Chapter 30

Answer to Checkpoint Questions

1. (a), (c), (b)

2. b, c, a

3. d, tie of a and c, then b

4. (d), (a), tie of (b) and (c) (zero)

Answer to Questions

1. (c), (d), then (a) and (b) tie

2. (a) into; (b) greater

3. 2 and 4

4. (c), (a), (b)

5. (a), (b), (c) (see Eq. 30-11)

6. (c), (a), (b)

7. (b), (d), (a) (zero)

8. (a) 1, 3, 2; (b) less than

9. (a) 1, $+x$; 2, $-y$; (b) 1, $+y$; 2, $+x$

10. a, b, c

11. outward

12. a, tie of b and d, then c

13. c and d tie, then b, a

14. b, a, d, c (zero)

15. (d), then tie of (a) and (e), then (b), (c)

16. (a) 2 opposite 4; (b) 2 and 4 opposite 6; (c) 1 and 5 opposite 3 and 6; (d) 1 and 5 opposite 2, 3, and 4

17. 0 (scalar product is zero)

18. (*a*) $4a$; (*b*) $3d$

Solutions to Exercises & Problems

1E

Solve i from Eq. 30-6:

$$i = \frac{2\pi r B}{\mu_0} = \frac{2\pi(88.0\times10^{-2}\,\mathrm{m})(7.30\times10^{-6}\,\mathrm{T})}{4\pi\times10^{-7}\,\mathrm{T\cdot m/A}} = 32.1\,\mathrm{A}\,.$$

2E

Use Eq. 30-6:

$$B = \frac{\mu_0 i}{2\pi r} = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(50\,\mathrm{A})}{2\pi(2.6\times10^{-3}\,\mathrm{m}/2)} = 7.7\times10^{-3}\,\mathrm{T}\,.$$

3E

(*a*) The magnitude of the magnetic field due to the current in the wire, at a point a distance r from the wire, is given by

$$B = \frac{\mu_0 i}{2\pi r}\,.$$

Put $r = 20\,\mathrm{ft} = 6.10\,\mathrm{m}$. Then

$$B = \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(100\ \mathrm{A})}{2\pi(6.10\,\mathrm{m})} = 3.3\times10^{-6}\,\mathrm{T} = 3.3\,\mu\mathrm{T}\,.$$

(*b*) This is about one-sixth the magnitude of the Earth's field. It will affect the compass reading.

4E

The current i due to the electron flow is $i = ne = (5.6\times10^{14}/\mathrm{s})(1.6\times10^{-19}\,\mathrm{C}) = 9.0\times10^{-5}\,\mathrm{A}$. Thus

$$B = \frac{\mu_0 i}{2\pi r} = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(9.0\times10^{-5}\,\mathrm{A})}{2\pi(1.5\times10^{-3}\,\mathrm{m})} = 1.2\times10^{-8}\,\mathrm{T}.$$

5E

Use $B(x,\, y,\, z) = (\mu_0/4\pi) i\, \Delta\mathbf{s} \times \mathbf{r}/r^3$, where $\Delta\mathbf{s} = \Delta s\,\mathbf{j}$ and $\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\mathbf{k}$. Thus

$$\mathbf{B}(x,\, y,\, z) = \left(\frac{\mu_0}{4\pi}\right) \frac{i\, \Delta s\,\mathbf{j} \times (x\mathbf{i} + y\,\mathbf{j} + z\mathbf{k})}{(x^2 + y^2 + z^2)^{3/2}} = \frac{\mu_0 i\, \Delta s\, (z\,\mathbf{i} - x\,\mathbf{k})}{4\pi(x^2 + y^s + z^2)^{3/2}} \,.$$

(a)

$$\begin{aligned}\mathbf{B}(0,\, 0,\, 5.0\,\mathrm{m}) &= \frac{(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A})(2.0\,\mathrm{A})(3.0 \times 10^{-2}\,\mathrm{m})(5.0\,\mathrm{m})\,\mathbf{i}}{4\pi[0^2 + 0^2 + (5.0\,\mathrm{m})^2]^{3/2}} \\ &= (2.4 \times 10^{-10}\,\mathbf{i})\,\mathrm{T}\,.\end{aligned}$$

(b) $\mathbf{B}(0,\, 6.0\,\mathrm{m},\, 0)$, since $x = z = 0$.

