Chapter 27

Answer to Checkpoint Questions

1. 8 A, rightward

2. (a) – (c) rightward

3. (a) and (c) tie, then (b)

4. device 2

5. (a) and (b) tie, then (d), then (c)

Answer to Questions

1. a, b, and c tie, then d (zero)

2. (a), (b), and (c) tie, then (d)

3. (b), (a), (c)

4. increase

5. tie of A, B, and C, then tie of $A + B$ and $B + C$, then $A + B + C$

6. (a) – (d) top–bottom, front–back, left–right

7. (a) – (c) 1 and 2 tie, then 3

8. C, then A and B tie, then D

9. C, A, B

10. (a) and (c) tie, then (b)

11. (b), (a), (c)

12. lower (for fixed V, smaller R gives larger P)

13. (a) conductors: 1 and 4; semiconductors: 2 and 3; (b) 2 and 3; (c) all four

Solutions to Exercises & Problems

1E

The number of electrons is given by

$$n = \frac{q}{e} = \frac{it}{e} = \frac{(200 \times 10^{-6}\,\mathrm{A})(1\,\mathrm{s})}{1.60 \times 10^{-19}\,\mathrm{C}} = 1.25 \times 10^{15}\,.$$

2E

(a) The charge that passes through any cross section is the product of the current and time. Since 4.0 min $= (4.0\,\mathrm{min})(60\,\mathrm{s/min}) = 240\,\mathrm{s}$, $q = it = (5.0\,\mathrm{A})(240\,\mathrm{s}) = 1200\,\mathrm{C}$.

(b) The number of electrons N is given by $q = Ne$, where e is the magnitude of the charge on an electron. Thus $N = q/e = (1200\,\mathrm{C})/(1.60 \times 10^{-19}\,\mathrm{C}) = 7.5 \times 10^{21}$.

3P

Use $i = \sigma v l$:

$$\sigma = \frac{i}{vl} = \frac{100 \times 10^{-6}\,\mathrm{A}}{(30\,\mathrm{m/s})(50 \times 10^{-2}\,\mathrm{m})} = 6.7 \times 10^{-6}\,\mathrm{C/m^2}\,.$$

4P

Suppose the charge on the sphere increases by Δq in time Δt. Then in that time its potential increases by

$$\Delta V = \frac{\Delta q}{4\pi\epsilon_0 r}\,,$$

where r is the radius of the sphere. This means

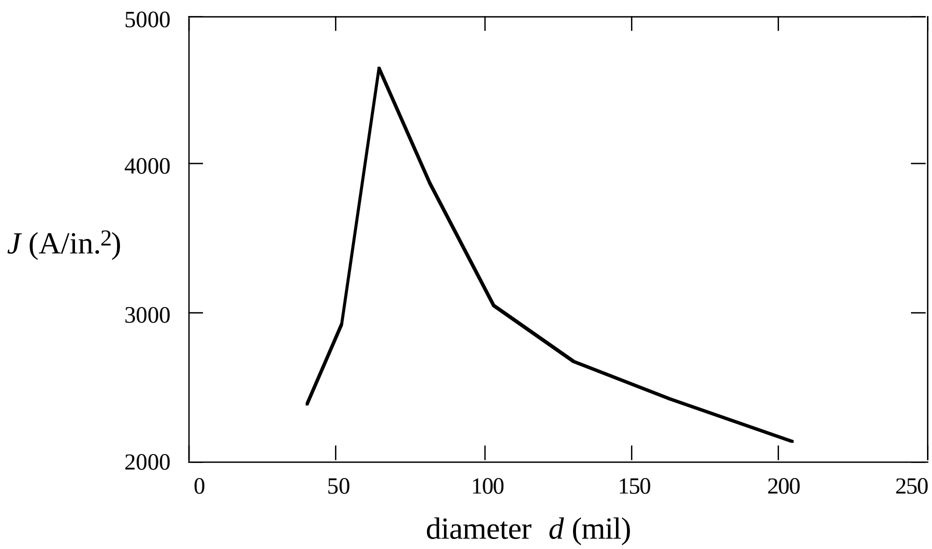
$$\Delta q = 4\pi\epsilon_0 r\,\Delta V\,.$$

Now $\Delta q = (i_{\mathrm{in}} - i_{\mathrm{out}})\,\Delta t$, where i_{in} is the current entering the sphere and i_{out} is the current leaving. Thus

$$\begin{aligned}\Delta t &= \frac{\Delta q}{i_{\mathrm{in}} - i_{\mathrm{out}}} = \frac{4\pi\epsilon_0 r\,\Delta V}{i_{\mathrm{in}} - i_{\mathrm{out}}}\\ &= \frac{(0.10\,\mathrm{m})(1000\,\mathrm{V})}{(8.99 \times 10^9\,\mathrm{F/m})(1.0000020\,\mathrm{A} - 1.0000000\,\mathrm{A})} = 5.6 \times 10^{-3}\,\mathrm{s}\,.\end{aligned}$$

5E

Use $J = i/A = 4i/\pi d^2 \propto i/d^2$, where i is the safe current and d is the diameter of the wire. You can easily verify that the 14-gauge wire has the highest value of i/d^2 so it has the maximum safe current density.



6E

(a) The magnitude of the current density is given by $J = nqv_d$, where n is the number of particles per unit volume, q is the charge on each particle, and v_d is the drift speed of the particles. The particle concentration is $n = 2.0 \times 10^8\,\text{cm}^{-3} = 2.0 \times 10^{14}\,\text{m}^{-3}$, the charge is $q = 2e = 2(1.60 \times 10^{-19}\,\text{C}) = 3.20 \times 10^{-19}\,\text{C}$, and the drift speed is $1.0 \times 10^5\,\text{m/s}$. Thus

$$J = (2 \times 10^{14}\,\text{m}^{-3})(3.2 \times 10^{-19}\,\text{C})(1.0 \times 10^5\,\text{m/s}) = 6.4\,\text{A/m}^2\,.$$

Since the particles are positively charged the current density is in the same direction as their motion, to the north.

