Chapter 28

Answer to Checkpoint Questions

1. (a) rightward; (b) all tie; (c) b, then a and c tie; (d) b, then a and c tie

2. (a) all tie; (b) R_1, R_2, R_3

3. (a) less; (b) greater; (c) equal

4. (a) $V/2$, i; (b) V, $i/2$

5. (a) 1, 2, 4, 3; (b) 4, tie of 1 and 2, then 3

Answer to Questions

1. 3, 4, 1, 2

2. (a) series; (b) parallel; (c) parallel

3. (a) no; (b) yes; (c) all tie (the circuits are the same)

4. (a) $3R$; (b) $R/3$; (c) equal

5. parallel, R_2, R_1, series

6. (a) parallel; (b) series

7. (a) equal; (b) more

8. (a) same; (b) same; (c) less; (d) more

9. (a) less; (b) less; (c) more

10. (a) same; (b) increase; (c) decrease, decrease

11. C_1, 15 V; C_2, 35 V; C_3, 20 V; C_4, 20 V; C_5, 30 V

12. 2.0 A

13. $60\,\mu$C

14. (a) 16, 14, 12; (b) 16, 14, 12; (c) all tie; (d) 12, 14, 16

15. c, b, a

16. (a) 1 and 3 tie, then 2; (b) 2, 1, 3

17. (a) all tie; (b) 1, 3, 2

18. 2 and 4 tie, then 1 and 3 tie, then 5

19. 1, 3, and 4 tie (8 V on each resistor), then 2 and 5 tie (4 V on each resistor)

20. (*a*) no; (*b*) yes; (*c*) no

Solutions to Exercises & Problems

1E

(*a*) The cost is $(100\,\mathrm{W}\cdot 8.0\,\mathrm{h}/2.0\,\mathrm{W}\cdot\mathrm{h})(\$0.80) = \$320$.
(*b*) The cost is $(100\,\mathrm{W}\cdot 8.0\mathrm{h}/10^3\,\mathrm{W}\cdot\mathrm{h})(\$0.06) = \$0.048 = 4.8\,\mathrm{cents}$.

2E

(*a*) $W = eV = e(12\,\mathrm{V}) = 12\,\mathrm{eV} = (12\,\mathrm{eV})(1.6 \times 10^{-19}\,\mathrm{J/eV}) = 1.9 \times 10^{-18}\,\mathrm{J}$.
(*b*) $P = iV = neV = (3.4 \times 10^{18}/\mathrm{s})(1.6 \times 10^{-19}\,\mathrm{C})(12\,\mathrm{V}) = 6.5\,\mathrm{W}$.

3E

The chemical energy of the battery is reduced by $\Delta E = q\mathcal{E}$, where q is the charge that passes through in time $\Delta t = 6.0\,\mathrm{min}$ and $\mathcal{E}$ is the emf of the battery. If i is the current, then $q = i\,\Delta t$ and $\Delta E = i\mathcal{E}\,\Delta t = (5.0\,\mathrm{A})(6.0\,\mathrm{V})(6.0\,\mathrm{min})(60\,\mathrm{s/min}) = 1.1 \times 10^4\,\mathrm{J}$. Notice the conversion of time from minutes to seconds.

4P

If P is the rate at which the battery delivers energy and Δt is the time, then $\Delta E = P\,\Delta t$ is the energy delivered in time Δt. If q is the charge that passes through the battery in time Δt and $\mathcal{E}$ is the emf of the battery, then $\Delta E = q\mathcal{E}$. Equate the two expressions for ΔE and solve for Δt:

$$\Delta t = \frac{q\mathcal{E}}{P} = \frac{(120\,\mathrm{A}\cdot\mathrm{h})(12\,\mathrm{V})}{100\,\mathrm{W}} = 14.4\,\mathrm{h} = 14\,\mathrm{h}\ 24\,\mathrm{min}\,.$$

5E

(*a*) Since $\mathcal{E}_1 > \mathcal{E}_2$ the current flows counterclockwise.
(*b*) Battery 1, since the current flows through it from its negative terminal to the positive one.
(*c*) Point B, since the current flows from B to A.

…t to be positive if it is to the left in R_1.

… $= 0$. Solve for i:

$$i = \frac{\ldots\,\mathrm{V} - 6.0\,\mathrm{V}}{4.0\,\Omega + 8.0\,\Omega} = 0.50\,\mathrm{A}\,.$$

… current is counterclockwise around the circuit.

… R, then the power dissipated by that resistor is given … $(\ldots\,\mathrm{A})^2(4.0\,\Omega) = 1.0\,\mathrm{W}$ and for R_2, $P_2 = (0.50\,\mathrm{A})^2(8.0\,\Omega) =$ …

… battery with emf $\mathcal{E}$, then the battery supplies energy at the … current and emf are in the same direction. The battery absorbs … $= i\mathcal{E}$ if the current and emf are in opposite directions. For $\mathcal{E}_1$, … $= 6.0\,\mathrm{W}$ and for $\mathcal{E}_2$, $P_2 = (0.50\,\mathrm{A})(6.0\,\mathrm{V}) = 3.0\,\mathrm{W}$. In battery 1 the … same direction as the emf so this battery supplies energy to the circuit. … discharging. The current in battery 2 is opposite the direction of the emf, … absorbs energy from the circuit. It is charging.

… The energy transferred is

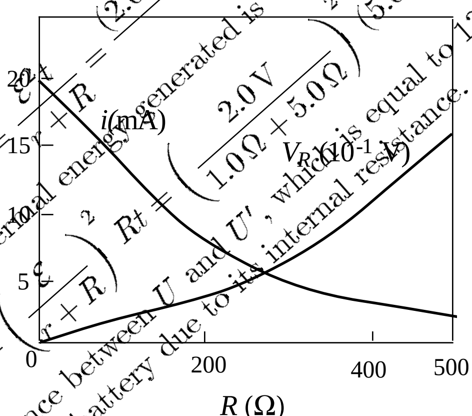
$$U = Pt = \frac{\mathcal{E}^2 t}{r + R} = \frac{(2.0\,\mathrm{V})^2(2.0\,\mathrm{min})(60\,\mathrm{s/min})}{1.0\,\Omega + 5.0\,\Omega} = 80\,\mathrm{J}\,.$$

(b) The amount of thermal energy generated is

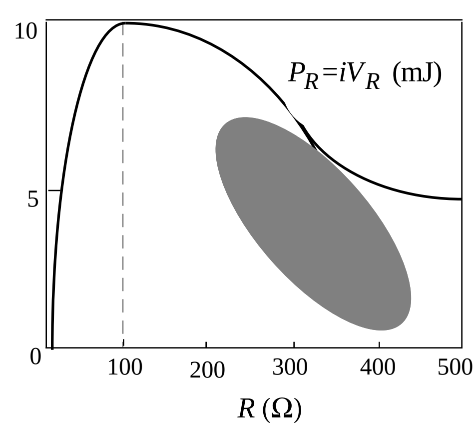
$$U' = i^2 Rt = \left(\frac{\mathcal{E}}{r + R}\right)^2 Rt = \left(\frac{2.0\,\mathrm{V}}{1.0\,\Omega + 5.0\,\Omega}\right)^2 (5.0\,\Omega)(2.0\,\mathrm{min})(60\,\mathrm{s/min}) = 67\,\mathrm{J}\,.$$

(c) The difference between U and U', which is equal to 13 J, is the thermal energy that is generated in the battery due to its internal resistance.

(a) and (b)



(c)



In the plot shown in the last page the curve in (c) is the power P_R consumed by R as a function of R.

9E

(a) The potential difference is $V = \mathcal{E} + ir = 12\,\text{V} + (0.040\,\Omega)(50\,\text{A}) = 14\,\text{V}$.
(b) $P = i^2r = (50\,\text{A})^2(0.040\,\Omega) = 100\,\text{W}$.
(c) $P' = iV = (50\,\text{A})(12\,\text{V}) = 600\,\text{W}$.
(d) In this case $V = \mathcal{E} - ir = 12\,\text{V} - (0.040\,\Omega)(50\,\text{A}) = 10\,\text{V}$ and $P = i^2r = 100\,\text{W}$.

10E

The current in the circuit is $i = (150\,\text{V} - 50\,\text{V})/(3.0\,\Omega + 2.0\,\Omega) = 20\,\text{A}$. So from $V_Q + 150\,\text{V} - (2.0\,\Omega)i = V_P$ we get $V_Q = 100\,\text{V} + (2.0\,\Omega)(20\,\text{A}) - 150\,\text{V} = -10\,\text{V}$.

11E

(a) If i is the current and ΔV is the potential difference then the power absorbed is given by $P = i\,\Delta V$. Thus

$$\Delta V = \frac{P}{i} = \frac{50\,\text{W}}{1.0\,\text{A}} = 50\,\text{V}\,.$$

Since the energy of the charge decreases, point A is at a higher potential than point B; that is $V_A - V_B = 50\,\text{V}$.

(b) The end-to-end potential difference is given by $V_A - V_B = +iR + \mathcal{E}$, where $\mathcal{E}$ is the emf of element C and is taken to be positive if it is to the left in the diagram. Thus $\mathcal{E} = V_A - V_B - iR = 50\,\text{V} - (1.0\,\text{A})(2.0\,\Omega) = 48\,\text{V}$.

(c) A positive value was obtained for $\mathcal{E}$, so it is toward the left. The negative terminal is at B.

