\mathrm{mm})} = 84.5\,\mathrm{pF}\,.$$

(b) Let the area required be A. Then $C = \epsilon_0 A/(b-a)$, or

$$A = \frac{C(b-a)}{\epsilon_0} = \frac{(84.5\,\mathrm{pF})(40.0\,\mathrm{mm} - 38.0\,\mathrm{mm})}{(8.85 \times 10^{-12}\,\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)} = 191\,\mathrm{cm}^2\,.$$

8E

The charge accumulated is given by

$$q = CV = 4\pi\epsilon_0 RV = \frac{(0.25\,\mathrm{m})(15 \times 10^3\,\mathrm{V})}{8.99 \times 10^9\,\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2} = 4.1 \times 10^{-7}\,\mathrm{C}\,.$$

9E

(a) Use $q = CV = \epsilon_0 AV/d$ to solve for A:

$$A = \frac{Cd}{\epsilon_0} = \frac{(10 \times 10^{-12}\,\mathrm{F})(1.0 \times 10^{-3}\,\mathrm{m})}{(8.85 \times 10^{-12}\,\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2)} = 1.1 \times 10^{-3}\,\mathrm{m}^2\,.$$

(b) Now

$$C' = C\left(\frac{d}{d'}\right) = (10\,\mathrm{pF})\left(\frac{1.0\,\mathrm{mm}}{0.9\,\mathrm{mm}}\right) = 11\,\mathrm{pF}\,.$$

(c) The new potential difference is $V' = q/C' = CV/C'$. Thus

$$\Delta V = V' - V = \frac{(10\,\mathrm{pF})(12\,\mathrm{V})}{11\,\mathrm{pF}} - 12\,\mathrm{V} = 1.2\,\mathrm{V}\,.$$

In a microphone, mechanical pressure applied to the aluminum foil as a result of sound can cause the capacitance of the foil to change, thereby inducing a variable ΔV in response to the sound signal.

10E

You want to find the radius of the combined spheres, then use $C = 4\pi\epsilon_0 R$ to find the capacitance. When the drops combine the volume is doubled. It is then $V = 2(4\pi/3)R^3$. The new radius R' is given by

$$\frac{4\pi}{3}(R')^3 = 2\frac{4\pi}{3}R^3\,,$$

so

$$R' = 2^{1/3}R\,.$$

The new capacitance is $C' = 4\pi\epsilon_0 R' = 4\pi\epsilon_0(2^{1/3}R) = 2^{1/3}C = 1.26C$.

11P

In Eq. 26-14, let $b = a + d$ with $d/a \ll 1$, we have

$$C = 2\pi\epsilon_0\frac{L}{\ln(b/a)} = 2\pi\epsilon_0\frac{L}{\ln(1+d/a)} \approx \frac{2\pi\epsilon_0 L}{d/a} = \frac{\epsilon_0(2\pi aL)}{d} \approx \frac{\epsilon_0 A}{d}\,,$$

where $A = 2\pi aL$ is roughly the surface area of either of the cylinders. Here the approximation $\ln(1+x) \approx x$ for $|x| \ll 1$ was used.

12P

Since $b - a - d$ and $a \approx b$, from Eq. 26-17

$$C = 4\pi\epsilon_0\frac{ab}{b-a} = \frac{\epsilon_0(4\pi ab)}{d} \approx \frac{\epsilon_0(4\pi a^2)}{d} = \frac{\epsilon_0 A}{d}\,.$$

13P

$$\begin{aligned}\frac{dC}{dT} &= \frac{d}{dT}\left(\epsilon_0\frac{A}{x}\right) = \frac{\epsilon_0}{x}\frac{dA}{dT} - \frac{\epsilon_0 A}{x^2}\frac{dx}{dT} = \frac{\epsilon_0 A}{x}\left(\frac{1}{A}\frac{dA}{dT} - \frac{1}{x}\frac{dx}{dT}\right)\\ &= C\left(\frac{1}{A}\frac{dA}{dT} - \frac{1}{x}\frac{dx}{dT}\right).\end{aligned}$$

(b) Set $dC/dT = 0$ to obtain

$$\frac{1}{A}\frac{dA}{dT} = \beta_{\rm Al} = 2\alpha_{\rm Al} = \alpha_x\,.$$

So $\alpha_x = 2\alpha_{\rm Al} = 2(23\times10^{-6}/\,{\rm C}^\circ) = 4.6\times10^{-5}/\,{\rm C}^\circ\,.$

14E

The equivalent capacitance is given by $C_{\rm eq} = q/V$, where q is the total charge on all the capacitors and V is the potential difference across any one of them. For N identical capacitors in parallel $C_{\rm eq} = NC$, where C is the capacitance of one of them. Thus $NC = q/V$ and

$$N = \frac{q}{VC} = \frac{1.00\,{\rm C}}{(110\,{\rm V})(1.00\times10^{-6}\,{\rm F})} = 9090\,.$$

15E

$$C_{\rm eq} = C_3 + \frac{C_1C_2}{C_1+C_2} = 4.00\,\mu{\rm F} + \frac{(10.0\,\mu{\rm F})(5.00\,\mu{\rm F})}{10.0\,\mu{\rm F}+5.00\,\mu{\rm F}} = 7.33\,\mu{\rm F}\,.$$

16P

$$C_{\rm eq} = \frac{(C_1+C_2)C_3}{C_1+C_2+C_3} = \frac{(10.0\,\mu{\rm F}+5.00\,\mu{\rm F})(4.00\,\mu{\rm F})}{10.0\,\mu{\rm F}+5.00\,\mu{\rm F}+4.00\,\mu{\rm F}} = 3.16\,\mu{\rm F}\,.$$

17E

The charge that passes through meter A is

$$q = C_{\rm eq}V = 3CV = 3(25.0\,\mu{\rm F})(4200\,{\rm V}) = 0.315\,{\rm C}\,.$$

18E

(a)

$$C_{\rm eq} = \frac{C_1C_2}{C_1+C_2} = \frac{(6.00\,\mu{\rm F})(4.00\,\mu{\rm F})}{6.00\,\mu{\rm F}+4.00\,\mu{\rm F}} = 2.40\,\mu{\rm F}\,.$$

(b) $q = C_{\rm eq}V = (2.40\,\mu{\rm F})(200\,{\rm V}) = 4.80\times10^4$ C.
(c) $V_1 = q/C_1 = 4.80\times10^4\,{\rm C}/2.40\,\mu{\rm F} = 120\,{\rm V}$, and $V_2 = V - V_1 = 200\,{\rm V} - 120\,{\rm V} = 80\,{\rm V}$.

19E

(a) Now $C_{\rm eq} = C_1 + C_2 = 6.00\,\mu{\rm F} + 4.00\,\mu{\rm F} = 10.0\,\mu{\rm F}$.

(b) $q_1 = C_1V = (6.00\,\mu\text{F})(200\,\text{V}) = 1.20 \times 10^{-3}\,\text{C}$, $q_2 = C_2V = (4.00\,\mu\text{F})(200\,\text{V}) = 8.00 \times 10^{-4}\,\text{C}$.
(c) $V_1 = V_2 = 200\,\text{V}$.