(b) The current cannot be calculated unless the cross-sectional area of the beam is known. Then $i = JA$ can be used.

7E

(a)

$$J = \frac{i}{A} = \frac{i}{\pi d^2/4} = \frac{4(1.2 \times 10^{-10}\,\text{A})}{\pi(2.5 \times 10^{-3}\,\text{m})^2} = 2.4 \times 10^{-5}\,\text{A/m}^2\,.$$

(b)

$$v_d = \frac{J}{ne} = \frac{2.4 \times 10^{-5}\,\text{A/m}^2}{(8.47 \times 10^{28}/\,\text{m}^3)(1.60 \times 10^{-19}\,\text{C})} = 1.8 \times 10^{-15}\,\text{m/s}\,.$$

8E

The cross-sectional area of wire is given by $A = \pi r^2$, where r is its radius. The magnitude of the current density is $J = i/A = i/\pi r^2$, so

$$r = \sqrt{\frac{i}{\pi J}} = \sqrt{\frac{0.50\,\mathrm{A}}{\pi(440 \times 10^4\,\mathrm{A/m^2})}} = 1.9 \times 10^{-4}\,\mathrm{m}\,.$$

The diameter is $d = 2r = 2(1.9 \times 10^{-4}\,\mathrm{m}) = 3.8 \times 10^{-4}\,\mathrm{m}$.

9E

The magnitude of the current is

$$i = (n_p + n_e)e = (1.1 \times 10^{18}/\mathrm{s} + 3.1 \times 10^{18}/\mathrm{s})(1.60 \times 10^{-19}\,\mathrm{C}) = 0.67\,\mathrm{A}\,.$$

The current flows toward the negative terminal.

10E

(a) $i = (n_h + n_e)e = (2.25 \times 10^{15}/\mathrm{s} + 3.50 \times 10^{15}/\mathrm{s})(1.60 \times 10^{-19}\,\mathrm{C}) = 9.20 \times 10^{-4}\,\mathrm{A}$.
(b) $J = i/A = 9.20 \times 10^{-4}\,\mathrm{A}/[\pi(0.165 \times 10^{-3}\,\mathrm{m})^2] = 1.08 \times 10^4\,\mathrm{A/m^2}$.

11P

(a) $J = n_p e v_p = (8.70/10^{-6}\,\mathrm{m^3})(1.60 \times 10^{-19}\,\mathrm{C})(470 \times 10^5\,\mathrm{m/s}) = 6.54 \times 10^{-7}\,\mathrm{A/m^2}$.
(b) $i = JA = \pi R_e^2 J = \pi(6.37 \times 10^6\,\mathrm{m})^2(6.54 \times 10^{-7}\,\mathrm{A/m^2}) = 8.34 \times 10^7\,\mathrm{A}$.

12P

(a) The charge that strikes the surface in time Δt is given by $\Delta q = i\,\Delta t$, where i is the current. Since each particle carries charge $2e$, the number of particles that strike the surface is

$$N = \frac{\Delta q}{2e} = \frac{i\,\Delta t}{2e} = \frac{(0.25 \times 10^{-6}\,\mathrm{A})(3.0\,\mathrm{s})}{2(1.6 \times 10^{-19}\,\mathrm{C})} = 2.3 \times 10^{12}\,.$$

(b) Now let N be the number of particles in a length L of the beam. They will all pass through the beam cross section at one end in time $t = L/v$, where v is the particle speed. The current is the charge that moves through the cross section per unit time. That is, $i = 2eN/t = 2eNv/L$. Thus $N = iL/2ev$.

Now find the particle speed. The kinetic energy of a particle is

$$K = 20\,\mathrm{MeV} = (20 \times 10^6\,\mathrm{eV})(1.60 \times 10^{-19}\,\mathrm{J/eV}) = 3.2 \times 10^{-12}\,\mathrm{J}\,.$$

Since $K = \frac{1}{2}mv^2$, $v = \sqrt{2K/m}$. The mass of an alpha particle is 4 times the mass of a proton or $m = 4(1.67 \times 10^{-27}\,\mathrm{kg}) = 6.68 \times 10^{-27}\,\mathrm{kg}$, so

$$v = \sqrt{\frac{2(3.2 \times 10^{-12}\,\mathrm{J})}{6.68 \times 10^{-27}\,\mathrm{kg}}} = 3.1 \times 10^7\,\mathrm{m/s}$$

and

$$N = \frac{iL}{2ev} = \frac{(0.25 \times 10^{-6})(20 \times 10^{-2}\,\mathrm{m})}{2(1.60 \times 10^{-19}\,\mathrm{C})(3.1 \times 10^{7}\,\mathrm{m/s})} = 5.0 \times 10^{3}\,.$$

(c) Use conservation of energy. The initial kinetic energy is zero, the final kinetic energy is $20\,\mathrm{MeV} = 3.2 \times 10^{-12}\,\mathrm{J}$, the initial potential energy is $qV = 2eV$, and the final potential energy is zero. Here V is the electric potential through which the particles are accelerated. Conservation of energy leads to $K_f = U_i = 2eV$, so

$$V = \frac{K_f}{2e} = \frac{3.2 \times 10^{-12}\,\mathrm{J}}{2(1.60 \times 10^{-19}\,\mathrm{C})} = 10 \times 10^{6}\,\mathrm{V}\,.$$

13P

Use $v_d = J/ne = i/Ane$. Thus

$$\begin{aligned} t &= \frac{L}{v_d} = \frac{L}{i/Ane} = \frac{LAne}{i} \\ &= \frac{(0.85\,\mathrm{m})(0.21 \times 10^{-4}\,\mathrm{m}^2)(8.47 \times 10^{28}/\,\mathrm{m}^3)(1.60 \times 10^{-19}\,\mathrm{C})}{300\,\mathrm{A}} \\ &= 8.1 \times 10^{2}\,\mathrm{s} = 13\,\mathrm{min}\,. \end{aligned}$$

14P

The current density is the sum of the contribution from the electrons and the α particles:

$$\begin{aligned} J &= J_\alpha + J_e = q_\alpha n_\alpha v_\alpha + e n_e v_e = (2e) n_\alpha v_\alpha + e(2n_\alpha) v_e = 2e n_\alpha (v_\alpha + v_e) \\ &= 2(1.60 \times 10^{-19}\,\mathrm{C})(2.8 \times 10^{15}/\mathrm{cm}^3)(25\,\mathrm{m/s} + 88\,\mathrm{m/s}) \\ &= 10\,\mathrm{A/cm}^2\,. \end{aligned}$$

The direction of the current is to the east.