12E

The potential difference across R_2 is

$$V_2 = iR_2 = \frac{\mathcal{E}R_2}{R_1 + R_2 + R_3} = \frac{(12\,\text{V})(4.0\,\Omega)}{3.0\,\Omega + 4.0\,\Omega + 5.0\,\Omega} = 4.0\,\text{V}\,.$$

13E

From $V_a - \mathcal{E}_1 - ir_1 = V_c$ and $i = (\mathcal{E}_2 - \mathcal{E}_1)/(R + r_1 + r_2)$, we get

$$\begin{aligned} V_a - V_c = \mathcal{E}_1 + ir_1 &= \mathcal{E}_1 + \frac{(\mathcal{E}_2 - \mathcal{E}_1)r_1}{R + r_1 + r_2} \\ &= 2.1\,\text{V} + \frac{(4.4\,\text{V} - 2.1\,\text{V})(1.8\,\Omega)}{5.5\,\Omega + 1.8\,\Omega + 2.3\,\Omega} = 2.5\,\text{V}\,. \end{aligned}$$

14E

(a) Since $R_{\text{tank}} = 140\,\Omega$, $i = 12\,\text{V}/(10\,\Omega + 140\,\Omega) = 8.0 \times 10^{-2}\,\text{A}$.
(b) Now $R_{\text{tank}} = (140\,\Omega + 20\,\Omega)/2 = 80\,\Omega$, so $i = 12\,\text{V}/(10\,\Omega + 80\,\Omega) = 0.13\,\text{A}$.
(c) Now $R_{\text{tank}} = 20\,\Omega$ so $i = 12\,\text{V}/(10\,\Omega + 20\,\Omega) = 0.40\,\text{A}$.

15P

(a) and (b) Denote $L = 10\,\text{km}$ and $\alpha = 13\,\Omega/\text{km}$. Measured from the east end we have $R_1 = 100\,\Omega = 2\alpha(L - x) + R$, and measured from the west end $R_2 = 200\,\Omega = 2\alpha x + R$. Solve for x and R:

$$x = \frac{R_2 - R_1}{4\alpha} + \frac{L}{2} = \frac{200\,\Omega - 100\,\Omega}{4(13\,\Omega/\text{km})} + \frac{10\,\text{km}}{2} = 6.9\,\text{km}\,,$$

$$R = \frac{R_1 + R_2}{2} - \alpha L = \frac{100\,\Omega + 200\,\Omega}{2} - (13\,\Omega/\text{km})(10\,\text{km}) = 20\,\Omega\,.$$

16P

(a) Solve $i = (\mathcal{E}_2 - \mathcal{E}_1)/(r_1 + r_2 + R)$ for R:

$$R = \frac{\mathcal{E}_2 - \mathcal{E}_1}{i} - r_1 - r_2 = \frac{3.0\,\text{V} - 2.0\,\text{V}}{1.0 \times 10^{-3}\,\text{A}} - 3.0\,\Omega - 3.0\,\Omega = 9.9 \times 10^2\,\Omega\,.$$

(b) $P = i^2 R = (1.0 \times 10^{-3}\,\text{A})^2 (9.9 \times 10^2\,\Omega) = 9.9 \times 10^{-4}\,\text{W}$.

17P

Let the emf be V. Then $V = iR = (5.0\,\text{A})R = i'(R + R') = (4.0\,\text{A})(R + 2.0\,\Omega)$. Solve for R: $R = 8.0\,\Omega$.

18P

(a) From $P = V^2/R$ we find $V = \sqrt{PR} = \sqrt{(10\,\text{W})(0.10\,\Omega)} = 1.0\,\text{V}$.
(b) From $i = V/R = (\mathcal{E} - V)/r$ we find $r = R(\mathcal{E} - V)/V = (0.10\,\Omega)(1.5\,\text{V} - 1.0\,\text{V})/(1.0\,\text{V}) = 0.050\,\Omega$.

19P

Let the power supplied be P_s and that dissipated be P_d. Since $P_d = i^2 R$ and $i = P_s/\mathcal{E}$, we have $P_d = P_s^2/\mathcal{E}^2 R \propto \mathcal{E}^{-2}$. The ratio is then

$$\frac{P_d(\mathcal{E} = 110,000\,\text{V})}{P_d(\mathcal{E} = 110\,\text{V})} = \left(\frac{110\,\text{V}}{110,000\,\text{V}}\right)^2 = 1.0 \times 10^{-6}\,.$$

20P

(a)

$$J_A = J_B = \frac{i}{A} = \frac{V}{(R_1 + R_2)A} = \frac{4\,\mathrm{V}}{(R_1 + R_2)\pi D^2}$$
$$= \frac{4(60.0\,\mathrm{V})}{\pi(0.127\,\Omega + 0.729\,\Omega)(2.60 \times 10^{-3}\,\mathrm{m})^2} = 1.32 \times 10^7\,\mathrm{A/m^2}\,.$$

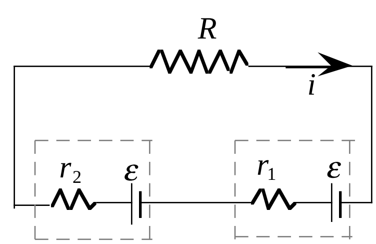
(b) $V_A = VR_1/(R_1 + R_2) = (60.0\,\mathrm{V})(0.127\,\Omega)/(0.127\,\Omega + 0.729\,\Omega) = 8.90\,\mathrm{V}$, and $V_B = V - V_A = 60.0\,\mathrm{V} - 8.9\,\mathrm{V} = 51.1\,\mathrm{V}$.
(c) Calculate the resistivity from $R = \rho L/A$ for both materials: $\rho_A = R_A A/L_A = \pi R_A D^2/4L_A = \pi(0.127\,\Omega)(2.60 \times 10^{-3}\,\mathrm{m})^2/[4(40.0\,\mathrm{m})] = 1.69 \times 10^{-8}\,\Omega{\cdot}\mathrm{m}$. So A is made of copper. Similarly we find $\rho_B = 9.68 \times 10^{-8}\,\Omega{\cdot}\mathrm{m}$, so B is made of iron.

21P

The internal resistance of the battery is $r = (12\,\mathrm{V} - 11.4\,\mathrm{V})/50\,\mathrm{A} = 0.012\,\Omega < 0.020\,\Omega$, so the battery is OK. The resistance of the cable is $R = 3.0\,\mathrm{V}/50\,\mathrm{A} = 0.060\,\Omega > 0.040\,\Omega$, so the cable is defective.

22P

(a) The circuit is shown in the diagram to the right. The current is taken to be positive if it is clockwise. The potential difference across battery 1 is given by $V_1 = \mathcal{E} - ir_1$ and for this to be 0 the current must be $i = \mathcal{E}/r_1$. Kirchhoff's loop rule gives $2\mathcal{E} - ir_1 - ir_2 - iR = 0$. Substitute $i = \mathcal{E}/r_1$ and solve for R. You should get $R = r_1 - r_2$.



(b) Since R must be positive, r_1 must be greater than r_2. The potential difference across the battery with the larger internal resistance can be made to vanish with the proper choice of R, the potential difference across the battery with the smaller potential difference cannot be made to vanish.

23P

(a) and (b) Let the emf of the solar cell be $\mathcal{E}$ and the output voltage be V, then $V = \mathcal{E} - ir = \mathcal{E} - (V/R)r$ for both cases. Numerically we get $0.10\,\mathrm{V} = \mathcal{E} - (0.10\,\mathrm{V}/500\,\Omega)r$ and $0.15\,\mathrm{V} = \mathcal{E} - (0.15\,\mathrm{V}/1000\,\Omega)r$. Solve for $\mathcal{E}$ and r: $\mathcal{E} = 0.30\,\mathrm{V}$, $r = 1000\,\Omega$.
(c) The efficiency is

$$e = \frac{V^2/R}{P_{\mathrm{received}}} = \frac{0.15\,\mathrm{V}}{(1000\,\Omega)(5.0\,\mathrm{cm^2})(2.0 \times 10^{-3}\,\mathrm{W/cm^2})} = 2.3 \times 10^{-3}\,.$$

24P

(a) The current in the circuit is $i = \mathcal{E}/(r+R)$, so the rate of energy dissipation in R is

$$P = i^2 R = \frac{\mathcal{E}^2 R}{(r+R)^2}\,.$$

You want to find the value of R that maximizes P. To do this find an expression for the derivative with respect to R, set it equal to 0, and solve for R. The derivative is

$$\frac{dP}{dR} = \frac{\mathcal{E}^2}{(r+R)^2} - \frac{2\mathcal{E}^2 R}{(r+R)^3} = \frac{\mathcal{E}^2(r-R)}{(r+R)^3}\,.$$

Thus $r - R = 0$ and $R = r$. For R very small P increases with R. For R large P decreases with increasing R. So $R = r$ is a maximum, not a minimum.

(b) Substitute $R = r$ into $P = \mathcal{E}^2 R/(r+R)^2$ to obtain $P = \mathcal{E}^2/4r$.

25P

(a) The power delivered by the motor is $P = (2.00\,\mathrm{V})(0.500\,\mathrm{m/s}) = 1.00\,\mathrm{W}$. From $P = i^2 R_{\mathrm{motor}}$ and $\mathcal{E} = i(r + R_{\mathrm{motor}})$ we then find $i^2 r - i\mathcal{E} + P = 0$ (which also follows directly from the conservation of energy principle). Solve for i:

$$i = \frac{\mathcal{E} \pm \sqrt{\mathcal{E}^2 - 4rP}}{2r} = \frac{2.00\,\mathrm{V} \pm \sqrt{(2.00\,\mathrm{V})^2 - 4(0.500\,\Omega)(1.00\,\mathrm{W})}}{2(0.500\,\Omega)}\,.$$

The answer is either $3.41\,\mathrm{A}$ or $0.586\,\mathrm{A}$.

(b) Use $V = \mathcal{E} - ir = 2.00\,\mathrm{V} - i(0.500\,\Omega)$. Substitute the two values of i obtained in (a) into the above formula to get $V = 0.293\,\mathrm{V}$ or $1.71\,\mathrm{V}$.

(c) The power P delivered by the motor is the same for either solution. Since $P = iV$ we may have a lower i and higher V or, reversely, a lower V and higher i, while keeping $P = iV$ a constant. In fact you should check for yourself that the two sets of solutions for i and V above do yield the same power $P = iV$.

26P

Denote silicon with subscript s and iron with i. Let $T_0 = 20°$. If $R(T) = R_s(T) + R_i(T) = R_s(T_0)[1 + \alpha(T - T_0)] + R_i(T_0)[1 + \alpha_i(T - T_0)] = [R_s(T)_0\alpha_s + R_i(T_0)\alpha_i]$+(temperature-independent terms) is to be temperature-independent, we must require that $R_s(T_0)\alpha_s + R_i(T_0)\alpha_i = 0$. Also note that $R_s(T_0) + R_i(T_0) = R = 1000\,\Omega$. Solve for $R_s(T_0)$ and $R_i(T_0)$ to obtain

$$R_s(T_0) = \frac{R\alpha_i}{\alpha_i - \alpha_s} = \frac{(1000\,\Omega)(6.5\times10^{-3})}{6.5\times10^{-3} + 70\times10^{-3}} = 85.0\,\Omega\,,$$

and $R_i(T_0) = 1000\,\Omega - 85.0\,\Omega = 915\,\Omega$.