20P

Let x be the separation of the plates in the lower capacitor. Then the plate separation in the upper capacitor is $a-b-x$. The capacitance of the lower capacitor is $C_\ell = \epsilon_0 A/x$ and the capacitance of the upper capacitor is $C_u = \epsilon_0 A/(a-b-x)$, where A is the plate area. Since the two capacitors are in series the equivalent capacitance is determined from

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_\ell} + \frac{1}{C_u} = \frac{x}{\epsilon_0 A} + \frac{a-b-x}{\epsilon_0 A} = \frac{a-b}{\epsilon_0 A}\,.$$

Thus the equivalent capacitance is given by $C_{\text{eq}} = \epsilon_0 A/(a-b)$ and is independent of x.

21P

(a) The equivalent capacitance of the three capacitors connected in parallel is $C_{\text{eq}} = 3C = 3\epsilon_0 A/d = \epsilon_0 A/(d/3)$. Thus the required spacing is $d/3$.
(b) Now $C_{\text{eq}} = C/3 = \epsilon_0 A/3d$, so the spacing should be $3d$.

22P

(a) The equivalent capacitance is $C_{\text{eq}} = C_1C_2/(C_1+C_2)$. Thus the charge q on each capacitor is

$$q = C_{\text{eq}}V = \frac{C_1C_2V}{C_1+C_2} = \frac{(2.0\,\mu\text{F})(8.0\,\mu\text{F})(300\,\text{V})}{2.0\,\mu\text{F} + 8.0\,\mu\text{F}} = 4.8 \times 10^{-4}\,\text{C}\,.$$

The potential differences are: $V_1 = q/C_1 = 4.8 \times 10^{-4}\,\text{C}/2.0\,\mu\text{F} = 240\,\text{V}$, $V_2 = V - V_1 = 300\,\text{V} - 240\,\text{V} = 60\,\text{V}$.
(b) Now we have $q_1'/C_1 = q_2'/C_2 = V'$ (V' being the new potential difference across each capacitor) and $q_1' + q_2' = 2q$. Solve for q_1', q_2' and V:

$$\begin{cases} q_1' = \dfrac{2C_1q}{C_1+C_2} = \dfrac{2(2.0\,\mu\text{F})(4.8 \times 10^{-4}\,\text{C})}{2.0\,\mu\text{F} + 8.0\,\mu\text{F}} = 1.9 \times 10^{-4}\,\text{C}\,, \\ q_2' = 2q - q_1 = 7.7 \times 10^{-4}\,\text{C}\,, \\ V' = \dfrac{q_1'}{C_1} = \dfrac{1.92 \times 10^{-4}\,\text{C}}{2.0\,\mu\text{F}} = 96\,\text{V}\,. \end{cases}$$

(c) Now the capacitors will simply discharge themselves, leaving $q_1 = q_2 = 0$ and $V_1 = V_2 = 0$.

23P

For maximum capacitance the two groups of plates must face each other with maximum area. In this case the whole capacitor consists of $(n-1)$ identical single capacitors connected in parallel. Each capacitor has surface area A and plate separation d so its capacitance is given by $C_0 = \epsilon_0 A/d$. Thus the total capacitance of the combination is

$$C = (n-1)C_0 = \frac{(n-1)\epsilon_0 A}{d}\,.$$

24P

(a) and (b) The original potential difference V_1 across C_1 is

$$V_1 = \frac{C_{\rm eq}V}{C_1 + C_2} = \frac{(3.16\,\mu\text{F})(100\,\text{V})}{10.0\,\mu\text{F} + 5.00\,\mu\text{F}} = 21.1\,\text{V}\,.$$

Thus $\Delta V_1 = 100\,\text{V} - 21.1\,\text{V} = 79\,\text{V}$ and $\Delta q_1 = C_1\Delta V_1 = (10.0\,\mu\text{F})(79\,\text{V}) = 7.9 \times 10^{-4}\,\text{C}$.

25P

(a) Put five such capacitors in series. Obviously the equivalent capacitance is $2.0\,\mu\text{F}/5 = 0.40\,\mu\text{F}$, and with each capacitor taking a 200-V potential difference the equivalent capacitor can withstand $1000\,\text{V}$.

(b) As one possibility, you can take three identical arrays of capacitors, each array being a five-capacitor combination described in part (a) above, and hook up the arrays in parallel. The equivalent capacitance is now $C_{\rm eq} = 3(0.40\,\mu\text{F}) = 1.2\,\mu\text{F}$, and with each capacitor taking a 200-V potential difference the equivalent capacitor can withstand $1000\,\text{V}$.

26P

(a) The potential difference across C_1 is $V_1 = 10\,\text{V}$. Thus $q_1 = C_1V_1 = (10\,\mu\text{F})(10\,\text{V}) = 1.0 \times 10^{-4}\,\text{V}$.

(b) Let $C = 10\,\mu\text{F}$. Consider first the three-capacitor combination consisting of C_2 and its two closest neighbors, each of capacitance C. The equivalent capacitance of this combination is

$$C_{\rm eq} = C + \frac{C_2C}{C + C_2} = 1.5C\,.$$

The voltage drop across this combination is then

$$V = \frac{CV_1}{C + C_{eq}} = \frac{CV_1}{C + 1.5C} = \frac{2}{5}V_1\,.$$

Since this voltage difference is divided equally between C_2 and the one connected in series with it, the voltage differenc across C_2 satisfies $V_2 = V/2 = V_1/5$. Thus

$$q_2 = C_2V_2 = (10\,\mu\text{F})\left(\frac{10\,\text{V}}{5}\right) = 2.0 \times 10^{-5}\,\text{V}\,.$$

27P

The charge initially carried by the 100-pF capacitor is $q_1 = C_1V_1 = (100\,\text{pF})(50\,\text{V}) = 5.0 \times 10^{-9}\,\text{C}$. When connected with the second capacitor, the charge on the first one reduces to $q_1' = C_1V_1' = (100\,\text{pF})(35\,\text{V}) = 3.5 \times 10^{-9}\,\text{C}$, which means that the second one now has a charge of $q_2 = q_1 - q_1' = 1.5 \times 10^{-9}\,\text{C}$. Thus its capacitance is $C_2 = q_2/V_2 = 1.5 \times 10^{-9}\,\text{C}/35\,\text{V} = 43\,\text{pF}$.