15P

(a)

$$i = \int_{\text{cylinder}} J\,dA = \int_0^R J_0\left(1 - \frac{r}{R}\right) 2\pi r dr = \frac{1}{3}\pi R^2 J_0 = \frac{1}{3} A J_0\,.$$

(b) Now

$$i = \int_{\text{cylinder}} J\,dA = \frac{J_0}{R}\int_0^R r \cdot 2\pi r dr = \frac{2}{3}\pi R^2 J_0 = \frac{2}{3} A J_0\,.$$

The result is different from that in part (a) because the current density in (b) is lower near the center of the cylinder (where the area is smaller for the same radial interval) and higher outward, resulting in a higher average current density over the cross section and consequently a greater current than that in part (*a*).

16E

$$R = \frac{\rho L}{A} = \frac{(3.00 \times 10^{-7}\,\Omega\cdot\text{m})(10.0 \times 10^3\,\text{m})}{56.0 \times 10^{-4}\,\text{m}^2} = 0.536\,\Omega\,.$$

17E

The resistance of the wire is given by $R = \rho L/A$, where ρ is the resistivity of the material, L is the length of the wire, and A is the cross-sectional area of the wire. The cross-sectional area is $A = \pi r^2 = \pi(0.50 \times 10^{-3}\,\text{m})^2 = 7.85 \times 10^{-7}\,\text{m}^2$. Here $r = 0.50\,\text{mm} = 0.50 \times 10^{-3}\,\text{m}$ is the radius of the wire. Thus

$$\rho = \frac{RA}{L} = \frac{(50 \times 10^{-3}\,\Omega)(7.85 \times 10^{-7}\,\text{m}^2)}{2.0\,\text{m}} = 2.0 \times 10^{-8}\,\Omega\cdot\text{m}\,.$$

18E

$$\sigma = \frac{1}{\rho} = \frac{L}{RA} = \frac{L}{(V/i)A} = \frac{Li}{VA} = \frac{(1.0\,\text{m})(4.0\,\text{A})}{(2.0\,\text{V})(1.0 \times 10^{-6}\,\text{m}^2)} = 2.0 \times 10^6(\Omega\cdot\text{m})^{-1}\,.$$

19E

Since the potential difference V and current i are related by $V = iR$, where R is the resistance of the electrician, the fatal voltage is $V = (50 \times 10^{-3}\,\text{A})(2000\,\Omega) = 100\,\text{V}$.

20E

The resistance of the coil is given by $R = \rho L/A$, where L is the length of the wire, ρ is the resistivity of copper, and A is the cross-sectional area of the wire. Since each turn of wire has length $2\pi r$, where r is the radius of the coil, $L = (250)2\pi r = (250)(2\pi)(0.12\,\text{m}) = 188.5\,\text{m}$. If r_w is the radius of the wire, its cross-sectional area is $A = \pi r_w^2 = \pi(0.65 \times 10^{-3}\,\text{m})^2 = 1.33 \times 10^{-6}\,\text{m}^2$. According to Table 27–1, the resistivity of copper is $1.69 \times 10^{-8}\,\Omega\text{·m}$. Thus

$$R = \frac{\rho L}{A} = \frac{(1.69 \times 10^{-8}\,\Omega\cdot\text{m})(188.5\,\text{m})}{1.33 \times 10^{-6}\,\text{m}^2} = 2.4\,\Omega\,.$$

21E

(*a*) $i = V/R = 23.0\,\text{V}/15.0 \times 10^{-3}\,\Omega = 1.53 \times 10^3\,\text{A}$.
(*b*) $J = i/A = 4i/\pi d^2 = 4(1.53 \times 10^{-3}\,\text{A})/[\pi(6.00 \times 10^{-3}\,\text{m})^2] = 5.41 \times 10^7\,\text{A/m}^2$.
(*c*) $\rho = RA/L = (15.0 \times 10^{-3}\,\Omega)(\pi)(6.00 \times 10^{-3}\,\text{m})^2/[4(4.00\,\text{m})] = 10.6 \times 10^{-8}\,\Omega\cdot\text{m}$.

The material is platinum.

22E
Use Eq. 27-17: $\rho - \rho_0 = \rho\alpha(T - T_0)$, and solve for T:

$$T = T_0 + \frac{1}{\alpha}\left(\frac{\rho}{\rho_0} - 1\right) = 20°\,\mathrm{C} + \frac{1}{4.3 \times 10^{-3}/\mathrm{K}}\left(\frac{58\,\Omega}{50\,\Omega} - 1\right) = 57\,℃\,.$$