27E

The potential difference across each resistor is $V = 25.0\,\mathrm{V}$. Since the resistors are identical, the current in each one is $i = V/R = (25.0\,\mathrm{V})/(18.0\,\Omega) = 1.39\,\mathrm{A}$. The total current through the battery is then $i_{\mathrm{total}} = 4(1.39\,\mathrm{A}) = 5.56\,\mathrm{A}$.

You might use the idea of equivalent resistance. For 4 identical resistors in parallel the equivalent resistance is given by

$$\frac{1}{R_{\text{eq}}} = \frac{4}{R}.$$

When a potential difference of 25.0 V is applied to the equivalent resistor the current through it is the same as the total current through the 4 resistors in parallel. Thus $i_{\text{total}} = V/R_{\text{eq}} = 4V/R = 4(25.0\,\text{V})/(18.0\,\Omega) = 5.56\,\text{A}$.

28E

Since $R_{\text{eq}} < R$ the two resistors ($R = 12.0\,\Omega$ and R_x) must be connceted in parallel: $R_{\text{eq}} = 3.00\,\Omega = R_xR/(R+R_x) = R_x(12.0\,\Omega)/(12.0\,\Omega+R_x)$. Solve for R_x: $R_x = R_{\text{eq}}R/(R-R_{\text{eq}}) = (3.00\,\Omega)(12.0\,\Omega)/(12.0\,\Omega - 3.00\,\Omega) = 4.00\,\Omega$.

29E

Let the resistances of the two resistors be R_1 and R_2, respectively. Note that the smallest value of the possible R_{eq} must be the result of connecting R_1 and R_2 in parallel, while the largest one must be that of connecting them in series. Thus $R_1R_2/(R_1 + R_2) = 3.0\,\Omega$ and $R_1 + R_2 = 16\,\Omega$. So R_1, R_2 must be $4.0\,\Omega$ and $12\,\Omega$.

30E

(*a*) $R_{\text{eq}}(AB) = 20.0\,\Omega/3 = 6.67\,\Omega$ (three 20.0-Ω resistors in parallel).
(*b*) $R_{\text{eq}}(AC) = 20.0\text{-}\Omega/3 = 6.67\,\Omega$ (Three 20.0-Ω resistors in parallel).
(*c*) $R_{\text{eq}}(BC) = 0$ (as B and C are connected by a conducting wire).

31E

$R_{\text{eq}} = 2.50\,\Omega + (4.00\,\Omega)(4.00\,\Omega)/(4.00\,\Omega + 4.00\,\Omega) = 4.50\,\Omega$.

32E

Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is upward. When the loop rule is applied to the lower loop, the result is

$$\mathcal{E}_2 - i_1R_1 = 0.$$

and when it is applied to the upper loop, the result is

$$\mathcal{E}_1 - \mathcal{E}_2 - \mathcal{E}_3 - i_2R_2 = 0.$$

The first equation yields

$$i_1 = \frac{\mathcal{E}_2}{R_1} = \frac{5.0\,\text{V}}{100\,\Omega} = 0.050\,\text{A}.$$

The second yields

$$i_2 = \frac{\mathcal{E}_1 - \mathcal{E}_2 - \mathcal{E}_3}{R_2} = \frac{6.0\,\mathrm{V} - 5.0\,\mathrm{V} - 4.0\,\mathrm{V}}{50\,\Omega} = -0.060\,\mathrm{A}\,.$$

The negative sign indicates that the current in R_2 is actually downward.

If V_b is the potential at point b, then the potential at point a is $V_a = V_b + \mathcal{E}_3 + \mathcal{E}_2$, so $V_a - V_b = \mathcal{E}_3 + \mathcal{E}_2 = 4.0\,\mathrm{V} + 5.0\,\mathrm{V} = 9.0\,\mathrm{V}$.

33E

S_1, S_2 and S_3 all open: $i_a = 0.00\,\mathrm{A}$.

S_1 closed, S_2 and S_3 open: $i_a = \mathcal{E}/2R_1 = 120\,\mathrm{V}/40.0\,\Omega = 3.00\,\mathrm{A}$.

S_2 closed, S_1 and S_3 open: $i_a = \mathcal{E}/(2R_1 + R_2) = 120\,\mathrm{V}/50.0\,\Omega = 2.40\,\mathrm{A}$.

S_3 closed, S_1 and S_2 open: $i_a = \mathcal{E}/(2R_1 + R_2) = 120\,\mathrm{V}/60.0\,\Omega = 2.00\,\mathrm{A}$.

S_1 open, S_2 and S_3 closed: $R_{\mathrm{eq}} = R_1 + R_2 + R_1(R_1 + R_2)/(2R_1 + R_2) = 20.0\,\Omega + 10.0\,\Omega + (20.0\,\Omega)(30.0\,\Omega)/(50.0\,\Omega) = 42.0\,\Omega$, so $i_a = \mathcal{E}/R_{\mathrm{eq}} = 120\,\mathrm{V}/42.0\,\Omega = 2.86\,\mathrm{A}$.

S_2 open, S_1 and S_3 closed: $R_{\mathrm{eq}} = R_1 + R_1(R_1 + 2R_2)/(2R_1 + 2R_2) = 20.0\,\Omega + (20.0\,\Omega)\times(40.0\,\Omega)/(60.0\,\Omega) = 33.3\,\Omega$, so $i_a = \mathcal{E}/R_{\mathrm{eq}} = 120\,\mathrm{V}/33.3\,\Omega = 3.60\,\mathrm{A}$.

S_3 open, S_1 and S_2 closed: $R_{\mathrm{eq}} = R_1 + R_1(R_1 + R_2)/(2R_1 + R_2) = 20.0\,\Omega + (20.0\,\Omega)\times(30.0\,\Omega)/(50.0\,\Omega) = 32.0\,\Omega$, so $i_a = \mathcal{E}/R_{\mathrm{eq}} = 120\,\mathrm{V}/32.0\,\Omega = 3.75\,\mathrm{A}$.

S_1, S_2 and S_3 all closed: $R_{\mathrm{eq}} = R_1 + R_1R'/(R_1 + R')$ where $R' = R_2 + R_1(R_1 + R_2)/(2R_1 + R_2) = 22.0\,\Omega$, i.e., $R_{\mathrm{eq}} = 20.0\,\Omega + (20.0\,\Omega)(22.0\,\Omega)/(20.0\,\Omega + 22.0\,\Omega) = 30.5\,\Omega$; so $i_a = \mathcal{E}/R_{\mathrm{eq}} = 120\,\mathrm{V}/30.5\,\Omega = 3.94\,\mathrm{A}$.

34E

(a) Let $\mathcal{E}$ be the emf of the battery. When the bulbs are connected in parallel the potential difference across them is the same and is the same as the emf of the battery. The power dissipated by bulb 1 is $P_1 = \mathcal{E}^2/R_1$ and the power dissipated by bulb 2 is $P_2 = \mathcal{E}^2/R_2$. Since R_1 is greater than R_2, bulb 2 dissipates energy at a greater rate than bulb 1 and is the brighter of the two.

(b) When the bulbs are connected in series the current in them is the same. The power dissipated by bulb 1 is now $P_1 = i^2R_1$ and the power dissipated by bulb 2 is $P_2 = i^2R_2$. Since R_1 is greater than R_2 greater power is dissipated by bulb 1 than by bulb 2 and bulb 1 is the brighter of the two.

35E

The currents i_1, i_2 and i_3 are obatined from Eqs. 28-15 through 28-17:

$$\begin{cases} i_1 = \dfrac{\mathcal{E}_1(R_2+R_3)-\mathcal{E}_2R_3}{R_1R_2+R_2R_3+R_1R_3} = \dfrac{(4.0\,\mathrm{V})(10\,\Omega+5.0\,\Omega)-(1.0\,\mathrm{V})(5.0\,\Omega)}{(10\,\Omega)(10\,\Omega)+(10\,\Omega)(5.0\,\Omega)+(10\,\Omega)(5.0\,\Omega)} \\ \quad = 0.275\,\mathrm{A}, \\ i_2 = \dfrac{\mathcal{E}_1R_3-\mathcal{E}_2(R_1+R_2)}{R_1R_2+R_2R_3+R_1R_3} = \dfrac{(4.0\,\mathrm{V})(5.0\,\Omega)-(1.0\,\mathrm{V})(10\,\Omega+5.0\,\Omega)}{(10\,\Omega)(10\,\Omega)+(10\,\Omega)(5.0\,\Omega)+(10\,\Omega)(5.0\,\Omega)} \\ \quad = 0.025\,\mathrm{A}, \\ i_3 = i_2 - i_1 = 0.025\,\mathrm{A} - 0.275\,\mathrm{A} = -0.250\,\mathrm{A}. \end{cases}$$

$V_d - V_c$ can now be calculated by taking various paths. Two examples: from $V_d - i_2R_2 = V_c$ we get $V_d - V_c = i_2R_2 = (0.0250\,\mathrm{A})(10\,\Omega) = +0.25\,\mathrm{V}$; or from $V_d + i_3R_3 + \mathcal{E}_2 = V_c$ we get $V_d - V_c = -i_3R_3 - \mathcal{E}_2 = -(-0.250\,\mathrm{A})(5.0\,\Omega) - 1.0\,\mathrm{V} = +0.25\,\mathrm{V}$.

36E

Let r be the resistance of each of the narrow wires. Since they are in parallel the resistance R of the composite is given by

$$\frac{1}{R} = \frac{9}{r},$$

or $R = r/9$. Now $r = 4\rho\ell/\pi d^2$ and $R = 4\rho\ell/\pi D^2$, where ρ is the resistivity of copper. $A = \pi d^2/4$ was used for the cross-sectional area of a single wire and a similar expression was used for the cross-sectional area of the thick wire. Since the single thick wire is to have the same resistance as the composite,

$$\frac{4\rho\ell}{\pi D^2} = \frac{4\rho\ell}{9\pi d^2}.$$

Solve for D to obtain $D = 3d$.