28P

(*a*) Firstly, the equivalent capacitance of C_2 and the the other 4.0-μF capacitor connected in series with it is given by $4.0\,\mu\text{F}/2 = 2.0\,\mu\text{F}$. This combination is then connected in parallel with each of the two 2.0-μF capacitors, resulting in an equivalent capacitance $C = 2.0\,\mu\text{F} + 2.0\,\mu\text{F} + 2.0\,\mu\text{F} = 6.0\,\mu\text{F}$. This four-capacitor combination is then connected in series with another combination, which consists of the two 3.0-μF capacitors connected in parallel, whose equivalent capacitance in turn is $C' = 3.0\,\mu\text{F} + 3.0\,\mu\text{F} = 6.0\,\mu\text{F}$. So finally the equivalent capacitance of the entire circuit is given by

$$C_{\text{eq}} = \frac{CC'}{C + C'} = \frac{(6.0\,\mu\text{F})(6.0\,\mu\text{F})}{6.0\,\mu\text{F} + 6.0\,\mu\text{F}} = 3.0\,\mu\text{F}\,.$$

(*b*) Let $V = 20\,\text{V}$ be the potential difference supplied by the battery. Then $q = C_{\text{eq}}V = (3.0\,\mu\text{F})(20\,\text{V}) = 6.0 \times 10^{-5}\,\text{C}$.
(*c*) The potential difference across C_1 is given by

$$V_1 = \frac{CV}{C + C'} = \frac{(6.0\,\mu\text{F})(20\,\text{V})}{6.0\,\mu\text{F} + 6.0\,\mu\text{F}} = 10\,\text{V}\,,$$

and the charge carried by C_1 is $q_1 = C_1V_1 = (3.0\,\mu\text{F})(10\,\text{V}) = 3.0 \times 10^{-5}\,\text{C}$.
(*d*) The potential difference across C_2 is given by $V_2 = V - V_1 = 20\,\text{V} - 10\,\text{V} = 10\,\text{V}$. Thus the charge carried by C_2 is $q_2 = C_2V_2 = (2.0\,\mu\text{F})(10\,\text{V}) = 2.0 \times 10^{-5}\,\text{C}$.
(*e*) Since this voltage difference V_2 is divided equally between C_3 and the other 4.0-μF capacitors connected in series with it, the voltage differenc across C_3 is given by $V_3 = V_2/2 = 10\,\text{V}/2 = 5.0\,\text{V}$. Thus $q_3 = C_3V_3 = (4.0\,\mu\text{F})(5.0\,\text{V}) = 2.0 \times 10^{-5}\,\text{C}$.

29P

(*a*) After the switches are closed the potential differences across the capacitors are the same and the two capacitors are in parallel. The potential difference from a to b is given by $V_{ab} = Q/C_{\text{eq}}$, where Q is the net charge on the combination and C_{eq} is the equivalent capacitance.

The equivalent capacitance is $C_{\text{eq}} = C_1 + C_2 = 4.0 \times 10^{-6}\,\text{F}$. The total charge on the combination is the net charge on either pair of connected plates. The charge on capacitor 1 is

$$q_1 = C_1 V = (1.0 \times 10^{-6}\,\text{F})(100\,\text{V}) = 1.0 \times 10^{-4}\,\text{C}$$

and the charge on capacitor 2 is

$$q_2 = C_2 V = (3.0 \times 10^{-6}\,\text{F})(100\,\text{V}) = 3.0 \times 10^{-4}\,\text{C}\,,$$

so the net charge on the combination is $3.0 \times 10^{-4}\,\text{C} - 1.0 \times 10^{-4}\,\text{C} = 2.0 \times 10^{-4}\,\text{C}$. The potential difference is

$$V_{ab} = \frac{2.0 \times 10^{-4}\,\text{C}}{4.0 \times 10^{-6}\,\text{F}} = 50\,\text{V}\,.$$

(*b*) The charge on capacitor 1 is now $q_1 = C_1 V_{ab} = (1.0 \times 10^{-6}\,\text{F})(50\,\text{V}) = 5.0 \times 10^{-5}\,\text{C}$.

(*c*) The charge on capacitor 2 is now $q_2 = C_2 V_{ab} = (3.0 \times 10^{-6}\,\text{F})(50\,\text{V}) = 1.5 \times 10^{-4}\,\text{C}$.

30P

The charges on capacitors 2 and 3 are the same, so these capacitors may be replaced by an equivalent capacitance determined from

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{C_2 + C_3}{C_2 C_3}\,.$$

Thus $C_{\text{eq}} = C_2 C_3/(C_2 + C_3)$. The charge on the equivalent capacitor is the same as the charge on either of the two capacitors in the combination and the potential difference across the equivalent capacitor is given by q_2/C_{eq}. The potential difference across capacitor 1 is q_1/C_1, where q_1 is the charge on this capacitor.

The potential difference across the combination of capacitors 2 and 3 must be the same as the potential difference across capacitor 1, so $q_1/C_1 = q_2/C_{\text{eq}}$. Now some of the charge originally on capacitor 1 flows to the combination of 2 and 3. If q_0 is the original charge, conservation of charge yields $q_1 + q_2 = q_0 = C_1 V_0$, where V_0 is the original potential difference across capacitor 1.

Solve the two equations $q_1/C_1 = q_2/C_{\text{eq}}$ and $q_1 + q_2 = C_1 V_0$ for q_1 and q_2. The second equation yields

$$q_2 = C_1 V_0 - q_1$$

and, when this is substituted into the first, the result is

$$\frac{q_1}{C_1} = \frac{C_1 V_0 - q_1}{C_{\text{eq}}}\,.$$

Solve for q_1. You should get

$$q_1 = \frac{C_1^2 V_0}{C_{\text{eq}} + C_1} = \frac{C_1^2 V_0}{\dfrac{C_2 C_3}{C_2 + C_3} + C_1} = \frac{C_1^2 (C_2 + C_3) V_0}{C_1 C_2 + C_1 C_3 + C_2 C_3}\,.$$

The charges on capacitors 2 and 3 are

$$q_2 = q_3 = C_1V_0 - q_1 = C_1V_0 - \frac{C_1^2(C_2+C_3)V_0}{C_1C_2+C_1C_3+C_2C_3} = \frac{C_1C_2C_3V_0}{C_1C_2+C_1C_3+C_2C_3}\,.$$

31P

(*a*)

$$q_1 = q_3 = \frac{C_1C_3V}{C_1+C_3} = \frac{(1.0\,\mu\mathrm{F})(3.0\,\mu\mathrm{F})(12\,\mathrm{V})}{1.0\,\mu\mathrm{F}+3.0\,\mu\mathrm{F}} = 9.0\,\mu\mathrm{C}\,,$$

$$q_2 = q_4 = \frac{C_2C_4V}{C_2+C_4} = \frac{(2.0\,\mu\mathrm{F})(4.0\,\mu\mathrm{F})(12\,\mathrm{V})}{2.0\,\mu\mathrm{F}+4.0\,\mu\mathrm{F}} = 16\,\mu\mathrm{C}\,.$$

(*b*) Now the voltage difference V_1 across C_1 and C_2 is

$$V_1 = \frac{C_3+C_4}{C_1+C_2+C_3+C_4}V = \frac{(3.0\,\mu\mathrm{F}+4.0\,\mu\mathrm{F})(12\,\mathrm{V})}{1.0\,\mu\mathrm{F}+2.0\,\mu\mathrm{F}+3.0\,\mu\mathrm{F}+4.0\,\mu\mathrm{F}} = 8.4\,\mathrm{V}\,.$$

Thus $q_1 = C_1V_1 = (1.0\,\mu\mathrm{F})(8.4\,\mathrm{V}) = 8.4\,\mu\mathrm{C}$, $q_2 = C_2V_1 = (2.0\,\mu\mathrm{F})(8.4\,\mathrm{V}) = 17\,\mu\mathrm{C}$, $q_3 = C_3(V-V_1) = (3.0\,\mu\mathrm{F})(12\,\mathrm{V}-8.4\,\mathrm{V}) = 11\,\mu\mathrm{C}$, and $q_4 = C_4(V-V_1) = (4.0\,\mu\mathrm{F})(12\,\mathrm{V}-8.4\,\mathrm{V}) = 14\,\mu\mathrm{C}$.