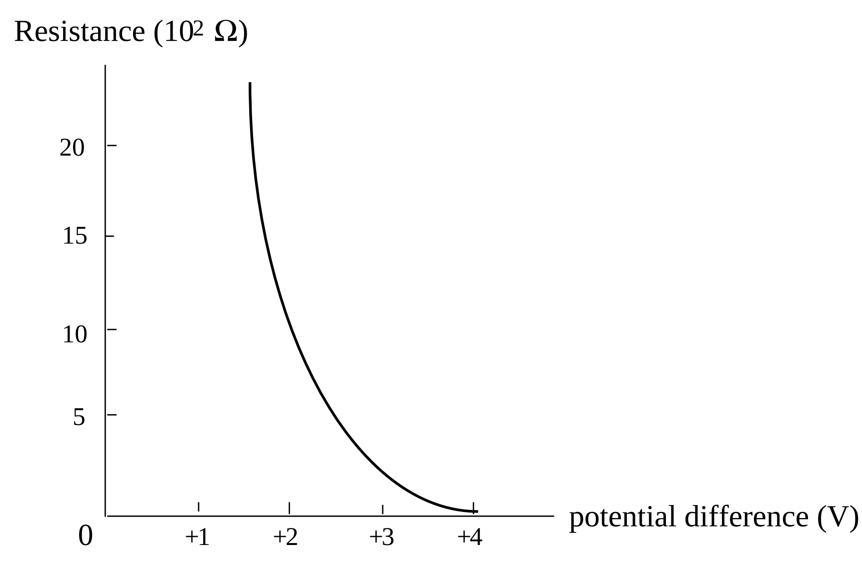
Here we noted that $\rho/\rho_0 = R/R_0$.

23E
In Eq. 27-17, let $\rho(T) = 2\rho_0$ where ρ_0 is the resistivity at 20 ℃: $\rho - \rho_0 = 2\rho_0 - \rho_0 = \rho_0\alpha(T - T_0)$, and solve for T:

$$T = T_0 + \frac{1}{\alpha} = 20\,℃ + \frac{1}{4.3 \times 10^{-3}/\mathrm{K}} = 2.5 \times 10^2\,℃\,.$$

This agrees well with Fig. 27-10, from which you can deduce $T = 5.2 \times 10^2$ K, or 2.5×10^2 ℃.

24E
Use $R = dV/di =$ local slope of the V vs i curve.



25E
(a)

$$V = iR = i\rho\frac{L}{A} = \frac{(12\,\mathrm{A})(1.69 \times 10^{-8}\,\Omega{\cdot}\mathrm{m})(4.0 \times 10^{-2}\,\mathrm{m})}{\pi(5.2 \times 10^{-3}\,\mathrm{m}/2)^2} = 3.8 \times 10^{-4}\,\mathrm{V}\,.$$

(b) Since it moves in the direction of the electron drift which is against the direction of the current, its tail is negative compared to its head.
(c) Use the result of 13P:

$$\begin{aligned} t &= \frac{L}{v_d} = \frac{lAne}{i} = \frac{\pi L d^2 ne}{4i} \\ &= \frac{\pi(1.0\times10^{-2}\,\mathrm{m})(5.2\times10^{-3}\,\mathrm{m})^2(8.47\times10^{28}/\mathrm{m}^3)(1.60\times10^{-19}\,\mathrm{C})}{4(12\,\mathrm{A})} \\ &= 238\,\mathrm{s} = 3\,\mathrm{min}\,58\,\mathrm{s}\,. \end{aligned}$$

26E
(a) The new area is $A' = AL/L' = A/2$.
(b) The new resistance is $R' = R(A/A')(L'/L) = 4R$.

27E
Since the mass and density of the material do not change, the volume remains the same. If L_0 is the original length, L is the new length, A_0 is the original cross-sectional area, and A is the new cross-sectional area, then $L_0A_0 = LA$ and $A = L_0A_0/L = L_0A_0/3L_0 = A_0/3$. The new resistance is

$$R = \frac{\rho L}{A} = \frac{\rho 3L_0}{A_0/3} = 9\frac{\rho L_0}{A_0} = 9R_0\,,$$

where R_0 is the original resistance. Thus $R = 9(6.0\,\Omega) = 54\,\Omega$.

28E
Use $R \propto L/A$ and note that $A = \frac{1}{4}\pi d^2 \propto d^2$. The resistance of the second wire is given by

$$R_2 = R\left(\frac{A_1}{A_2}\right)\left(\frac{L_2}{L_1}\right) = R = \left(\frac{d_1}{d_2}\right)^2\left(\frac{L_2}{L_1}\right) = R(2)^2(1/2) = 2R\,.$$

29P
The resistance of conductor A is given by

$$R_A = \frac{\rho L}{\pi r_A^2}\,,$$

where r_A is the radius of the conductor. If r_o is the outside diameter of conductor B and r_i is its inside diameter, then its cross-sectional area is $\pi(r_o^2 - r_i^2)$ and its resistance is

$$R_B = \frac{\rho L}{\pi(r_o^2 - r_i^2)}\,.$$

The ratio is

$$\frac{R_A}{R_B} = \frac{r_o^2 - r_i^2}{r_A^2} = \frac{(1.0\,\text{mm})^2 - (0.50\,\text{mm})^2}{(0.50\,\text{mm})^2} = 3.0\,.$$

30P

(*a*) Denote the copper wire with subscript c and the iron wire with subscript i. Let $i_c = V/R_c = i_i = V/R_i$, or $R_c = R_i$, we get $\rho_c L_c/\pi r_c^2 = \rho_i L_i/\pi r_i^2$, i.e.,

$$\frac{r_i}{r_c} = \sqrt{\frac{\rho_i}{\rho_c}} = \sqrt{\frac{9.68 \times 10^8}{1.69 \times 10^8}} = 2.39\,.$$

(*b*) If $J_i = i/A_i = J_c = i/A_c$ then $A_i = A_c$, or $r_i = r_c$, which is inconsistent with the result in (*a*). So this is impossible.

31P

Denote the copper wire with subscript c and the aluminum wire with subscript a.