(c)

$$\begin{aligned}\mathbf{B}(7.0\,\mathrm{m},\, 7.0\,\mathrm{m},\, 0) &= \frac{(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A})(2.0\,\mathrm{A})(3.0 \times 10^{-2}\,\mathrm{m})(-7.0\,\mathrm{m})\,\mathbf{k}}{4\pi[(7.0\,\mathrm{m})^2 + (7.0\,\mathrm{m})^2 + 0^2]^{3/2}} \\ &= (4.3 \times 10^{-11}\,\mathbf{k})\,\mathrm{T}\,.\end{aligned}$$

(d)

$$\begin{aligned}\mathbf{B}(-3.0\,\mathrm{m},\, -4.0\,\mathrm{m},\, 0) &= \frac{(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A})(2.0\,\mathrm{A})(3.0 \times 10^{-2}\,\mathrm{m})(3.0\,\mathrm{m})\,\mathbf{k}}{4\pi[(-3.0\,\mathrm{m})^2 + (-4.0\,\mathrm{m})^2 + 0^2]^{3/2}} \\ &= (1.4 \times 10^{-10}\,\mathbf{k})\,\mathrm{T}\,.\end{aligned}$$

6E

The points must be along a line parallel to the wire and a distance r from it, where r satisfies

$$B_{\text{wire}} = \frac{\mu_0 i}{2\pi r} = B_{\text{ext}}\,,$$

or

$$r = \frac{\mu_0 i}{2\pi B_{\text{ext}}} = \frac{(1.26 \times 10^{-6}\,\mathrm{T{\cdot}\,m/A})(100\,\mathrm{A})}{2\pi(5.0 \times 10^{-3}\,\mathrm{T})} = 4.0 \times 10^{-3}\,\mathrm{m}\,.$$

7E

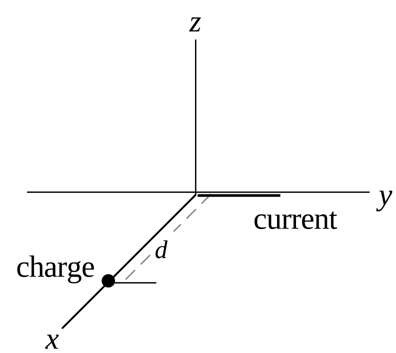
(a) The field due to the wire, at a point $8.0\,\mathrm{cm}$ from the wire, must be $39\,\mu\mathrm{T}$ and must be directed due south. Since $B = \mu_0 i/2\pi r$,

$$i = \frac{2\pi r B}{\mu_0} = \frac{2\pi(0.080\,\mathrm{m})(39 \times 10^{-6}\,\mathrm{T})}{4\pi \times 10^{-7}\,\mathrm{T{\cdot}\,m/A}} = 16\,\mathrm{A}\,.$$

(b) The current must be from west to east to produce a field to the south at points below it.

8E

Set up a coordinate system as shown to the right. The $\mathbf{B}$-field at the location of the charge q is



$$\mathbf{B} = \frac{\mu_0 i}{2\pi d}(-\mathbf{k}).$$

Thus

$$\mathbf{F}_q = q\mathbf{v} \times \mathbf{B} = \frac{\mu_0 i}{2\pi d}(\mathbf{k} \times \mathbf{v}).$$

(a) Now $\mathbf{v} = v(-\mathbf{i})$ so

$$\mathbf{F}_q = \frac{\mu_0 iv}{2\pi d}\,\mathbf{k} \times (-\mathbf{i}) = \frac{\mu_0 iqv}{2\pi d}(-\mathbf{j}),$$

i.e., $\mathbf{F}_q$ has a magnitude of $\mu_0 iqv/2\pi d$ and is directed against the current direction.
(b) Now the direction $\mathbf{v}$ is reversed so $\mathbf{F}_q = \mu_0 iqv\,\mathbf{j}/2\pi d$.

9E

The straight segment of the wire produces no magnetic fields at C. The field from the two semi-circular loop cancel at C. So $B_C = 0$.

10E

Use the same coordinated system as in 8E. Then $\mathbf{F}_e = (-e\mu_0 iv/2\pi d)(\mathbf{k} \times \mathbf{v})$, as obtained there.
(a) Now $\mathbf{v} = v(-\mathbf{i})$ so

$$\begin{aligned}\mathbf{F}_e &= \frac{\mu_0 iev\,\mathbf{j}}{2\pi d} = \frac{(1.26 \times 10^{-6}\,\mathrm{T \cdot m/A})(50\,\mathrm{A})(1.6 \times 10^{-19}\,\mathrm{C})(1.0 \times 10^{7}\,\mathrm{m/s})\mathbf{j}}{2\pi(5.0 \times 10^{-2}\,\mathrm{m})} \\ &= (3.2 \times 10^{-16}\,\mathrm{N})\,\mathbf{j}.\end{aligned}$$

(c) Now $\mathbf{v} = \pm v\,\mathbf{k}$ so $\mathbf{F}_e \propto \mathbf{k} \times \mathbf{v} = 0$.