32P

In the first case the two capacitors are effectively connected in series so the output potential difference is $V_{\mathrm{out}} = CV_{\mathrm{in}}/2C = V_{\mathrm{in}}/2 = 50.0\,\mathrm{V}$. In the second case the lower diode acts as a wire so $V_{\mathrm{out}} = 0$.

33E

Let $v = 1.00\,\mathrm{m}^3$. The energy stored is

$$\begin{aligned} U &= uv = \frac{1}{2}\epsilon_0E^2v \\ &= \frac{1}{2}(8.85\times10^{-12}\,\mathrm{C^2/N{\cdot}m^2})(150\,\mathrm{V/m})^2(1.00\,\mathrm{m}^3) = 9.96\times10^{-8}\,\mathrm{J}\,. \end{aligned}$$

34E

(*a*)

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(61.0\times10^{-3}\,\mathrm{F})(10.0\times10^3\,\mathrm{V})^2 = 3.05\times10^6\,\mathrm{J}\,.$$

(*b*)

$$U = (3.05\times10^6\,\mathrm{J})/(3.6\times10^6\,\mathrm{J/\,kW\cdot h}) = 0.847\,\mathrm{kW{\cdot}h}\,.$$

35E

The energy stored by a capacitor is given by $U = \frac{1}{2}CV^2$, where V is the potential difference across its plates. You must convert the given value of the energy to joules. Since a joule is a watt·second, simply multiply by $(10^3\,\mathrm{W/kW})(3600\,\mathrm{s/h})$ to obtain $10\,\mathrm{kW{\cdot}h} = 3.6 \times 10^7\,\mathrm{J}$. Thus

$$C = \frac{2U}{V^2} = \frac{2(3.6 \times 10^7\,\mathrm{J})}{(1000\,\mathrm{V})^2} = 72\,\mathrm{F}\,.$$

36E

(*a*)

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(130 \times 10^{-12}\,\mathrm{F})(56.0\,\mathrm{V})^2 = 2.04 \times 10^{-7}\,\mathrm{J}\,.$$

(*b*) No, because we don't know the volume of the space inside the capacitor where the electric field is present.

37E

Use $U = \frac{1}{2}CV^2$. As V is increased by ΔV the energy stored in the capacitor increases correspondingly from U to $U + \Delta U$: $U + \Delta U = \frac{1}{2}C(V + \Delta V)^2$. Thus $(1 + \Delta V/V)^2 = 1 + \Delta U/U$, or

$$\frac{\Delta V}{V} = \sqrt{1 + \frac{\Delta U}{U}} - 1 = \sqrt{1 + 10\%} - 1 = 4.9\%\,.$$

38E

(*a*)

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\,\mathrm{C^2/N{\cdot}m^2})(40 \times 10^{-4}\,\mathrm{m^2})}{1.0 \times 10^{-3}\,\mathrm{m}} = 3.5 \times 10^{-11}\,\mathrm{F} = 35\,\mathrm{pF}\,.$$

(*b*) $q = CV = (35\,\mathrm{pF})(600\,\mathrm{V}) = 2.1 \times 10^{-8}\,\mathrm{C} = 21\,\mathrm{nC}$.
(*c*) $U = \frac{1}{2}CV^2 = \frac{1}{2}(35\,\mathrm{pF})(21\,\mathrm{nC})^2 = 6.3 \times 10^{-6}\,\mathrm{J} = 6.3\,\mu\mathrm{J}$.
(*d*) $E = V/d = 600\,\mathrm{V}/1.0 \times 10^{-3}\,\mathrm{m} = 6.0 \times 10^5\,\mathrm{V/m}$.
(*e*)

$$u = \frac{U}{v} = \frac{6.3 \times 10^{-6}\,\mathrm{J}}{(40 \times 10^{-4}\,\mathrm{m^2})(1.0 \times 10^{-3}\,\mathrm{m})} = 1.6\,\mathrm{J/m^3}\,.$$

39E

The total energy is the sum of the energies stored in the individual capacitors. Since they are connected in parallel the potential difference V across the capacitors is the same and the total energy is

$$U = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}(2.0 \times 10^{-6}\,\mathrm{F} + 4.0 \times 10^{-6}\,\mathrm{F})(300\,\mathrm{V})^2 = 0.27\,\mathrm{J}\,.$$

40E

(a)

$$u=\frac{1}{2}\epsilon_0E^2=\frac{1}{2}\epsilon_0\left(\frac{e}{4\pi\epsilon_0r^2}\right)^2=\frac{e^2}{32\pi^2\epsilon_0r^4}\,.$$

(b) From the expression above $u\propto r^{-4}$. So for $r\to 0$ $u\to\infty$.

41P

Use $E=q/4\pi\epsilon_0R^2=V/R$. Thus

$$u=\frac{1}{2}\epsilon_0E^2=\frac{1}{2}\epsilon_0\left(\frac{V}{R}\right)^2=\frac{1}{2}(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})\left(\frac{8000\,\mathrm{V}}{0.050\,\mathrm{m}}\right)^2=0.11\,\mathrm{J/m^3}.$$

42P

The total energy stored in the capacitor bank is

$$U=\frac{1}{2}C_{\mathrm{total}}V^2=\frac{1}{2}(2000)(5.00\times10^{-6}\,\mathrm{F})(50,000\,\mathrm{V})^2=1.3\times10^7\,\mathrm{J}\,.$$

Thus the cost is

$$\frac{(1.3\times10^7\,\mathrm{J})(3.0\,\mathrm{cent/\,kW\cdot h})}{3.6\times10^6\,\mathrm{J/\,kW\cdot h}}=10\,\mathrm{cents}\,.$$

43P

(a) In the first case $U=q^2/2C$; and in the second case $U=2(q/2)^2/2C=q^2/4C$. So the energy is now $4.0\,\mathrm{J}/2=2.0\,\mathrm{J}$.

(b) It becomes the thermal energy generated in the wire connecting the capacitors during the discharging process.

44P

(a)

$$U_1=\frac{q^2}{2C_1}=\frac{(4.8\times10^{-4}\,\mathrm{C})^2}{2(2.0\times10^{-6}\,\mathrm{F})}=5.8\times10^{-2}\,\mathrm{J}\,,$$

$$U_2=\frac{q^2}{2C_2}=\frac{(4.8\times10^{-4}\,\mathrm{C})^2}{2(8.0\times10^{-6}\,\mathrm{F})}=1.4\times10^{-2}\,\mathrm{J}\,.$$

(b)

$$U_1'=\frac{q_1'^2}{2C_1}=\frac{(1.9\times10^{-4}\,\mathrm{C})^2}{2(2.0\times10^{-6}\,\mathrm{F})}=9.2\times10^{-3}\,\mathrm{J}\,,$$

$$U_2'=\frac{q_2'^2}{2C_2}=\frac{(7.7\times10^{-4}\,\mathrm{C})^2}{2(8.0\times10^{-6}\,\mathrm{F})}=3.7\times10^{-2}\,\mathrm{J}\,.$$

(c) $U_1''=U_2''=0$.

Note that $U_1 + U_2 > U_1' + U_2' > U_1'' + U_2''$, since the system partially discharges as it goes from situation (a) to (b), and completely discharges in (c).