(*a*)

$$R = \rho_a \frac{L}{A} = \frac{(2.75 \times 10^{-8}\,\Omega\!\cdot\!\text{m})(1.3\,\text{m})}{(5.2 \times 10^{-3}\,\text{m})^2} = 1.3 \times 10^{-3}\,\Omega\,.$$

(*b*) Let $R = \rho_c L/(\pi d^2/4)$ and solve for the diameter d of the copper wire:

$$d = \sqrt{\frac{4\rho_c L}{\pi R}} = \sqrt{\frac{4(1.69 \times 10^{-8}\,\Omega\!\cdot\!\text{m})(1.3\,\text{m})}{\pi(1.3 \times 10^{-3}\,\Omega)}} = 4.6 \times 10^{-3}\,\text{m}\,.$$

32P

(*a*) Since $\rho = RA/L = \pi R d^2/4L = \pi(1.09 \times 10^{-3}\,\Omega)(5.50 \times 10^{-3}\,\text{m})^2/[4(1.60\,\text{m})] = 1.62 \times 10^{-8}\,\Omega\!\cdot\!\text{m}$, the material is silver.

(*b*)

$$R = \rho\frac{L}{A} = \frac{4\rho L}{\pi d^2} = \frac{4(1.62 \times 10^{-8}\,\Omega\!\cdot\!\text{m})(1.00 \times 10^{-3}\,\text{m})}{\pi(2.00 \times 10^{-2}\,\text{m})^2} = 5.16 \times 10^{-8}\,\Omega\,.$$

33P

(*a*) The current in each strand is $i = 0.750\,\text{A}/125 = 6.00 \times 10^{-3}\,\text{A}$.

(*b*) The potential difference is $V = iR = (6.00 \times 10^{-3}\,\text{A})(2.65 \times 10^{-6}\,\Omega) = 1.59 \times 10^{-8}\,\text{V}$.

(*c*) The resistance is $R_{\text{total}} = 2.65 \times 10^{-6}\,\Omega/125 = 2.12 \times 10^{-8}\,\Omega$.

34P

Use $J = E/\rho$, where E is the magnitude of the electric field in the wire, J is the magnitude of the current density, and ρ is the resistivity of the material. The electric field is given by $E = V/L$, where V is the potential difference along the wire and L is the length of the wire. Thus $J = V/L\rho$ and

$$\rho = \frac{V}{LJ} = \frac{115\,\mathrm{V}}{(10\,\mathrm{m})(1.4\times10^{4}\,\mathrm{A/m^2})} = 8.2\times10^{-4}\,\Omega{\cdot}\mathrm{m}\,.$$

35P

The resistance at operating temperature T is $R = V/i = 2.9\,\mathrm{V}/0.30\,\mathrm{A} = 9.67\,\Omega$. Thus from $R - R_0 = R_0\alpha(T - T_0)$ we find

$$\begin{aligned} T &= T_0 + \frac{1}{\alpha}\left(\frac{R}{R_0} - 1\right) \\ &= 20\,^\circ\mathrm{C} + \left(\frac{1}{4.5\times10^{-3}/\mathrm{K}}\right)\left(\frac{9.67\,\Omega}{1.1\,\Omega} - 1\right) = 1.8\times10^{3}\,^\circ\mathrm{C}\,. \end{aligned}$$

36P

(*a*) $i = V/R = 35.8\,\mathrm{V}/935\,\Omega = 3.83\times10^{-2}\,\mathrm{A}$.
(*b*) $J = i/A = 3.83\times10^{-2}\,\mathrm{A}/(3.50\times10^{-4}\,\mathrm{m^2}) = 109\,\mathrm{A/m^2}$.
(*c*) $v_d = J/ne = (109\,\mathrm{A/m^2})/[(5.33\times10^{22}/\mathrm{m^3})(1.60\times10^{-19}\,\mathrm{C})] = 1.28\times10^{-2}\,\mathrm{m/s}$.
(*d*) $E = V/L = 35.8\,\mathrm{V}/0.158\,\mathrm{m} = 227\,\mathrm{V/m}$.

37P

Use $R/L = \rho/A = 0.150\,\Omega/\mathrm{km}$.
(*a*) For copper $J = i/A = (60.0\,\mathrm{A})(0.150\,\Omega/\mathrm{km})/(1.69\times10^{-8}\,\Omega{\cdot}\mathrm{m}) = 5.32\times10^{5}\,\mathrm{A/m^2}$; and for aluminum $J = (60.0\,\mathrm{A})(0.150\,\Omega/\mathrm{km})/(2.75\times10^{-8}\,\Omega{\cdot}\mathrm{m}) = 3.27\times10^{5}\,\mathrm{A/m^2}$.
(*b*) Denote the mass densities as ρ_m. For copper $(m/L)_c = (\rho_m A)_c = (8960\,\mathrm{kg/m^3})(1.69\times10^{-8}\,\Omega{\cdot}\mathrm{m})/(0.150\,\Omega/\mathrm{km}) = 1.01\,\mathrm{kg/m}$; and for aluminum $(m/L)_a = (\rho_m A)_a = (2700\,\mathrm{kg/m^3})(2.75\times10^{-8}\,\Omega{\cdot}\mathrm{m})/(0.150\,\Omega/\mathrm{km}) = 0.495\,\mathrm{kg/m}$.

38P

Use $J = \sigma E = (n_+ + n_-)ev_d$.
(*a*)

$$v_d = \frac{\sigma E}{(n_+ + n_-)e} = \frac{(2.70\times10^{-14}/\,\Omega{\cdot}\mathrm{m})(120\,\mathrm{V/m})}{[(620+550)/\mathrm{cm^3}](1.60\times10^{-19}\,\mathrm{C})} = 1.73\,\mathrm{cm/s}\,.$$

(*b*) $J = \sigma E = (2.70\times 10{-}14/\,\Omega{\cdot}\mathrm{m})(120\,\mathrm{V/m}) = 3.24\times10^{-12}\,\mathrm{A/m^2}$.

39P

Use $R \propto L/A$. The diameter of a 22-gauge wire is 1/4 that of a 10-gauge wire. Thus from $R = \rho L/A$ we find the resistance of 25 ft of 22-gauge copper wire to be $R = (1.00\,\Omega)(25\,\mathrm{ft}/1000\,\mathrm{ft})(4)^2 = 0.40\,\Omega$.