37E

The maximum power output is $(120\,\mathrm{V})(15\,\mathrm{A}) = 1800\,\mathrm{W}$. Since $1800\,\mathrm{W}/500\,\mathrm{W} = 3.6$, the maximum number of 500-W lamps allowed is 3.

38E

Consider the lowest branch with the two resistors $R_1\,(=3.0\,\Omega)$ and the $R_2\,(=5.0\,\Omega)$. The voltage difference across the 5.0-Ω resistor is

$$V = i_2R_2 = \frac{\mathcal{E}R_2}{R_1+R_2} = \frac{(120\,\mathrm{V})(5.0\,\Omega)}{3.0\,\Omega+5.0\,\Omega} = 7.5\,\mathrm{V}.$$

39P

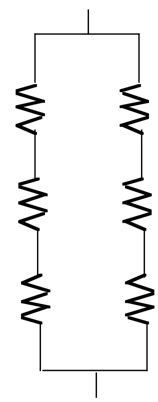
(a) $R_{\text{eq}}(FH) = (10.0\,\Omega)(10.0\,\Omega)(5.00\,\Omega)/[(10.0\,\Omega)(10.0\,\Omega) + 2(10.0\,\Omega)(5.00\,\Omega)] = 2.50\,\Omega$.
(b) $R_{\text{eq}}(FG) = (5.00\,\Omega)R/(R + 5.00\,\Omega)$, where $R = 5.00\,\Omega + (5.00\,\Omega)(10.0\,\Omega)/(5.00\,\Omega + 10.0\,\Omega) = 8.33\,\Omega$. So $R_{\text{eq}}(FG) = (5.00\,\Omega)(8.33\,\Omega)/(5.00\,\Omega + 8.33\,\Omega) = 3.13\,\Omega$.

40P

When connected in series, the rate at which electric energy dissipates is $P_s = \mathcal{E}^2/(R_1+R_2)$. When connected in parallel the corresponding rate is $P_p = \mathcal{E}^2(R_1+R_2)/R_1R_2$. Let $P_p/P_s = 5$, we get $(R_1 + R_2)^2/R_1R_2 = 5$, where $R_1 = 100\,\Omega$. Solve for R_2: $R_2 = 38\,\Omega$ or $260\,\Omega$.

41P

Divide the resistors into groups of n resistors each, with all the resistors in the same group connected in series. Suppose there are m such groups that are connected in parallel with each other. The scheme is shown in the diagram on the right for $n = 3$ and $m = 2$. Let R be the resistance of any one of the resistors. Then the equivalent resistance of any group is nR, and R_{eq}, the equivalent resistance of the whole array, satisfies



$$\frac{1}{R_{\text{total}}} = \frac{m}{nR}\,.$$

Since we want $R_{\text{total}} = R$, we must select $n = m$.

The current is the same in every resistor and there are n^2 resistors, so the maximum total power that can be dissipated is $P_{\text{total}} = n^2P$, where P is the maximum power that can be dissipated by any one of the resistors.

You want $P_{\text{total}} > 5.0P$, so n^2 must be larger than 5.0. Since n must be an integer, the smallest it can be is 3. The least number of resistors is $n^2 = 9$.

42P

(a) The batteries are identical and, because they are connected in parallel, the potential differences across them are the same. This means the currents in them are the same. Let i be the current in either battery and take it to be positive to the left. According to the junction rule the current in R is $2i$ and it is positive to the right. The loop rule applied to either loop containing a battery and R yields $\mathcal{E} - ir - 2iR = 0$, so

$$i = \frac{\mathcal{E}}{r + 2R}.$$

The power dissipated in R is

$$P = (2i)^2R = \frac{4\mathcal{E}^2R}{(r + 2R)^2}\,.$$

Find the maximum by setting the derivative with respect to R equal to zero. The derivative is

$$\frac{dP}{dR}=\frac{4\mathcal{E}^2}{(r+2R)^2}-\frac{16\mathcal{E}^2R}{(r+2R)^3}=\frac{4\mathcal{E}^2(r-2R)}{(r+2R)^3}\,.$$

It vanishes and P is a maximum if $R=r/2$.
(b) Substitute $R=r/2$ into $P=4\mathcal{E}^2R/(r+2R)^2$ to obtain

$$P_{\max}=\frac{4\mathcal{E}^2(r/2)}{[r+2(r/2)]^2}=\frac{\mathcal{E}^2}{2r}\,.$$

43P

(a) Since $\mathcal{E}_2=\mathcal{E}_3$ and $R_2=2R_1$, from symmetry we know that the currents through $\mathcal{E}_2$ and $\mathcal{E}_3$ are the same: $i_2=i_3=i$. Therefore the current through $\mathcal{E}_1$ is $i_1=2i$. Then from $V_b-V_a=\mathcal{E}_2-iR_2=\mathcal{E}_1+(2R_1)(2i)$ we get

$$i=\frac{\mathcal{E}_2-\mathcal{E}_1}{4R_1+R_2}=\frac{4.0\,\mathrm{V}-2.0\,\mathrm{V}}{4(1.0\,\Omega)+2.0\,\Omega}=0.33\,\mathrm{A}\,.$$

So the current through $\mathcal{E}_1$ is $i_1=2i=0.67\,\mathrm{A}$, flowing downward; the current through either $\mathcal{E}_2$ and $\mathcal{E}_3$ is $0.33\,\mathrm{A}$, flowing upward.
(b) $V_a-V_b=-iR_2+\mathcal{E}_2=-(0.333\,\mathrm{A})(2.0\,\Omega)+4.0\,\mathrm{V}=3.3\,\mathrm{V}$.

44P

By symmetry, when the two batteries are connected in parallel the current i going through either one is the same. So from $\mathcal{E}=ir+(2i)R$ we get $i_R=2i=2\mathcal{E}/(r+2R)$.
When connected in series $2\mathcal{E}-i_Rr-i_Rr-i_RR=0$, or $i_R=2\mathcal{E}/(2r+R)$.
(b) In series, since $R>r$.
(c) In parallel, since $R<r$.

45P

When all the batteries are connected in parallel, each supplies a current i and therefore $i_R=Ni$. Then from $\mathcal{E}=ir+i_RR=ir+Nir$ we get $i_R=N\mathcal{E}/[(N+1)r]$.
When all the batteries are connected in series $i_r=i_R$ and $\mathcal{E}_{\text{total}}=N\mathcal{E}=Ni_rr+i_RR=Ni_Rr+i_Rr$, so $i_R=N\mathcal{E}/[(N+1)r]$.

46P

(a) Since $P=\mathcal{E}^2/R_{\text{eq}}$, the higher the power rating the smaller the value of R_{eq}. To achieve this, we can let the low position connect to the larger resistance (R_1), middle position connect to the smaller resistance (R_2), and the high position connect to both of them in parallel.

(b) For $P = 100\,\mathrm{W}$, $R_{\mathrm{eq}} = R_1 = \mathcal{E}^2/P = (120\,\mathrm{V})^2/100\,\mathrm{W} = 144\,\Omega$; for $P = 300\,\mathrm{W}$, $R_{\mathrm{eq}} = R_1R_2/(R_1+R_2) = (144\,\Omega)R_2/(144\,\Omega + R_2) = (120\,\mathrm{V})^2/300\,\mathrm{W}$. Solve for R_2: $R_2 = 72\,\Omega$.

47P

(a) R_2, R_3 and R_4 are in parallel with an equivalent resistance of

$$R = \frac{R_2R_3R_4}{R_2R_3 + R_2R_4 + R_3R_4} = \frac{(50\,\Omega)(50\,\Omega)(75\,\Omega)}{(50\,\Omega)(50\,\Omega) + (50\,\Omega)(75\,\Omega) + (50\,\Omega)(75\,\Omega)} = 19\,\Omega\,.$$

Thus $R_{\mathrm{eq}} = R_1 + R = 100\,\Omega + 19\,\Omega = 1.2 \times 10^2\,\Omega$.
(b) $i_1 = \mathcal{E}/R_{\mathrm{eq}} = 6.0\,\mathrm{V}/(1.1875 \times 10^2\,\Omega) = 5.1 \times 10^{-2}\,\mathrm{A}$;
$i_2 = (\mathcal{E} - V_1)/R_2 = (\mathcal{E} - i_1R_1)/R_2 = [6.0\,\mathrm{V} - (5.05 \times 10^{-2}\,\mathrm{A})(100\,\Omega)]/50\,\Omega = 1.9 \times 10^{-2}\,\mathrm{A}$;
$i_3 = (\mathcal{E} - V_1)/R_3 == i_2R_2/R_3 = (1.9 \times 10^{-2}\,\mathrm{A})(50\,\Omega/50\,\Omega) = 1.9 \times 10^{-2}\,\mathrm{A}$;
$i_4 = i_1 - i_2 - i_3 = 5.0 \times 10^{-2}\,\mathrm{A} - 2(1.895 \times 10^{-2}\,\mathrm{A}) = 1.2 \times 10^{-2}\,\mathrm{A}$.