45P

(a) and (b). The voltage difference across C_1 and C_2 is given by

$$V_1 = V_2 = \frac{C_3 V}{C_1 + C_2 + C_3} = \frac{(4.00\,\mu\text{F})(100\,\text{V})}{10.0\,\mu\text{F} + 5.00\,\mu\text{F} + 4.00\,\mu\text{F}} = 21.1\,\text{V}\,.$$

Also, $V_3 = V - V_1 = V - V_2 = 100\,\text{V} - 21.1\,\text{V} = 78.9\,\text{V}$. Thus $q_1 = C_1 V_1 = (10.0\,\mu\text{F})(21.1\,\text{V}) = 2.11 \times 10^{-4}\,\text{C}$, $q_2 = C_2 V_2 = (5.00\,\mu\text{F})(21.1\,\text{V}) = 1.05 \times 10^{-4}\,\text{C}$, and $q_3 = q_1 + q_2 = 2.11 \times 10^{-4}\,\text{C} + 1.05 \times 10^{-4}\,\text{C} = 3.16 \times 10^{-4}\,\text{C}$.

(c) $U_1 = \frac{1}{2}C_1 V_1^2 = \frac{1}{2}(10.0\,\mu\text{F})(21.1\,\text{V})^2 = 2.22 \times 10^{-3}\,\text{J}$, $U_2 = \frac{1}{2}C_2 V_2^2 = \frac{1}{2}(5.00\,\mu\text{F}) \times (21.1\,\text{V})^2 = 1.11 \times 10^{-3}\,\text{J}$, and $U_3 = \frac{1}{2}C_3 V_3^2 = \frac{1}{2}(4.00\,\mu\text{F})(78.9\,\text{V})^2 = 1.25 \times 10^{-2}\,\text{J}$.

46P

(a) Let q be the charge on the positive plate. Since the capacitance of a parallel-plate capacitor is given by $\epsilon_0 A/d$, the charge is $q = CV = \epsilon_0 AV/d$. After the plates are pulled apart their separation is $2d$ and the potential difference is V'. Then $q = \epsilon_0 AV'/2d$ and

$$V' = \frac{2d}{\epsilon_0 A}\, q = \frac{2d}{\epsilon_0 A}\, \frac{\epsilon_0 A}{d}\, V = 2V\,.$$

(b) The initial energy stored in the capacitor is

$$U_i = \frac{1}{2} CV^2 = \frac{\epsilon_0 AV^2}{2d}$$

and the final energy stored is

$$U_f = \frac{1}{2}\,\frac{\epsilon_0 A}{2d}\,(V')^2 = \frac{1}{2}\,\frac{\epsilon_0 A}{2d}\,4V^2 = \frac{\epsilon_0 AV^2}{d}\,.$$

This is twice the initial energy.

(c) The work done to pull the plates apart is the difference in the energy: $W = U_f - U_i = \epsilon_0 AV^2/2d$.

47P

(a)

$$q_3 = C_3 V = (4.00\,\mu\text{F})(100\,\text{V}) = 4.00 \times 10^{-4}\,\text{mC}\,,$$

$$q_1 = q_2 = \frac{C_1 C_2 V}{C_1 + C_2} = \frac{(10.0\,\mu\text{F})(5.00\,\mu\text{F})(100\,\text{V})}{10.0\,\mu\text{F} + 5.00\,\mu\text{F}} = 3.33 \times 10^{-4}\,\text{C}\,.$$

(b) $V_1 = q_1/C_1 = 3.33 \times 10^{-4}\,\mathrm{C}/10.0\,\mu\mathrm{F} = 33.3\,\mathrm{V}$, $V_2 = V - V_1 = 100\,\mathrm{V} - 33.3\,\mathrm{V} = 66.7\,\mathrm{V}$, and $V_3 = V = 100\,\mathrm{V}$.
(c) Use $U_i = \frac{1}{2}C_iV_i^2$, where $i = 1, 2, 3$. The answers are $U_1 = 5.6\,\mathrm{mJ}$, $U_1 = 11\,\mathrm{mJ}$, and $U_1 = 20\,\mathrm{mJ}$.

48P

You first need to find an expression for the energy stored in a cylinder of radius R and length L, whose surface lies between the inner and outer cylinders of the capacitor ($a < R < b$). The energy density at any point is given by $u = \frac{1}{2}\epsilon_0 E^2$, where E is the magnitude of the electric field at that point. If q is the charge on the surface of the inner cylinder then the magnitude of the electric field at a point a distance r from the cylinder axis is given by

$$E = \frac{q}{2\pi\epsilon_0 Lr}$$

(see Eq. 26-12) and the energy density at that point is given by

$$u = \frac{1}{2}\epsilon_0 E^2 = \frac{q^2}{8\pi^2\epsilon_0 L^2 r^2}\,.$$

The energy in the cylinder is the volume integral

$$U_R = \int u\,dV\,.$$

Now $dV = 2\pi rL\,dr$, so

$$U_R = \int_a^R \frac{q^2}{8\pi^2\epsilon_0 L^2 r^2}\,2\pi r\,dr = \frac{q^2}{4\pi\epsilon_0 L^2}\int_a^R \frac{dr}{r} = \frac{q^2}{4\pi\epsilon_0 L^2}\ln\frac{R}{a}\,.$$

To find an expression for the total energy stored in the capacitor, replace R with b:

$$U_b = \frac{q^2}{4\pi\epsilon_0 L^2}\ln\frac{b}{a}\,.$$

You want the ratio U_R/U_b to be $1/2$, so

$$\ln\frac{R}{a} = \frac{1}{2}\ln\frac{b}{a}$$

or, since $\frac{1}{2}\ln(b/a) = \ln(\sqrt{b/a})$, $\ln(R/a) = \ln(\sqrt{b/a})$. This means $R/a = \sqrt{b/a}$ or $R = \sqrt{ab}$.

49P

The charge is held constant while the plates are being separated, so write the expression for the stored energy as $U = q^2/2C$, where q is the charge and C is the capacitance. The capacitance of a parallel-plate capacitor is given by $C = \epsilon_0 A/x$, where A is the plate area and x is the plate separation, so

$$U = \frac{q^2 x}{2\epsilon_0 A}\,.$$

If the plate separation increases by dx the energy increases by $dU = (q^2/2\epsilon_0 A)dx$. Suppose the agent pulling the plate apart exerts force F. Then the agent does work $F dx$ and if the plates begin and end at rest this must equal the increase in stored energy. Thus

$$F dx = \left(\frac{q^2}{2\epsilon_0 A}\right) dx$$

and

$$F = \frac{q^2}{2\epsilon_0 A}\,.$$

The net force on a plate is zero so this must also be the magnitude of the force one plate exerts on the other.

The force can also be computed as the product of the charge q on one plate and the electric field E_1 due to the charge on the other plate. Recall that the field produced by a uniform plane surface of charge is $E_1 = q/2\epsilon_0 A$. Thus $F = q^2/2\epsilon_0 A$.