40P

(a) Let ΔT be the change in temperature and β be the coefficient of linear expansion for copper. Then $\Delta L = \beta L\,\Delta T$ and

$$\frac{\Delta L}{L} = \beta\,\Delta T = (1.7\times10^{-5}/\mathrm{C}^\circ)(1.0\,^\circ\mathrm{C}) = 1.7\times10^{-5}\,.$$

This is 0.0017%.

The fractional change in area is

$$\frac{\Delta A}{A} = 2\beta\,\Delta T = 2(1.7\times10^{-5}/\mathrm{C}^\circ)(1.0\,^\circ\mathrm{C}) = 3.4\times10^{-5}\,.$$

This is 0.0034%.

For small changes in the resistivity ρ, length L, and area A of a wire, the change in the resistance is given by

$$\Delta R = \frac{\partial R}{\partial \rho}\,\Delta\rho + \frac{\partial R}{\partial L}\,\Delta L + \frac{\partial R}{\partial A}\,\Delta A\,.$$

Since $R = \rho L/A$, $\partial R/\partial\rho = L/A = R/\rho$, $\partial R/\partial L = \rho/A = R/L$, and $\partial R/\partial A = -\rho L/A^2 = -R/A$. Furthermore, $\Delta\rho/\rho = \alpha\,\Delta T$, where α is the temperature coefficient of resistivity for copper ($4.3\times10^{-3}/\mathrm{C}^\circ$, according to Table 27–1). Thus

$$\begin{aligned}\frac{\Delta R}{R} &= \frac{\Delta\rho}{\rho} + \frac{\Delta L}{L} - \frac{\Delta A}{A} = (\alpha+\beta-2\beta)\,\Delta T = (\alpha-\beta)\,\Delta T\\ &= (4.3\times10^{-3}/\mathrm{C}^\circ - 1.7\times10^{-5}/\mathrm{C}^\circ)\,(1.0\,\mathrm{C}^\circ) = 4.3\times10^{-3}\,.\end{aligned}$$

This is 0.43%.

(b) The fractional change in resistivity is much larger than the fractional change in length and area. Changes in length and area affect the resistance much less than changes in resistivity.

41P

(a) Assume, as in Fig. 27-25, that the cone has current i, from bottom to top. The current is the same through every cross-section. We can find an expression for the electric field at every cross section, in terms of the current and then use this expression to find the potential difference V from bottom to top of the cone. The resistance of the cone is given by $R = V/i$.

Consider any cross section of the cone. Let J denote the current density at that cross section and assume it is uniform over the cross section. Then the current through the cross section is given by $i = \int J\,dA = \pi r^2 J$, where r is the radius of the cross section. Now $J = E/\rho$, where ρ is the resistivity and E is the magnitude of the electric field at the cross section. Thus $i = \pi r^2 E/\rho$ and $E = i\rho/\pi r^2$. The current density and electric field have different values on different cross sections because different cross sections have different radii.

Let x measure distance downward from the upper surface of the cone. The radius increases linearly with x, so we may write

$$r = a + \frac{b-a}{L}\,x\,.$$

The coefficients in this function have been chosen so $r = a$ when $x = 0$ and $r = b$ when $x = L$. Thus

$$E = \frac{i\rho}{\pi}\left(a + \frac{b-a}{L}\,x\right)^{-2}.$$

The potential difference between the upper and lower surfaces of the cone is given by

$$\begin{aligned} V &= -\int_0^L E\,dx = -\frac{i\rho}{\pi}\int_0^L \left(a + \frac{b-a}{L}\,x\right)^{-2} dx \\ &= \frac{i\rho}{\pi}\,\frac{L}{b-a}\left(a + \frac{b-a}{L}\,x\right)^{-1}\Bigg|_0^L = \frac{i\rho}{\pi}\,\frac{L}{b-a}\left(\frac{1}{a} - \frac{1}{b}\right) \\ &= \frac{i\rho}{\pi}\,\frac{L}{b-a}\,\frac{b-a}{ab} = \frac{i\rho L}{\pi ab}\,. \end{aligned}$$

The resistance is

$$R = \frac{V}{i} = \frac{\rho L}{\pi ab}\,.$$

(b) If $b = a$ then $R = \rho L/\pi a^2 = \rho L/A$, where $A = \pi a^2$ is the cross-sectional area of the cylinder.

42P

From Eq. 27-20 $\rho \propto T^{-1} \propto \bar{v}$. But according to Chapter 20 $\bar{v} \propto \sqrt{T}$. Thus $\rho \propto \sqrt{T}$.

43E

Since $P = iV$, $q = it = Pt/V = (7.0\,\mathrm{W})(5.0\,\mathrm{h})(3600\,\mathrm{s/h})/9.0\,\mathrm{V} = 1.4 \times 10^4\,\mathrm{C}$.

44E

The power dissipated is given by the product of the current and the potential difference:

$$P = iV = (7.0 \times 10^{-3}\,\mathrm{A})(80 \times 10^3\,\mathrm{V}) = 560\,\mathrm{W}\,.$$

45E

$R = P/i^2 = 100\,\mathrm{W}/(3.00\,\mathrm{A})^2 = 11.1\,\Omega$.

46E

The horse power required is

$$P = \frac{iV}{80\%} = \frac{(10\,\mathrm{A})(12\,\mathrm{V})}{(80\%)(746\,\mathrm{W/hp})} = 0.20\,\mathrm{hp}\,.$$

47E

(a) Electrical energy is transferred to heat at a rate given by

$$P = \frac{V^2}{R}\,,$$

where V is the potential difference across the heater and R is the resistance of the heater. Thus

$$P = \frac{(120\,\mathrm{V})^2}{14\,\Omega} = 1.0 \times 10^3\ \mathrm{W} = 1.0\ \mathrm{kW}\,.$$

(b) The cost is given by

$$C = (1.0\,\mathrm{kW})(5.0\,\mathrm{h})(5.0\,\mathrm{cents/kW\cdot h}) = 25\ \mathrm{cents}\,.$$

48E

(a) $R = V^2/P = (120\,\mathrm{V})^2/500\,\mathrm{W} = 28.8\,\Omega$.
(b) The rate is given by $i/e = P/eV = 500\,\mathrm{W}/[1.60 \times 10^{-19}\ \mathrm{C}(120\,\mathrm{V})] = 2.60 \times 10^{19}/\mathrm{s}$.