48P

(a) First find the currents. Let i_1 be the current in R_1 and take it to be positive if it is upward. Let i_2 be the current in R_2 and take it to be positive if it is to the left. Let i_3 be the current in R_3 and take it to be positive if it is to the right. The junction rule produces

$$i_1 + i_2 + i_3 = 0\,.$$

The loop rule applied to the left-hand loop produces

$$\mathcal{E}_1 - i_3R_3 + i_1R_1 = 0$$

and applied to the right-hand loop produces

$$\mathcal{E}_2 - i_2R_2 + i_1R_1 = 0\,.$$

Substitute $i_1 = -i_2 - i_3$, from the first equation, into the other two to obtain

$$\mathcal{E}_1 - i_3R_3 - i_2R_1 - i_3R_1 = 0$$

and

$$\mathcal{E}_2 - i_2R_2 - i_2R_1 - i_3R_1 = 0\,.$$

The first of these yields

$$i_3 = \frac{\mathcal{E}_1 - i_2R_1}{R_1 + R_3}\,.$$

Substitute this into the second equation and solve for i_2. You should obtain

$$\begin{aligned} i_2 &= \frac{\mathcal{E}_2(R_1+R_3) - \mathcal{E}_1R_1}{R_1R_2 + R_1R_3 + R_2R_3} \\ &= \frac{(1.00\,\mathrm{V})(5.00\,\Omega + 4.00\,\Omega) - (3.00\,\mathrm{V})(5.00\,\Omega)}{(5.00\,\Omega)(2.00\,\Omega) + (5.00\,\Omega)(4.00\,\Omega) + (2.00\,\Omega)(4.00\,\Omega)} = -0.158\,\mathrm{A}\,. \end{aligned}$$

Substitute into the expression for i_3 to obtain

$$i_3 = \frac{\mathcal{E}_1 - i_2 R_1}{R_1 + R_3} = \frac{3.00\,\mathrm{V} - (-0.158\,\mathrm{A})(5.00\,\Omega)}{5.00\,\Omega + 4.00\,\Omega} = 0.421\,\mathrm{A}\,.$$

Finally,

$$i_1 = -i_2 - i_3 = -(-0.158\,\mathrm{A}) - (0.421\,\mathrm{A}) = -0.263\,\mathrm{A}\,.$$

Note that the current in R_1 is actually downward and the current in R_2 is to the right. The current in R_3 is also to the right.

The power dissipated in R_1 is $P_1 = i_1^2 R_1 = (-0.263\,\mathrm{A})^2(5.00\,\Omega) = 0.346\,\mathrm{W}$, the power dissipated in R_2 is $P_2 = i_2^2 R_2 = (-0.158\,\mathrm{A})^2(2.00\,\Omega) = 0.0499\,\mathrm{W}$, and the power dissipated in R_3 is $P_3 = i_3^2 R_3 = (0.421\,\mathrm{A})^2(4.00\,\Omega) = 0.709\,\mathrm{W}$.

(*b*) The power supplied by $\mathcal{E}_1$ is $i_3\mathcal{E}_1 = (0.421\,\mathrm{A})(3.00\,\mathrm{V}) = 1.26\,\mathrm{W}$ and the power supplied by $\mathcal{E}_2$ is $i_2\mathcal{E}_2 = (-0.158\,\mathrm{A})(1.00\,\mathrm{V}) = -0.158\,\mathrm{W}$. The negative sign indicates that $\mathcal{E}_2$ is actually absorbing energy from the circuit.

49P

(*a*) Use $P = \mathcal{E}^2/R_{\mathrm{eq}}$, where

$$R_{\mathrm{eq}} = 7.00\,\Omega + \frac{(12.0\,\Omega)(4.00\,\Omega)R}{(12.0\,\Omega)(4.0\,\Omega) + (12.0\,\Omega)R + (4.00\,\Omega)R}\,.$$

Put $P = 60.0\,\mathrm{W}$ and $\mathcal{E} = 24.0\,\mathrm{V}$ and solve for R: $R = 19.5\,\Omega$.

(*b*) Since $P \propto R_{\mathrm{eq}}$, we must minimize R_{eq}, which means $R = 0$.

(*c*) Now we must maximize R_{eq}, or set $R = \infty$.

(*d*) Since $R_{\mathrm{eq,\,max}} = 7.00\,\Omega + (12.0\,\Omega)(4.00\,\Omega)/(12.0\,\Omega + 4.00\,\Omega) = 10.0\,\Omega$, $P_{\min} = \mathcal{E}^2/R_{\mathrm{eq,\,max}} = (24.0\,\mathrm{V})^2/10.0\,\Omega = 57.6\,\mathrm{W}$.

Since $R_{\mathrm{eq,\,min}} = 7.00\,\Omega$, $P_{\max} = \mathcal{E}^2/R_{\mathrm{eq,\,min}} = (24.0\,\mathrm{V})^2/7.00\,\Omega = 82.3\,\mathrm{W}$.

50P

The voltage difference across R is $V_R = \mathcal{E}R'/(R' + 2.00\,\Omega)$, where $R' = (5.00\,\Omega R)/(5.00\,\Omega + R)$. So

$$\begin{aligned} P_R = \frac{V_R^2}{R} &= \frac{1}{R}\left(\frac{\mathcal{E}R'}{R' + 2.00\,\Omega}\right)^2 = \frac{1}{R}\left(\frac{\mathcal{E}}{1 + 2.00\,\Omega/R'}\right)^2 \\ &= \frac{\mathcal{E}^2}{R}\left[1 + \frac{(2.00\,\Omega)(5.00\,\Omega + R)}{(5.00\,\Omega)R}\right]^{-2} \equiv \frac{\mathcal{E}^2}{f(R)}\,. \end{aligned}$$

To maximize P_R we need to minimize the expression $f(R)$. Set

$$\frac{df(R)}{dR} = -\frac{4.00\,\Omega^2}{R^2} + \frac{49}{25} = 0$$

to obtain $R = \sqrt{(4.00\,\Omega^2)(25)/49} = 1.43\,\Omega$.

51P

(*a*) The copper wire and the aluminum sheath are connected in parallel, so the potential difference is the same for them. Since the potential difference is the product of the current and the resistance, $i_C R_C = i_A R_A$, where i_C is the current in the copper, I_A is the current in the aluminum, R_C is the resistance of the copper, and R_A is the resistance of the aluminum. The resistance of either component is given by $R = \rho L/A$, where ρ is the resistivity, L is the length, and A is the cross-sectional area. The resistance of the copper wire is $R_C = \rho_C L/\pi a^2$ and the resistance of the aluminum sheath is $R_A = \rho_A L/\pi(b^2 - a^2)$. Substitute these expressions into $i_C R_C = i_A R_A$, and cancel the common factors L and π to obtain

$$\frac{i_C \rho_C}{a^2} = \frac{i_A \rho_A}{b^2 - a^2}\,.$$

Solve this equation simultaneously with $i = i_C + i_A$, where i is the total current. You should get

$$i_C = \frac{r_C^2 \rho_C i}{(r_A^2 - r_C^2)\rho_C + r_C^2 \rho_A}$$

and

$$i_A = \frac{(r_A^2 - r_C^2)\rho_C i}{(r_A^2 - r_C^2)\rho_C + r_C^2 \rho_A}\,.$$

The denominators are the same and each has the value

$$\begin{aligned}(b^2 - a^2)\rho_C + a^2\rho_A &= \left[(0.380 \times 10^{-3}\,\mathrm{m})^2 - (0.250 \times 10^{-3}\,\mathrm{m})^2\right](1.69 \times 10^{-8}\,\Omega{\cdot}\mathrm{m}) \\ &\quad + (0.250 \times 10^{-3}\,\mathrm{m})^2(2.75 \times 10^{-8}\,\Omega{\cdot}\mathrm{m}) \\ &= 3.10 \times 10^{-15}\,\Omega{\cdot}\mathrm{m}^3\,.\end{aligned}$$

Thus

$$i_C = \frac{(0.250 \times 10^{-3}\,\mathrm{m})^2(2.75 \times 10^{-8}\Omega{\cdot}\mathrm{m})(2.00\,\mathrm{A})}{3.10 \times 10^{-15}\,\Omega{\cdot}\mathrm{m}^3} = 1.11\,\mathrm{A}$$

and

$$\begin{aligned}i_A &= \frac{\left[(0.380 \times 10^{-3}\,\mathrm{m})^2 - (0.250 \times 10^{-3}\,\mathrm{m})^2\right](1.69 \times 10^{-8}\,\Omega{\cdot}\mathrm{m})(2.00\,\mathrm{A})}{3.10 \times 10^{-15}\,\Omega{\cdot}\mathrm{m}^3} \\ &= 0.893\,\mathrm{A}\,.\end{aligned}$$

(*b*) Consider the copper wire. If V is the potential difference, then the current is given by $V = i_C R_C = i_C \rho_C L/\pi a^2$, so

$$L = \frac{\pi a^2 V}{i_C \rho_C} = \frac{\pi \times (0.250 \times 10^{-3}\,\mathrm{m})^2(12.0\,\mathrm{V})}{(1.11\,\mathrm{A})(1.69 \times 10^{-8}\,\Omega{\cdot}\mathrm{m})} = 126\,\mathrm{m}\,.$$

52P

The part of R_0 connected in parallel with R is given by $R_1 = R_0x/L$, where $L = 10\,\mathrm{cm}$. The voltage difference across R is then $V_R = \mathcal{E}R'/R_{\mathrm{eq}}$, where $R' = RR_1/(R+R_1)$ and $R_{\mathrm{eq}} = R_0(1-x/L)+R'$. Thus

$$P_R = \frac{V_R^2}{R} = \frac{1}{R}\left[\frac{\mathcal{E}RR_1/(R+R_1)}{R_0(1-x/L)+RR_1/(R+R_1)}\right]^2 .$$

Some simple algebra then leads to

$$P_R = \frac{100R(\mathcal{E}x/R_0)^2}{(100R/R_0+10x-x^2)^2} ,$$

where x is measured in cm.

53P

Since $i = \mathcal{E}/(r+R_{\mathrm{ext}})$ and $i_{\mathrm{max}} = \mathcal{E}/r$, we have $R_{\mathrm{ext}} = R(i_{\mathrm{max}}/i - 1)$ where $r = 1.50\,\mathrm{V}/1.00\,\mathrm{mA} = 1.50\times 10^3\,\Omega$. Thus
(*a*) $R_{\mathrm{ext}} = (1.5\times10^3\,\Omega)(1/10\% - 1) = 1.35\times10^4\,\Omega$;
(*b*) $R_{\mathrm{ext}} = (1.5\times10^3\,\Omega)(1/50\% - 1) = 1.50\times10^3\,\Omega$;
(*c*) $R_{\mathrm{ext}} = (1.5\times10^3\,\Omega)(1/90\% - 1) = 167\,\Omega$.
(*d*) Since $r = 20.0\,\Omega + R$, $R = 1.50\times10^3\,\Omega - 20.0\,\Omega = 1.48\times10^3\,\Omega$.