50P

According to the result of 49P the force on either capacitor plate is given by $F = q^2/2\epsilon_0 A$, where q is the charge on one plate and A is the area of a plate. The electric field between the plates is $E = q/\epsilon_0 A$, so $q = \epsilon_0 AE$ and

$$F = \frac{\epsilon_0^2 A^2 E^2}{2\epsilon_0 A} = \frac{1}{2}\epsilon_0 AE^2\,.$$

The force per unit area of plate is

$$\frac{F}{A} = \frac{1}{2}\epsilon_0 E^2\,.$$

Note that the field E that enters this equation is the total field, due to charges on both plates.

51P

According to the result of 50P the electrostatic force acting on a small area ΔA is $F_e = \frac{1}{2}\epsilon_0 E^2\,\Delta A$. The electric field at the surface is $E = q/4\pi\epsilon_0 R^2$, where q is the charge on the bubble. Thus

$$F_e = \frac{1}{2}\epsilon_0 \left(\frac{q}{4\pi\epsilon_0 R^2}\right)^2 \Delta A = \frac{q^2\,\Delta A}{32\pi^2\epsilon_0 R^4}\,.$$

This force is outward. The force of the gas inside is the product of the pressure inside and the area: $F_g = p(V_0/V)\,\Delta A$. Since $V_0 = (4\pi/3)R_0^3$ and $V = (4\pi/3)R^3$,

$$F_g = p\left(\frac{R_0^3}{R^3}\right)\Delta A\,.$$

This force is outward. The force of the air outside is $F_a = p\,\Delta A$. This force is inward. Since the bubble surface is in equilibrium, the sum of the forces must vanish: $F_e + F_g - F_a = 0$. This means

$$\frac{q^2}{32\pi^2\epsilon_0 R^4} + p\frac{R_0^3}{R^3} - p = 0\,.$$

Solve for q^2. You should get

$$q^2 = 32\pi^2\epsilon_0 R^4 p\left(1 - \frac{R_0^3}{R^3}\right) = 32\pi^2\epsilon_0 pR(R^3 - R_0^3)\,.$$

52E

If the original capacitance is given by $C = \epsilon_0 A/d$, then the new capacitance is $C' = \epsilon_0\kappa A/2d$. Thus $C'/C = \kappa/2$ or $\kappa = 2C'/C = 2(2.6\,\text{pF}/1.3\,\text{pF}) = 4.0$.

53E

The capacitance with the dielectric in place is given by $C = \kappa C_0$, where C_0 is the capacitance before the dielectric is inserted. The energy stored is given by $U = \frac{1}{2}CV^2 = \frac{1}{2}\kappa C_0 V^2$, so

$$\kappa = \frac{2U}{C_0V^2} = \frac{2(7.4\times 10^{-6}\,\text{J})}{(7.4\times 10^{-12}\,\text{F})(652\,\text{V})^2} = 4.7\,.$$

According to Table 26–1 you should use pyrex.

54E

Use $C = \epsilon_0\kappa A/d \propto \kappa/d$. To maximize C we need to choose the material with the greatest value of κ/d. It follows that the mica sheet should be chosen.

55E

(*a*) Use $C = \epsilon_0 A/d$ to solve for d:

$$d = \frac{\epsilon_0 A}{C} = \frac{(8.85\times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(0.35\,\text{m}^2)}{50\times 10^{-12}\,\text{F}} = 6.2\times 10^{-2}\,\text{m}\,.$$

(*b*) Use $C \propto \kappa$. The new capacitance is $C' = C(\kappa/\kappa_{\text{air}}) = (50\,\text{pf})(5.6/1.0) = 280\,\text{pF}$.

56E

The capacitance of a cylindrical capacitor is given by

$$C = \kappa C_0 = \frac{2\pi\kappa\epsilon_0 L}{\ln(b/a)},$$

where C_0 is the capacitance without the dielectric, κ is the dielectric constant, L is the length, a is the inner radius, and b is the outer radius. The capacitance per unit length of the cable is

$$\frac{C}{L} = \frac{2\pi\kappa\epsilon_0}{\ln(b/a)} = \frac{2\pi(2.6)(8.85\times10^{-12}\,\mathrm{F/m})}{\ln[(0.60\,\mathrm{mm})/(0.10\,\mathrm{mm})]} = 8.1\times10^{-11}\ \mathrm{F/m} = 81\,\mathrm{pF/m}\,.$$

57P

The capacitance is given by $C = \kappa C_0 = \kappa\epsilon_0 A/d$, where C_0 is the capacitance without the dielectric, κ is the dielectric constant, A is the plate area, and d is the plate separation. The electric field between the plates is given by $E = V/d$, where V is the potential difference between the plates. Thus $d = V/E$ and $C = \kappa\epsilon_0 AE/V$. Solve for A:

$$A = \frac{CV}{\kappa\epsilon_0 E}\,.$$

For the area to be a minimum, the electric field must be the greatest it can be without breakdown occurring. That is,

$$A = \frac{(7.0\times10^{-8}\,\mathrm{F})(4.0\times10^{3}\,\mathrm{V})}{2.8(8.85\times10^{-12}\,\mathrm{F/m})(18\times10^{6}\,\mathrm{V/m})} = 0.63\,\mathrm{m}^2\,.$$

58P

(a) Use Eq. 26-14:

$$C = 2\pi\epsilon_0\kappa\frac{L}{\ln(b/a)} = \frac{(4.7)(0.15\,\mathrm{m})}{2(8.99\times10^{9}\,\mathrm{N{\cdot}m^2/C^2})\ln(3.8\,\mathrm{cm}/3.6\,\mathrm{cm})} = 0.73\,\mathrm{nF}\,.$$

(b) The breakdown potential is $(14\,\mathrm{kV/mm})(3.8\,\mathrm{cm} - 3.6\,\mathrm{cm}) = 28\,\mathrm{kV}$.

59P

(a) Since $u = \frac{1}{2}\kappa\epsilon_0 E^2$, we select the material with the greatest value of $\kappa E^2_{\mathrm{max}}$, when E_{max} is its dielectric strength. Thus we choose strontium titanate, with the corresponding minimum volume

$$V_{\mathrm{min}} = \frac{U}{U_{\mathrm{max}}} = \frac{2U}{\kappa\epsilon_0 E^2_{\mathrm{max}}} = \frac{2(250\,\mathrm{kJ})}{(310)(8.85\times10^{-12}\,\mathrm{C^2/N{\cdot}m^2})(8\,\mathrm{kV/mm})^2} = 2.85\,\mathrm{m}^3\,.$$

(b) Solve κ' from $U = \frac{1}{2}\kappa'\epsilon_0 E_{\rm max}^2 V'_{\rm min}$:

$$\kappa' = \frac{2U}{\epsilon_0 V'_{\rm min} E_{\rm max}^2} = \frac{2(250\,{\rm kJ})}{(8.85\times10^{-12}\,{\rm C^2/N\cdot m^2})(0.0870\,{\rm m^3})(8\,{\rm kV/mm})^2} = 1.01\times10^4\,.$$

60P

(a) Label the capacitor whose dielectric constant is $\kappa+\Delta\kappa$ as capacitor 1, and the other one as capacitor 2. For parallel connection

$$C_{\rm eq} = C_1 + C_2 = \frac{\epsilon_0 A(\kappa+\Delta\kappa)}{d} + \frac{\epsilon_0 A(\kappa-\Delta\kappa)}{d} = \frac{2\epsilon_0 A\kappa}{d}\,.$$

(b) Since $C_1 > C_2$ we need to evaluate q_1, the charge on capacitor 1:

$$q_1 = \frac{C_1 Q}{C_1+C_2} = \frac{(\epsilon_0 A/d)(\kappa+\Delta\kappa)Q}{(\epsilon_0 A/d)(\kappa+\Delta\kappa)+(\epsilon_0 A/d)(\kappa-\Delta\kappa)} = \frac{1}{2}Q\left(1+\frac{\Delta\kappa}{\kappa}\right).$$

61P

(a) The length d is effectively shortened by b so $C' = \epsilon_0 A/(d-b)$.