49E

Use $P = V^2/R$. The power dissipated in the second case is

$$P = (1.50\,\mathrm{V}/3.00\,\mathrm{V})^2(0.540\,\mathrm{W}) = 0.135\,\mathrm{W}\,.$$

50E

(a)

$$J = \frac{i}{A} = \frac{4(25\,\mathrm{A})}{\pi[(0.10\,\mathrm{in.})(2.54 \times 10^{-2}\ \mathrm{m/in.})]^2} = 4.9 \times 10^6\ \mathrm{A/m^2}\,.$$

(b) $E = J/\sigma = \rho J = (1.69 \times 10^{-8}\ \Omega\cdot\mathrm{m})(4.9 \times 10^6\ \mathrm{A/m^2}) = 8.3 \times 10^{-2}\ \mathrm{V/m}$.
(c) $V = EL = (8.3 \times 10^{-2}\ \mathrm{V/m})(1000\,\mathrm{ft})(0.3048\,\mathrm{m/ft}) = 25\,\mathrm{V}$.
(d) $P = Vi = (25\,\mathrm{V})(25\,\mathrm{A}) = 6.3 \times 10^2\ \mathrm{W}$.

51E

(*a*)

$$\begin{aligned} i &= \frac{V}{R} = \frac{V}{\rho L/A} = \frac{\pi V d^2}{4\rho L} \\ &= \frac{\pi(1.20\,\mathrm{V})[(0.0400\,\mathrm{in.})(2.54\times 10^{-2}\,\mathrm{m/in.})]^2}{4(1.69\times 10^{-8}\,\Omega\cdot\mathrm{m})(33.0\,\mathrm{m})} = 1.74\,\mathrm{A}\,. \end{aligned}$$

(*b*)

$$J = \frac{i}{A} = \frac{4i}{\pi d^2} = \frac{4(1.74\,\mathrm{A})}{\pi[(0.0400\,\mathrm{in.})(2.54\times 10^{-2}\,\mathrm{m/in.})]^2} = 2.15\times 10^6\,\mathrm{A/m^2}.$$

(*c*) $E = V/L = 1.20\,\mathrm{V}/33.0\,\mathrm{m} = 3.63\times 10^{-2}\,\mathrm{V/m}$.
(*d*) $P = Vi = (1.20\,\mathrm{V})(1.74\,\mathrm{A}) = 2.09\,\mathrm{W}$.

52P

Use $P = i^2 R = i^2\rho L/A$, or $L/A = P/i^2\rho$. So the new values of L and A satisfy

$$\left(\frac{L}{A}\right)_{\mathrm{new}} = \left(\frac{P}{i^2\rho}\right)_{\mathrm{new}} = \frac{30}{4^2}\left(\frac{P}{i^2\rho}\right)_{\mathrm{old}} = \frac{30}{16}\left(\frac{L}{A}\right)_{\mathrm{old}},$$

i.e., $(L/A)_{\mathrm{new}} = 1.875(L/A)_{\mathrm{old}}$. Also note that $(LA)_{\mathrm{new}} = (LA)_{\mathrm{old}}$. Solve the above two equations for L_{new} and A_{new}: $L_{\mathrm{new}} = \sqrt{1.875}L_{\mathrm{old}} = 1.369L_{\mathrm{old}}$, $A_{\mathrm{new}} = \sqrt{1/1.875}A_{\mathrm{old}} = 0.730A_{\mathrm{old}}$.

53P

(*a*) Since $P = i^2R = J^2A^2R$, the current density is

$$\begin{aligned} J &= \frac{1}{A}\sqrt{\frac{P}{R}} = \frac{1}{A}\sqrt{\frac{P}{\rho L/A}} = \sqrt{\frac{P}{\rho LA}} \\ &= \sqrt{\frac{1.0\,\mathrm{W}}{\pi(3.5\times 10^{-5}\,\Omega\cdot\mathrm{m})(2.0\times 10^{-2}\,\mathrm{m})(5.0\times 10^{-3}\,\mathrm{m})^2}} = 1.3\times 10^5\,\mathrm{A/m^2}\,. \end{aligned}$$

(*b*) From $P = iV = JAV$ we get

$$V = \frac{P}{JA} = \frac{P}{\pi r^2 J} = \frac{1.0\,\mathrm{W}}{\pi(5.0\times 10^{-3}\,\mathrm{m})^2(1.3\times 10^5\,\mathrm{A/m^2})} = 9.4\times 10^{-2}\,\mathrm{V}\,.$$

54P

(*a*) From $P = V^2/R = AV^2/\rho L$ we solve for L:

$$L = \frac{AV^2}{\rho P} = \frac{(2.60\times 10^{-6}\,\mathrm{m^2})(75.0\,\mathrm{V})^2}{(5.00\times 10^{-7}\,\Omega\cdot\mathrm{m})(5000\,\mathrm{W})} = 5.85\,\mathrm{m}\,.$$

(b) Since $L \propto V^2$ the new length should be

$$L' = L(V'/V)^2 = (5.85\,\mathrm{m})(100\,\mathrm{V}/75.0\,\mathrm{V})^2 = 10.4\,\mathrm{m}\,.$$

55P

(a) The monthly cost is $(100\,\mathrm{W})(24\,\mathrm{h/d})(31\,\mathrm{d/month})(6\,\mathrm{cents/kW\cdot h}) = 446\,\mathrm{cents} = \$4.46$, assuming a 31-day month.
(b) $R = V^2/P = (120\,\mathrm{V})^2/100\,\mathrm{W} = 144\,\Omega$.
(c) $i = P/V = 100\,\mathrm{W}/120\,\mathrm{V} = 0.833\,\mathrm{A}$.