54P

(*a*) Put r roughly in the middle of its range; adjust current roughly with B; make fine adjustment with A.
(*b*) Relatively large percentage changes in A causes only small percentage charges in the resistance of the parallel combination, thus permitting fine adjustment; any change in A causes half as much change in this combination.

55P

(*a*) The current in R_1 is given by

$$i_1 = \frac{\mathcal{E}}{R_1+R_2R_3/(R_2+R_3)} = \frac{5.0\,\mathrm{V}}{2.0\,\Omega+(4.0\,\Omega)(6.0\,\Omega)/(4.0\,\Omega+6.0\,\Omega)} = 1.14\,\mathrm{A}\,.$$

Thus

$$i_3 = \frac{\mathcal{E}-V_1}{R_3} = \frac{\mathcal{E}-i_1R_1}{R_3} = \frac{5.0\,\mathrm{V}-(1.14\,\mathrm{A})(2.0\,\Omega)}{6.0\,\Omega} = 0.45\,\mathrm{A}\,.$$

(*b*) All we need to do is to interchange the subscripts 1 and 3 in the equation above. Now $i_3 = \mathcal{E}/[R_3+R_2R_1/(R_2+R_1)] = (5.0\,\mathrm{V})/[6.0\,\Omega+(2.0\,\Omega)(4.0\,\Omega)/(2.0\,\Omega+4.0\,\Omega)] = 0.6818\,\mathrm{A}$ and $i_1 = [(5.0\,\mathrm{V})-(0.6818\,\mathrm{A})(6.0\,\Omega)]/2.0\,\Omega = 0.45\,\mathrm{A}$, the same as before.

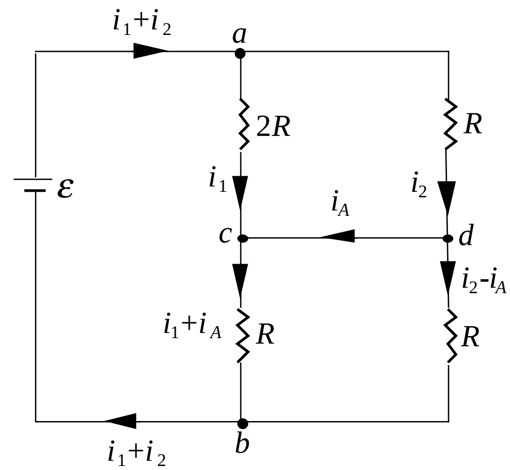
56P
From

$$V_{ab} = \mathcal{E} = 2Ri_1 + R(i_1 + i_A)$$
$$= Ri_2 + R(i_2 - i_A)$$

and

$$V_{ac} = 2i_1 R = V_{ad} = i_2 R\,,$$

we solve for i_A to obtain $i_A = \mathcal{E}/7R$.



57P
(a) $\mathcal{E} = V + ir = 12\,\mathrm{V} + (10\,\mathrm{A})(0.050\,\Omega) = 12.5\,\mathrm{V}$.
(b) Now $\mathcal{E} = V' + (i_{\mathrm{motor}} + 8.0\,\mathrm{A})r$, where $V' = i'_A R_{\mathrm{light}} = (8.0\,\mathrm{A})(12\,\mathrm{V}/10\,\mathrm{A}) = 9.6\,\mathrm{V}$. Solve for i_{motor}: $i_{\mathrm{motor}} = (\mathcal{E} - V')/r - 8.0\,\mathrm{A} = (12.5\,\mathrm{V} - 9.6\,\mathrm{V})/0.050\,\Omega - 8.0\,\mathrm{A} = 50\,\mathrm{A}$.

58P
The current in R_2 is i. Let i_1 be the current in R_1 and take it to be downward. According to the junction rule the current in the voltmeter is $i - i_1$ and it is downward. Apply the loop rule to the left-hand loop to obtain

$$\mathcal{E} - iR_2 - i_1 R_1 - ir = 0\,.$$

Apply the loop rule to the right-hand loop to obtain

$$i_1 R_1 - (i - i_1)R_V = 0\,.$$

Solve these equations for i_1. The second equation yields

$$i = \frac{R_1 + R_V}{R_V}\, i_1\,.$$

Substitute this into the first equation to obtain

$$\mathcal{E} - \frac{(R_2 + r)(R_1 + R_V)}{R_V}\, i_1 + R_1 i_1 = 0\,.$$

This has the solution

$$i_1 = \frac{\mathcal{E} R_V}{(R_2 + r)(R_1 + R_V) + R_1 R_V}\,.$$

The reading on the voltmeter is

$$
\begin{aligned}
i_1 R_1 &= \frac{\mathcal{E} R_V R_1}{(R_2 + r)(R_1 + R_V) + R_1 R_V} \\
&= \frac{(3.0\,\mathrm{V})(5.0 \times 10^3\,\Omega)(250\,\Omega)}{(300\,\Omega + 100\,\Omega)(250\,\Omega + 5.0 \times 10^3\,\Omega) + (250\,\Omega)(5.0 \times 10^3\,\Omega)} = 1.12\,\mathrm{V}\,.
\end{aligned}
$$

The current in the absence of the voltmeter can be obtained by taking the limit as R_V becomes infinitely large. Then

$$
i_1 R_1 = \frac{\mathcal{E} R_1}{R_1 + R_2 + r} = \frac{(3.0\,\mathrm{V})(250\,\Omega)}{250\,\Omega + 300\,\Omega + 100\,\Omega} = 1.15\,\mathrm{V}\,.
$$

The fractional error is $(1.15 - 1.12)/(1.15) = 0.030$. This is 3.0%.

59P

The current in the ammeter is given by $i_A = \mathcal{E}/(r + R_1 + R_2 + R_A)$. The current in R_1 and R_2 without the ammeter is $i = \mathcal{E}/(r + R_1 + R_2)$. The percent error is then

$$
\begin{aligned}
\frac{\Delta i}{i} &= \frac{i - i_A}{i} = 1 - \frac{r + R_1 + R_2}{r + R_1 + R_2 + R_A} = \frac{R_A}{r + R_1 + R_2 + R_A} \\
&= \frac{0.10\,\Omega}{2.0\,\Omega + 5.0\,\Omega + 4.0\,\Omega + 0.10\,\Omega} = 0.90\%\,.
\end{aligned}
$$

60P

The currents in R and R_V are i and $i' - i$, respectively. Since $V = iR = (i' - i)R_V$ we have, by dividing both sides with V, $1 = (i'/V - i/V)R_V = (1/R' - 1/R)R_V$, i.e.

$$
\frac{1}{R} = \frac{1}{R'} - \frac{1}{R_V}\,.
$$

61P

Let the current in the ammeter be i'. We have $V = i'(R + R_A)$, or $R = V/i' - R_A = R' - R_A$, where $R' = V/i'$ is the apparent reading of the resistance.

62P

(*a*) In the first case $i' = \mathcal{E}/R_{\mathrm{eq}} = \mathcal{E}/[R_A + R_0 + R_V R/(R + R_V)] = 12.0\,\mathrm{V}/[3.00\,\Omega + 100\,\Omega + (300\,\Omega)(85.0\,\Omega)/(300\,\Omega + 85.0\,\Omega)] = 7.09 \times 10^{-2}\,\mathrm{A}$, and $V = \mathcal{E} - i'(R_A + R_0) = 12.0\,\mathrm{V} - (0.0709\,\mathrm{A})(103.00\,\Omega) = 4.70\,\mathrm{V}$. In the second case $V = \mathcal{E}R'/(R' + R_0)$, where $R' = R_V(R + R_A)/(R_V + R + R_A) = (300\,\Omega)(300\,\Omega + 85.0\,\Omega)/(300\,\Omega + 85.0\,\Omega + 3.00\,\Omega) = 68.0\,\Omega$.

So $V = (12.0\,\mathrm{V})(68.0\,\Omega)/(68.0\,\Omega + 100\,\Omega) = 4.86\,\mathrm{V}$, and $i' = V(R + R_A) = 4.86\,\mathrm{V}/(300\,\Omega + 85.0\,\Omega) = 5.52 \times 10^{-2}\,\mathrm{A}$.
(b) In the first case $R' = V/i' = 4.70\,\mathrm{V}/(7.09 \times 10^{-2}\,\mathrm{A}) = 66.3\,\Omega$; and in the second case $R' = V/i' = 4.86\,\mathrm{V}/(5.52 \times 10^{-2}\,\mathrm{A}) = 88.0\,\Omega$.

63P

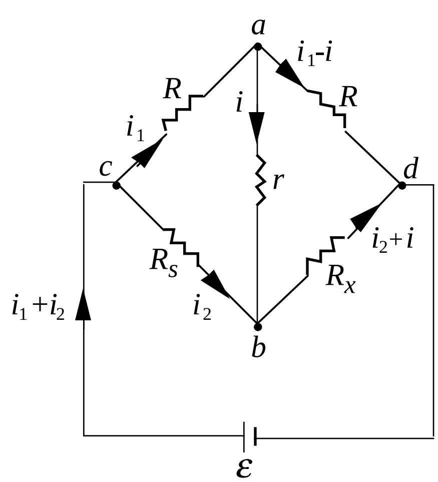
Let i_1 be the current in R_1 and R_2 and take it to be positive if it is toward point a in R_1. Let i_2 be the current in R_s and R_x and take it to be positive if it is toward b in R_s. The loop rule yields $(R_1 + R_2)i_1 - (R_x + R_s)i_2 = 0$. Since points a and b are at the same potential $i_1R_1 = i_2R_s$. The second equation gives $i_2 = i_1R_1/R_s$. This expression is substituted into the first equation to obtain

$$(R_1 + R_2)i_1 = (R_x + R_s)\,\frac{R_1}{R_s}\,i_1\,.$$

Solve for R_x. You should get $R_x = R_2R_s/R_1$.

64P

(a) Refer to the diagram to the right. From



$$\begin{aligned}\mathcal{E} = V_c - V_d &= i_1R + (i_1 - i)R \\ &= i_2R_s + (i_2 + i)R_x\end{aligned}$$

and

$$\begin{aligned}V_a - V_b &= (V_a - V_c) - (V_b - V_c) \\ &= -i_1R + i_2R_s = ir\,,\end{aligned}$$

we solve for i:

$$i = \frac{\mathcal{E}(R_s - R_x)}{(R + 2r)(R_s + R_x) + 2R_sR_x}\,.$$

(b) The condition for $i = 0$ is $R_s = R_x$. Since $R_1 = R_2 = R$, this is equivalent to $R_x = R_s(R_2/R_1)$, consistent with the result of Problem 63.