(b)

$$\frac{U}{U'} = \frac{q^2/2C}{q^2/2C'} = \frac{C'}{C} = \frac{\epsilon_0 A/(d-b)}{\epsilon_0 A/d} = \frac{d}{d-b}\,.$$

(c) The work done is

$$W = \Delta U = U' - U = \frac{q^2}{2}\left(\frac{1}{C'}-\frac{1}{C}\right) = \frac{q^2}{2\epsilon_0 A}(d-b-d) = -\frac{q^2 b}{2\epsilon_0 A}\,.$$

Since $W<0$ the slab is sucked in.

62P

(a) $C' = \epsilon_0 A/(d-b)$, the same as part (a) in 61P.

(b)

$$\frac{U}{U'} = \frac{\frac{1}{2}CV^2}{\frac{1}{2}C'V^2} = \frac{C}{C'} = \frac{\epsilon_0 A/d}{\epsilon_0 A/(d-b)} = \frac{d-b}{d}\,.$$

(c) The work done is

$$W = \Delta U = U' - U = \frac{1}{2}(C'-C)V^2 = \frac{\epsilon_0 A}{2}\left(\frac{1}{d-b}-\frac{1}{d}\right)V^2 = \frac{\epsilon_0 AbV^2}{2d(d-b)}\,.$$

Since $W>0$ the slab must be pushed in.

63P

The capacitor can be viewed as two capacitors C_1 and C_2 in parallel, each with surface area $A/2$ and plate separation d, filled with dielectric materials with dielectric constans κ_1 and κ_2, respectively. Thus

$$C = C_1 + C_2 = \frac{\epsilon_0 (A/2)\kappa_1}{d} + \frac{\epsilon_0 (A/2)\kappa_2}{d} = \frac{\epsilon_0 A}{d}\left(\frac{\kappa_1 + \kappa_2}{2}\right).$$

64P

Assume there is charge q on one plate and charge $-q$ on the other. Calculate the electric field at points between the plates and use the result to find an expression for the potential difference V between the plates, in terms of q. The capacitance is $C = q/V$.

The electric field in the upper half of the region between the plates is

$$E_1 = \frac{q}{\kappa_1 \epsilon_0 A},$$

where A is the plate area. The electric field in the lower half is

$$E_2 = \frac{q}{\kappa_2 \epsilon_0 A}.$$

Take $d/2$ to be the thickness of each dielectric. Since the field is uniform in each region the potential difference between the plates is

$$V = \frac{E_1 d}{2} + \frac{E_2 d}{2} = \frac{qd}{2\epsilon_0 A}\left(\frac{1}{\kappa_1} + \frac{1}{\kappa_2}\right) = \frac{qd}{2\epsilon_0 A}\,\frac{\kappa_1 + \kappa_2}{\kappa_1 \kappa_2},$$

so

$$C = \frac{q}{V} = \frac{2\epsilon_0 A}{d}\,\frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}.$$

Notice that this expression is exactly the same as the expression for the equivalent capacitance of two capacitors in series, one with dielectric constant κ_1 and the other with dielectric constant κ_2. Each has plate area A and plate separation $d/2$. Also notice that if $\kappa_1 = \kappa_2$ the expression reduces to $C = \kappa_1 \epsilon_0 A/d$, the correct result for a parallel plate capacitor with plate area A, plate separation d, and dielectric constant κ_1.

65P

Let $C_1 = \epsilon_0 (A/2)\kappa_1/2d = \epsilon_0 A\kappa_1/4d$, $C_2 = \epsilon_0 (A/2)\kappa_2/d = \epsilon_0 A\kappa_2/2d$, and $C_3 = \epsilon_0 A\kappa_3/2d$. Note that C_2 and C_3 are effectively connected in series, while C_1 is effectively connected in parallel with the C_2-C_3 combination. Thus

$$\begin{aligned} C &= C_1 + \frac{C_2 C_3}{C_2 + C_3} = \frac{\epsilon_0 A\kappa_1}{4d} + \frac{(\epsilon_0 A/d)(\kappa_2/2)(\kappa_3/2)}{\kappa_2/2 + \kappa_3/2} \\ &= \frac{\epsilon_0 A}{4d}\left(\kappa_1 + \frac{2\kappa_2\kappa_3}{\kappa_2 + \kappa_3}\right). \end{aligned}$$

66E

(a) The electric field in the region between the plates is given by $E = V/d$, where V is the potential difference between the plates and d is the plate separation. The capacitance is given by $C = \kappa\epsilon_0 A/d$, where A is the plate area and κ is the dielectric constant, so $d = \kappa\epsilon_0 A/C$ and

$$E = \frac{VC}{\kappa\epsilon_0 A} = \frac{(50\,\text{V})(100\times 10^{-12}\,\text{F})}{5.4(8.85\times 10^{-12}\,\text{F/m})(100\times 10^{-4}\,\text{m}^2)} = 1.0\times 10^4\,\text{V/m}\,.$$

(b) The free charge on the plates is $q_f = CV = (100\times 10^{-12}\,\text{F})(50\,\text{V}) = 5.0\times 10^{-9}\,\text{C}$.

(c) The electric field is produced by both the free and induced charge. Since the field of a large uniform layer of charge is $q/2\epsilon_0 A$, the field between the plates is

$$E = \frac{q_f}{2\epsilon_0 A} + \frac{q_f}{2\epsilon_0 A} - \frac{q_i}{2\epsilon_0 A} - \frac{q_i}{2\epsilon_0 A}\,,$$

where the first term is due to the positive free charge on one plate, the second is due to the negative free charge on the other plate, the third is due to the positive induced charge on one dielectric surface, and the fourth is due to the negative induced charge on the other dielectric surface. Note that the field due to the induced charge is opposite the field due to the free charge, so they tend to cancel. The induced charge is therefore

$$\begin{aligned} q_i &= q_f - \epsilon_0 AE \\ &= 5.0\times 10^{-9}\,\text{C} - (8.85\times 10^{-12}\,\text{F/m})(100\times 10^{-4}\,\text{m}^2)(1.0\times 10^4\,\text{V/m}) \\ &= 4.1\times 10^{-9}\,\text{C} = 4.1\,\text{nC}\,. \end{aligned}$$

67E

(a) The electric field E_1 in the free space between the two plates is $E_1 = q/\epsilon_0 A$ while that inside the slab is $E_2 = E_1/\kappa = q/\kappa\epsilon_0 A$. Thus $V_0 = E_1(d-b) + E_2 b = (q/\epsilon_0 A)(d - b + b/\kappa)$, and the capacitance is

$$\begin{aligned} C &= \frac{q}{V_0} = \frac{\epsilon_0 A\kappa}{\kappa(d-b)+b} \\ &= \frac{(8.85\times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(115\times 10^{-4}\,\text{m}^2)(2.61)}{(2.61)(1.24-0.780)(10^{-2}\,\text{m}) + (0.780\times 10^{-2})} = 13.4\,\text{pF}\,. \end{aligned}$$

(b) $q = CV = (13.4\times 10^{-12}\,\text{F})(85.5\,\text{V}) = 1.15\,\text{nC}$.