56P

(a) Let P be the power dissipated, i be the current in the heater, and V be the potential difference across the heater. They are related by $P = iV$. Solve for i:

$$i = \frac{P}{V} = \frac{1250\,\mathrm{W}}{115\,\mathrm{V}} = 10.9\,\mathrm{A}\,.$$

(b) According to Ohm's law $V = iR$, where R is the resistance of the heater. Solve for R:

$$R = \frac{V}{i} = \frac{115\,\mathrm{V}}{10.9\,\mathrm{A}} = 10.6\,\Omega\,.$$

(c) The thermal energy E generated by the heater in time t ($= 1.0\,\mathrm{h} = 3600\,\mathrm{s}$) is

$$E = Pt = (1250\,\mathrm{W})(3600\,\mathrm{s}) = 4.5\times10^6\,\mathrm{J}\,.$$

57P

Let R_H be the resistance at the higher temperature (800 ℃) and let R_L be the resistance at the lower temperature (200 ℃). Since the potential difference is the same for the two temperatures, the power dissipated at the lower temperature is $P_L = V^2/R_L$, and the power dissipated at the higher temperature is $P_H = V^2/R_H$, so $P_L = (R_H/R_L)P_H$. Now $R_L = R_H + \alpha R_H\,\Delta T$, where ΔT is the temperature difference $T_L - T_H = -600\,\mathrm{C}^\circ$. Thus

$$P_L = \frac{R_H}{R_H + \alpha R_H\,\Delta T}\,P_H = \frac{P_H}{1+\alpha\,\Delta T} = \frac{500\,\mathrm{W}}{1 + (4.0\times10^{-4}/\mathrm{C}^\circ)(-600\,\mathrm{C}^\circ)} = 660\,\mathrm{W}\,.$$

58P

(a) The rate n is given by $n = i/e = (15\times10^{-6}\,\mathrm{A})/(1.60\times10^{-19}\,\mathrm{C}) = 9.4\times10^{13}/\mathrm{s}$.
(b) The rate is $P = (9.4\times10^{13}/\mathrm{s})(16\times10^6\,\mathrm{eV})(1.60\times10{-}19\,\mathrm{J/eV}) = 2.4\times10^2\,\mathrm{W}$.

59P

(a) The charge q that flows past any cross section of the beam in time Δt is given by $q = i\,\Delta t$ and the number of electrons is $N = q/e = (i/e)\,\Delta t$. This is the number of electrons that are accelerated. Thus

$$N = \frac{(0.50\,\mathrm{A})(0.10 \times 10^{-6}\,\mathrm{s})}{1.60 \times 10^{-19}\,\mathrm{C}} = 3.1 \times 10^{11}\,.$$

(b) Over a long time t the total charge is $Q = nqt$, where n is the number of pulses per unit time and q is the charge in one pulse. The average current is given by $\bar{i} = Q/t = nq$. Now $q = i\,\Delta t = (0.50\,\mathrm{A})(0.10 \times 10^{-6}\,\mathrm{s}) = 5.0 \times 10^{-8}\,\mathrm{C}$, so

$$\bar{i} = (500\,\mathrm{s}^{-1})(5.0 \times 10^{-8}\,\mathrm{C}) = 2.5 \times 10^{-5}\,\mathrm{A}\,.$$

(c) The accelerating potential difference is $V = K/e$, where K is the final kinetic energy of an electron. Since $K = 50\,\mathrm{MeV}$, the accelerating potential is $V = 50\,\mathrm{kV} = 5.0 \times 10^{7}\,\mathrm{V}$. During a pulse the power output is

$$P = iV = (0.50\,\mathrm{A})(5.0 \times 10^{7}\,\mathrm{V}) = 2.5 \times 10^{7}\,\mathrm{W}\,.$$

This is the peak power. The average power is

$$\bar{P} = \bar{i}V = (2.5 \times 10^{-5}\,\mathrm{A})(5.0 \times 10^{7}\,\mathrm{V}) = 1.3 \times 10^{3}\,\mathrm{W}\,.$$

60P

Let the heat of vaporization be L_v. Then $mL_v = P = iV$, where $m = 21\,\mathrm{mg/s}$. So

$$L_v = \frac{iV}{m} = \frac{(5.2\,\mathrm{A})(12\,\mathrm{V})}{21 \times 10^{-6}\,\mathrm{kg/s}} = 3.0 \times 10^{6}\,\mathrm{J/kg}\,.$$

61P

The rate of change of mechanical energy of the piston-Earth system, mgv, must be equal to the rate at which heat is generated from the coil, i^2R: $mgv = i^2R$. Thus

$$v = \frac{i^2R}{mg} = \frac{(0.240\,\mathrm{A})^2(550\,\Omega)}{(12\,\mathrm{kg})(9.80\,\mathrm{m/s^2})} = 0.27\,\mathrm{m/s}\,.$$

62P

Use $P = V^2/R \propto V^2$, which gives $\Delta P \propto \Delta V^2 \approx 2V\Delta V$. The percentage change is roughly $\Delta P/P = 2\Delta V/V = 2(110 - 115)/115 = -8.6\%$.

(b) A drop in V causes a drop in P, which in turn lowers the temperature of the resistor in the coil. At a lower temperature R is also decreased. Since $P \propto R^{-1}$ a decrease in R will result in an increase in P, which partially offsets the decrease in P due to the drop in V. Thus the actual drop in P will be smaller when the temperature dependency of the resistance is taken into consideration.

63P

(a) $120\,\Omega$;
(b) $107\,\Omega$;
(c) 5.3×10^{-3}/K;
(d) 5.9×10^{-3}/K;
(e) $276\,\Omega$