65E

Use $q = q_0e^{-t/\tau}$, or $t = \tau\ln(q_0/q)$.
(a) $t_{1/3} = \tau\ln[q_0/(2q_0/3)] = \tau\ln(3/2) = 0.41\tau$.
(b) $t_{2/3} = \tau\ln[q_0/(q_0/3)] = \tau\ln 3 = 1.1\tau$.

66E

(a) $\tau = RC = (1.40 \times 10^6\,\Omega)(1.80 \times 10^{-6}\,\mathrm{F}) = 2.52\,\mathrm{s}$.
(b) $q_o = \mathcal{E}C = (12.0\,\mathrm{V})(1.80\,\mu\,\mathrm{F}) = 21.6\,\mu\mathrm{C}$.
(c) The time t satisfies $q = q_0(1 - e^{-t/RC})$, or

$$t = RC\ln\left(\frac{q_0}{q_0 - q}\right) = (2.52\,\mathrm{s})\ln\left(\frac{21.6\,\mu\mathrm{C}}{21.6\,\mu\mathrm{C} - 16.0\,\mu\mathrm{C}}\right) = 3.40\,\mathrm{s}\,.$$

67E

During charging the charge on the positive plate of the capacitor is given by

$$q = C\mathcal{E}(1 - e^{-t/\tau})\,,$$

where C is the capacitance, $\mathcal{E}$ is applied emf, and τ is the time constant. The equilibrium charge is $q_{\mathrm{eq}} = C\mathcal{E}$. You want $q = 0.99q_{\mathrm{eq}} = 0.99C\mathcal{E}$, so

$$0.99 = 1 - e^{-t/\tau}\,.$$

Thus

$$e^{-t/\tau} = 0.01\,.$$

Take the natural logarithm of both sides to obtain $t/\tau = -\ln 0.01 = 4.6$ and $t = 4.6\tau$.

68E

(a) The voltage difference V across the capacitor varies with time as $V(t) = \mathcal{E}(1 - e^{-t/RC})$. At $t = 1.30\,\mu\,\mathrm{s}$ we have $V(t) = 5.00\,\mathrm{V}$, so $5.00\,\mathrm{V} = (12.0\,\mathrm{V})(1 - e^{-1.30\,\mu\mathrm{s}/RC})$, which gives $\tau = (1.30\,\mu\,\mathrm{s})/\ln(12/7) = 2.41\,\mu\mathrm{s}$.
(b) $C = \tau/R = 2.41\,\mu\mathrm{s}/15.0\,\mathrm{k}\Omega = 161\,\mathrm{pF}$.

69P

(a) The charge on the positive plate of the capacitor is given by

$$q = C\mathcal{E}\,(1 - e^{-t/\tau})\,,$$

where $\mathcal{E}$ is the emf of the battery, C is the capacitance, and τ is the time constant. The value of τ is $\tau = RC = (3.00 \times 10^6\,\Omega)(1.00 \times 10^{-6}\,\mathrm{F}) = 3.00\,\mathrm{s}$. At $t = 1.00\,\mathrm{s}$, $t/\tau = (1.00\,\mathrm{s})/(3.00\,\mathrm{s}) = 0.333$ and the rate at which the charge is increasing is

$$\frac{dq}{dt} = \frac{C\mathcal{E}}{\tau}\,e^{-t/\tau} = \frac{(1.00 \times 10^{-6})(4.00\,\mathrm{V})}{3.00\,\mathrm{s}}\,e^{-0.333} = 9.55 \times 10^{-7}\,\mathrm{C/s}\,.$$

(*b*) The energy stored in the capacitor is given by

$$U_C = \frac{q^2}{2C}\,.$$

and its rate of change is

$$\frac{dU_C}{dt} = \frac{q}{C}\,\frac{dq}{dt}\,.$$

Now

$$q = C\mathcal{E}\,(1 - e^{-t/\tau}) = (1.00\times 10^{-6})(4.00\,\mathrm{V})(1 - e^{-0.333}) = 1.13\times 10^{-6}\,\mathrm{C}\,,$$

so

$$\frac{dU_C}{dt} = \left(\frac{1.13\times 10^{-6}\,\mathrm{C}}{1.00\times 10^{-6}\,\mathrm{F}}\right)(9.55\times 10^{-7}\,\mathrm{C/s}) = 1.08\times 10^{-6}\,\mathrm{W}\,.$$

(*c*) The rate at which energy is being dissipated in the resistor is given by $P = i^2R$. The current is 9.55×10^{-7} A, so

$$P = (9.55\times 10^{-7}\,\mathrm{A})^2(3.00\times 10^6\,\Omega) = 2.74\times 10^{-6}\,\mathrm{W}\,.$$

(*d*) The rate at which energy is delivered by the battery is

$$i\mathcal{E} = (9.55\times 10^{-7}\,\mathrm{A})(4.00\,\mathrm{V}) = 3.82\times 10^{-6}\,\mathrm{W}\,.$$

The energy delivered by the battery is either stored in the capacitor or dissipated in the resistor. Conservation of energy requires that $i\mathcal{E} = (q/C)\,(dq/dt) + i^2R$. Except for some round-off error the numerical results support the conservation principle.

70P

(*a*) The potential difference V across the plates of a capacitor is related to the charge q on the positive plate by $V = q/C$, where C is capacitance. Since the charge on a discharging capacitor is given by $q = q_0\,e^{-t/\tau}$, where q_0 is the charge at time $t = 0$ and τ is the time constant, this means

$$V = V_0\,e^{-t/\tau}\,,$$

where q_0/C was replaced by V_0, the initial potential difference. Solve for τ by dividing the equation by V_0 and taking the natural logarithm of both sides. The result is:

$$\tau = -\frac{t}{\ln{(V/V_0)}} = -\frac{10.0\,\mathrm{s}}{\ln{[(1.00\,\mathrm{V})/(100\,\mathrm{V})]}} = 2.17\,\mathrm{s}\,.$$

(*b*) At $t = 17.0\,\mathrm{s}$, $t/\tau = (17.0\,\mathrm{s})/(2.17\,\mathrm{s}) = 7.83$, so

$$V = V_0\,e^{-t/\tau} = (100\,\mathrm{V})\,e^{-7.83} = 3.96\times 10^{-2}\,\mathrm{V}\,.$$

71P

The time it takes for the voltage difference across the capacitor to reach V_L is given by $V_L = \mathcal{E}(1 - e^{-t/RC})$. Solve for R:

$$R = \frac{t}{C\ln[\mathcal{E}/(\mathcal{E} - V_L)]} = \frac{0.500\,\text{s}}{(0.150\times 10^{-6}\,\text{F})\ln[95.0\,\text{V}/(95.0\,\text{V} - 72.0\,\text{V})]} = 2.35\times 10^{6}\,\Omega\,,$$

where we used $t = 0.500\,\text{s}$ given in the problem.

72P

(a) The initial energy stored in a capacitor is given by

$$U_C = \frac{q_0^2}{2C}\,,$$

where C is the capacitance and q_0 is the initial charge on one plate. Thus

$$q_0 = \sqrt{2CU_C} = \sqrt{2(1.0\times 10^{-6}\,\text{F})(0.50\,\text{J})} = 1.0\times 10^{-3}\,\text{C}\,.$$

(b) The charge as a function of time is given by $q = q_0\,e^{-t/\tau}$, where g is the time constant. The current is the derivative of the charge:

$$i = -\frac{dq}{dt} = \frac{q_0}{\tau}\,e^{-t/\tau}$$

and the initial current is given by this function evaluated for $t = 0$: $i_0 = q_0/\tau$. The time constant is $\tau = RC = (1.0\times 10^{-6}\,\text{F})(1.0\times 10^{6}\,\Omega) = 1.0\,\text{s}$. Thus $i_0 = (1.0\times 10^{-3}\,\text{C})/(1.0\,\text{s}) = 1.0\times 10^{-3}\,\text{A}$.

(c) Substitute $q = q_0\,e^{-t/\tau}$ into $V_C = q/C$ to obtain

$$V_C = \frac{q_0}{C}\,e^{-t/\tau} = \left(\frac{1.0\times 10^{-3}\,\text{C}}{1.0\times 10^{-6}\,\text{F}}\right)e^{-t/1.0\,\text{s}} = (1.0\times 10^{3}\,\text{V})\,e^{-1.0t}\,,$$

where t is measured in seconds. Substitute $i = (q_0/\tau)\,e^{-t/\tau}$ into $V_R = iR$ to obtain

$$V_R = \frac{q_0R}{\tau}\,e^{-t/\tau} = \frac{(1.0\times 10^{-3}\,\text{C})(1.0\times 10^{6}\,\Omega)}{1.0\,\text{s}}\,e^{-t/1.0\,\text{s}} = (1.0\times 10^{3}\,\text{V})\,e^{-1.0t}\,,$$

where t is measured in seconds.

(d) Substitute $i = (q_0/\tau)\,e^{-t/\tau}$ into $P = i^2R$ to obtain

$$P = \frac{q_0^2R}{\tau^2}\,e^{-2t/\tau} = \frac{(1.0\times 10^{-3}\,\text{C})^2(1.0\times 10^{6}\,\Omega)}{(1.0\,\text{s})^2}\,e^{-2t/1.0\,\text{s}} = (1.0\,\text{W})\,e^{-2.0t}\,,$$

where t is again measured in seconds.

73P

The potential difference across the capacitor varies as a function of time t as $V(t) = V_0 e^{-t/RC}$, which gives

$$R = \frac{t}{C\ln(V_0/V)} = \frac{2.0\,\text{s}}{(2.0\times10^{-6}\,\text{F})\ln 4} = 7.2\times10^5\,\Omega\,.$$

Here we used $V = V_0/4$ at $t = 2.0\,\text{s}$.