(c)

$$E_1 = \frac{q}{\epsilon_0 A} = \frac{1.15\times 10^{-9}\,\text{C}}{(8.85\times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(115\times 10^{-4}\,\text{m}^2)} = 1.13\times 10^4\,\text{N/C}\,.$$

(d)

$$E_2 = \frac{E_1}{\kappa} = \frac{1.13\times 10^4\,\text{N/C}}{2.61} = 4.33\times 10^3\,\text{N/C}\,.$$

68P

(a) Use $C = C_0\kappa$, where C_0 is given by Eq. 26-17:

$$C = 4\pi\epsilon_0\kappa\frac{ab}{b-a}.$$

(b) $q = CV = 4\pi\epsilon_0\kappa Vab/(b-a)$.

(c)

$$q' = q\left(1-\frac{1}{\kappa}\right) = \frac{4\pi\epsilon_0(\kappa-1)Vab}{b-a}.$$

69P

(a) Apply Gauss's law with dielectric: $q/\epsilon_0 = \kappa EA$, and solve for κ:

$$\kappa = \frac{q}{\epsilon_0 EA} = \frac{8.9\times10^{-7}\,\mathrm{C}}{(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})(1.4\times10^{-6}\,\mathrm{V/m})(100\times10^{-4}\,\mathrm{m^2})} = 7.2\,.$$

(b) The charge induced is

$$q' = q\left(1-\frac{1}{\kappa}\right) = (8.9\times10^{-7}\,\mathrm{C})\left(1-\frac{1}{7.2}\right) = 7.7\times10^{-7}\,\mathrm{C}\,.$$

70P

(a)

$$C_0 = \frac{\epsilon_0 A}{d} = \frac{(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})(0.12\,\mathrm{m^2})}{1.2\times10^{-2}\,\mathrm{m}} = 89\,\mathrm{pF}\,.$$

(b) Use the result of Exercise 67, part (a):

$$C = \frac{\epsilon_0 A\kappa}{\kappa(d-b)+b} = \frac{(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})(0.12\,\mathrm{m^2})(4.8)}{(4.8)(1.2-0.40)(10^{-2}\,\mathrm{m})+(4.0\times10^{-3}\,\mathrm{m})} = 120\,\mathrm{pF}\,.$$

(c) Before the insertion: $q = C_0V(89\,\mathrm{pF})(120\,\mathrm{V}) = 11\,\mathrm{nC}$. Since the battery is disconnected, q will remain the same after the insertion of the slab.

(d) $E = q/\epsilon_0 A = 11\,\mathrm{nC}/(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})(0.12\,\mathrm{m^2}) = 10\,\mathrm{kV/m}$.

(e) $E' = E/\kappa = (10\,\mathrm{kV/m})/4.8 = 2.1\,\mathrm{kV/m}$.

(f) $V = E(d-b)+E'b = (10\,\mathrm{kV/m})(1.2-0.40)(10^{-2}\,\mathrm{m})+(2.1\,\mathrm{kV/m})(0.40\times10^{-3}\,\mathrm{m}) = 88\,\mathrm{V}$.

(g) The work done is

$$\begin{aligned} W_{\mathrm{ext}} = \Delta U &= \frac{q^2}{2}\left(\frac{1}{C}-\frac{1}{C_0}\right) \\ &= \frac{(11\times10^{-9}\,\mathrm{C})^2}{2}\left(\frac{1}{89\times10^{-12}\,\mathrm{F}}-\frac{1}{120\times10^{-12}\,\mathrm{F}}\right) = -1.7\times10^{-7}\,\mathrm{J}\,. \end{aligned}$$

71P

(a) Since $u = \frac{1}{2}\kappa\epsilon_0 E^2$, $E_{\text{slab}} = E_{\text{air}}/\kappa_{\text{slab}}$, and $U = uv$, the fraction of energy stored in the air gaps is

$$\begin{aligned}\text{frac} &= \frac{E_{\text{air}}^2 A(d-b)}{E_{\text{air}}^2 A(d-b) + \kappa_{\text{slab}} E_{\text{slab}}^2 Ab} = \frac{1}{1+\kappa_{\text{slab}}(E_{\text{slab}}/E_{\text{air}})^2[b/(d-b)]} \\ &= \frac{1}{1+(2.61)(1/2.61)^2[0.780/(1.24-0.780)]} = 0.606\,.\end{aligned}$$

(b) The fraction of energy stored in the slab is $1 - 0.606 = 0.394$.

72P

Assume the charge on one plate is $+q$ and the charge on the other plate is $-q$. Find an expression for the electric field in each region, in terms of q, then use the result to find an expression for the potential difference V between the plates. The capacitance is $C = q/V$.

The electric field in the dielectric is $E_d = q/\kappa\epsilon_0 A$, where κ is the dielectric constant and A is the plate area. Outside the dielectric (but still between the capacitor plates) the field is $E = q/\epsilon_0 A$. The field is uniform in each region so the potential difference across the plates is

$$V = E_d b + E(d-b) = \frac{qb}{\kappa\epsilon_0 A} + \frac{q(d-b)}{\epsilon_0 A} = \frac{q}{\epsilon_0 A}\,\frac{b+\kappa(d-b)}{\kappa}\,.$$

The capacitance is

$$C = \frac{q}{V} = \frac{\kappa\epsilon_0 A}{\kappa(d-b)+b} = \frac{\kappa\epsilon_0 A}{\kappa d - b(\kappa-1)}\,.$$

The result does not depend on where the dielectric is located between the plates; it might be touching one plate or it might have a vacuum gap on each side.

For the capacitor of Sample Problem 27–10, $\kappa = 2.61$, $A = 115\,\text{cm}^2 = 115\times10^{-4}\,\text{m}^2$, $d = 1.24\,\text{cm} = 1.24\times10^{-2}\,\text{m}$, and $b = 0.78\,\text{cm} = 0.78\times10^{-2}\,\text{m}$, so

$$\begin{aligned}C &= \frac{2.61(8.85\times10^{-12}\,\text{F/m})(115\times10^{-4}\,\text{m}^2)}{2.61(1.24\times10^{-2}\,\text{m} - 0.78\times10^{-2}\,\text{m}) + 0.78\times10^{-2}\,\text{m}} \\ &= 1.34\times10^{-11}\,\text{F} = 13.4\,\text{pF}\,,\end{aligned}$$

in agreement with the result found in the sample problem.

If $b = 0$ and $\kappa = 1$, then the expression derived above yields $C = \epsilon_0 A/d$, the correct expression for a parallel-plate capacitor with no dielectric. If $b = d$ then the derived expression yields $C = \kappa\epsilon_0 A/d$, the correct expression for a parallel-plate capacitor completely filled with a dielectric.