74P

(a) The charge q on the capacitor as a function of time is $q(t) = (\mathcal{E}C)(1 - e^{-t/RC})$, so the charging current is $i(t) = dq/dt = (\mathcal{E}/R)e^{-t/RC}$. The energy supplied by the emf is then

$$U = \int_0^\infty \mathcal{E}i dt = \frac{\mathcal{E}^2}{R}\int_0^\infty e^{-t/RC}dt = C\mathcal{E}^2 = 2U_C\,,$$

where $U_C = \frac{1}{2}C\mathcal{E}^2$ is the energy stored in the capacitor.
(b)

$$U_R = \int_0^\infty i^2 R dt = \frac{\mathcal{E}^2}{R}\int_0^\infty e^{-2t/RC}\,dt = \frac{1}{2}C\mathcal{E}^2\,.$$

75P

Use the result of 73P: $R = t/[C\ln(V_0/V)]$. Then for $t_{\text{min}} = 10.0\,\mu\text{s}$

$$R_{\text{min}} = \frac{10.0\,\mu\text{s}}{(0.220\,\mu\text{F})\ln(5.00/0.800)} = 24.8\,\Omega\,;$$

and for $t_{\text{max}} = 6.00\,\text{ms}$

$$R_{\text{max}} = \left(\frac{6.00\,\text{ms}}{10.0\,\mu\text{s}}\right)(24.8\,\Omega) = 1.49\times10^4\,\Omega\,,$$

where in the last equation we used $\tau = RC$.

76P

When S is open for a long time, the charge on C is $q_i = \mathcal{E}_2 C$. When S is closed for a long time, the current i in R_1 and R_2 is $i = (\mathcal{E}_2 - \mathcal{E}_1)/(R_1 + R_2) = (3.0\,\text{V} - 1.0\,\text{V})/(0.20\,\Omega + 0.40\,\Omega) = 3.33\,\text{A}$. The voltage difference V across the capacitor is then $V = \mathcal{E}_2 - iR_2 = 3.0\,\text{V} - (3.33\,\text{A})(0.40\,\Omega) = 1.67\,\text{V}$. Thus the final charge on C is $q_f = VC$. So the change in the change on the capacitor is $\Delta q = q_f - q_i = (V - \mathcal{E}_2)C = (1.67\,\text{V} - 3.0\,\text{V})(10\,\mu\text{F}) = -13\,\mu\text{C}$.

77P

(a) At $t = 0$ the capacitor is completely uncharged and the current in the capacitor branch is as it would be if the capacitor were replaced by a wire. Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is downward. Let i_3 be the current in R_3 and take it to be positive if it is downward. The junction rule produces

$$i_1 = i_2 + i_3 \,,$$

the loop rule applied to the left-hand loop produces

$$\mathcal{E} - i_1 R_1 - i_2 R_2 = 0 \,,$$

and the loop rule applied to the right-hand loop produces

$$i_2 R_2 - i_3 R_3 = 0 \,.$$

Since the resistances are all the same you can simplify the mathematics by replacing R_1, R_2, and R_3 with R. The solution to the three simultaneous equations is

$$i_1 = \frac{2\mathcal{E}}{3R} = \frac{2(1.2 \times 10^3\,\text{V})}{3(0.73 \times 10^6\,\Omega)} = 1.1 \times 10^{-3}\,\text{A}$$

and

$$i_2 = i_3 = \frac{\mathcal{E}}{3R} = \frac{1.2 \times 10^3\,\text{V}}{3(0.73 \times 10^6\,\Omega)} = 5.5 \times 10^{-4}\,\text{A} \,.$$

At $t = \infty$ the capacitor is fully charged and the current in the capacitor branch is 0. Then $i_1 = i_2$ and the loop rule yields

$$\mathcal{E} - i_1 R_1 - i_1 R_2 = 0 \,.$$

The solution is

$$i_1 = i_2 = \frac{\mathcal{E}}{2R} = \frac{1.2 \times 10^3\,\text{V}}{2(0.73 \times 10^6\,\Omega)} = 8.2 \times 10^{-4}\,\text{A} \,.$$

(*b*) Take the upper plate of the capacitor to be positive. This is consistent with current into that plate. The junction and loop equations are

$$i_1 = i_2 + i_3 \,,$$

$$\mathcal{E} - i_1 R - i_2 R = 0 \,,$$

and

$$-\frac{q}{C} - i_3 R + i_2 R = 0 \,.$$

Use the first equation to substitute for i_1 in the second and obtain $\mathcal{E} - 2i_2 R - i_3 R = 0$. Thus $i_2 = (\mathcal{E} - i_3 R)/2R$. Substitute this expression into the third equation above to obtain $-(q/C) - (i_3 R) + (\mathcal{E}/2) - (i_3 R/2) = 0$. Now replace i_3 with dq/dt and obtain

$$\frac{3R}{2}\frac{dq}{dt} + \frac{q}{C} = \frac{\mathcal{E}}{2} \,.$$

This is just like the equation for an RC series circuit, except that the time constant is $\tau = 3RC/2$ and the impressed potential difference is $\mathcal{E}/2$. The solution is

$$q = \frac{C\mathcal{E}}{2}\left[1 - e^{-2t/3RC}\right] .$$

The current in the capacitor branch is

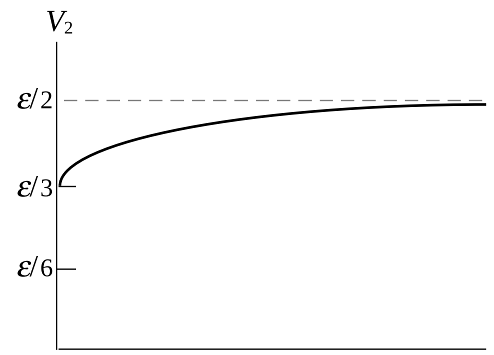
$$i_3 = \frac{dq}{dt} = \frac{\mathcal{E}}{3R}\, e^{-2t/3RC} .$$

The current in the center branch is

$$\begin{aligned} i_2 &= \frac{\mathcal{E}}{2R} - \frac{i_3}{2} = \frac{\mathcal{E}}{2R} - \frac{\mathcal{E}}{6R}\, e^{-2t/3RC} \\ &= \frac{\mathcal{E}}{6R}\left[3 - e^{-2t/3RC}\right] \end{aligned}$$

and the potential difference across R_2 is

$$V_2 = i_2 R = \frac{\mathcal{E}}{6}\left[3 - e^{-2t/3RC}\right] .$$



The graph is shown to the right.

(c) For $t = 0$, $e^{-2t/3RC}$ is 1 and $V_R = \mathcal{E}/3 = (1.2 \times 10^3\,\mathrm{V})/3 = 400\,\mathrm{V}$. For $t = \infty$, $e^{-2t/3RC}$ is 0 and $V_R = \mathcal{E}/2 = (1.2 \times 20^3\,\mathrm{V})/2 = 600\,\mathrm{V}$.

(d) "A long time" means after several time constants. Then the current in the capacitor branch is very small and can be approximated by 0.

78P

(a) $i = 0:$ current in $\mathcal{E}_1 = 0.665\,\mathrm{A}$ up, current in $\mathcal{E}_2 = 1.26\,\mathrm{A}$ up; $V_{AB} = 6.29\,\mathrm{V}$;

$i = 4.0\,\mathrm{A}:$ current in $\mathcal{E}_1 = 0.248\,\mathrm{A}$ up, current in $\mathcal{E}_2 = 3.21\,\mathrm{A}$ up; $V_{AB} = -14.2\,\mathrm{V}$;

$i = 8.0\,\mathrm{A}:$ current in $\mathcal{E}_1 = 0.169\,\mathrm{A}$ down, current in $\mathcal{E}_2 = 5.17\,\mathrm{A}$ up; $V_{AB} = -34.6\,\mathrm{V}$;

$i = 12\,\mathrm{A}:$ current in $\mathcal{E}_1 = 0.587\,\mathrm{A}$ down, current in $\mathcal{E}_2 = 7.13\,\mathrm{A}$ up; $V_{AB} = -55.1\,\mathrm{V}$;

$\mathcal{E}_1$ is discharging for $i = 0$ and $4.0\,\mathrm{A}$; it is charging for $i = 6.0$ and $12\,\mathrm{A}$; $\mathcal{E}_2$ is discharging for all values of i;

(b) the emf is $6.29\,\mathrm{V}$ and the resistance is $2.56\,\Omega$

79P

(a) $V_T = -ri + \mathcal{E}$;

(b) $13.6\,\mathrm{V}$;

(c) $0.060\,\Omega$

80P

(a) $i_2 = i_1 + i_4 + i_5$, $i_3 + i_4 + i_5 = 0$, $-16\,\mathrm{V} + (7\,\Omega)i_1 + (5\,\Omega)i_2 + 4\,\mathrm{V} = 0$, $10\,\mathrm{V} + (8\,\Omega)i_3 - (9\,\Omega)i_4 - 4\,\mathrm{V} - (5\,\Omega)i_2 = 0$, $12\,\mathrm{V} - (4\,\Omega)i_5 + (9\,\Omega)i_4 = 0$;

(b)

$$[A] = \begin{bmatrix} 1 & -1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 7\,\Omega & 5\,\Omega & 0 & 0 & 0 \\ 0 & -5\,\Omega & 8\,\Omega & -9\,\Omega & 0 \\ 0 & 0 & 0 & 9\,\Omega & -4\,\Omega \end{bmatrix};$$

$$[C] = \begin{bmatrix} 0 \\ 0 \\ 12\,\mathrm{V} \\ -6\,\mathrm{V} \\ -12\,\mathrm{V} \end{bmatrix}$$

(c) $\{i_1, i_2, i_3, i_4, i_5\} = \{0.717\,\mathrm{A}, 1.40\,\mathrm{A}, -0.680\,\mathrm{A}, -0.714\,\mathrm{A}, 1.39\,\mathrm{A}\}$

81P

(a) $6.4\,\mathrm{V}$;
(b) $3.6\,\mathrm{W}$;
(c) $17\,\mathrm{W}$;
(d) $-5.6\,\mathrm{W}$;
(e) a

82P

(a) $V_c = \mathcal{E}_0 e^{-t/\tau}$;
(b) $12\,\mathrm{V}$;
(c) $0.77\,\mathrm{s}$;
(d) $3.8\,\mu\mathrm{F}$