11P

(a) The straight segment of the wire produces no magnetic field at C.
(b) For the semicircular loop

$$\begin{aligned}B_C &= \frac{\mu_0}{4\pi}\int_{\mathrm{loop}} \frac{|d\mathbf{s} \times \mathbf{r}|}{r^3} = \frac{\mu_0 i}{4\pi}\int_{\mathrm{loop}} \frac{r\,ds}{r^3} \\ &= \frac{(\mu_0 i)(\pi r)}{4\pi R^2} = \frac{\mu_0 i}{4R}.\end{aligned}$$

$\mathbf{B}$ points into the page.
(c) The same as (b), since the straight segments do not contribute to $\mathbf{B}$.

12P

Since sections AH and JD do not contribute to $\mathbf{B}_C$, we only need to consider the two arcs. For the smaller arc

$$\mathbf{B}_{C1} = \left(\frac{\mu_0}{4\pi}\right)\int \frac{i\, d\mathbf{s}_1 \times \mathbf{r}}{r^3} = \frac{\mu_0 i}{4\pi}\int \frac{R_1\, ds_1}{R_1^3}\,\mathbf{e} = \frac{\mu_0 i \mathbf{e}}{4R_1}\,,$$

where $\mathbf{e}$ is a unit vector pointing into the page. Similarly, for the larger arc

$$\mathbf{B}_{C2} = -\frac{\mu_0 i \mathbf{e}}{4R_2}\,.$$

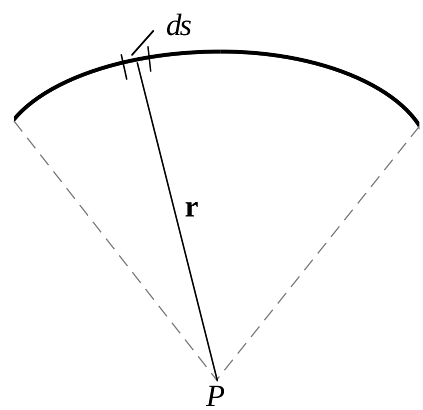
Thus

$$\mathbf{B}_C = \mathbf{B}_{C1} + \mathbf{B}_{C2} = \frac{\mu_0\, i(R_2 - R_1)\mathbf{e}}{4R_1 R_2}\,.$$

13P

First find the magnetic field of a circular arc at its center. Let d$\mathbf{s}$ be an infinitesimal segment of the arc and $\mathbf{r}$ be the vector from the segment to the arc center. $d\mathbf{s}$ and $\mathbf{r}$ are perpendicular to each other, so the contribution of the segment to the field at the center has magnitude

$$dB = \frac{\mu_0 i ds}{4\pi r^2}\,.$$



The field is into the page if the current is from left to right in the diagram and out of the page if the current is from right to left. All segments contribute magnetic fields in the same direction. Furthermore r is the same for all of them. Thus the magnitude of the total field at the center is given by

$$B = \frac{\mu_0 i s}{4\pi r^2} = \frac{\mu_0 i \theta}{4\pi r}\,.$$

Here s is the arc length and θ is the angle (in radians) subtended by the arc at its center. The second expression was obtained by replacing s with $r\theta$. θ must be in radians for this expression to be valid.

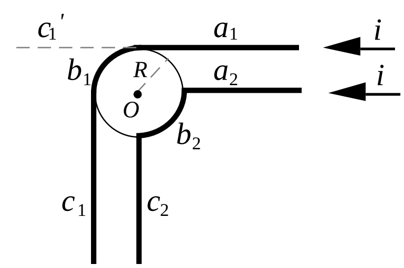
Now consider the circuit of Fig. 30–41a. The magnetic field produced by the inner arc has magnitude $\mu_0 i\theta/4\pi b$ and is out of the page. The field produced by the outer arc has magnitude $\mu_0 i\theta/4\pi a$ and is into the page. The two straight segments of the circuit do not produce fields at the center of the arcs because the vector $\mathbf{r}$ from any point on them to the center is parallel or antiparallel to the current at that point. If the positive direction is out of the page then the total magnetic field at the center is

$$B = \frac{\mu_0 i\theta}{4\pi}\left(\frac{1}{b} - \frac{1}{a}\right).$$

Since $b < a$ the total field is out of the page.

14P

Label the various sections of the wires as shown. Firstly, the sections a_2 and c_2 do not coutribute to $\mathbf{B}$ at point O, as point O lies on the straight line coinciding with a_2 and c_2. Secondly, the $\mathbf{B}$-field due to the curved sections b_1 and b_2 cancel each other at point O. This leaves us just a_1 and c_1. Finally, if we relocate c_1 to c_1', its contribution to the $\mathbf{B}$-field at O will not change. Note that a_1 and c_1' together do form an infinite straight wire carrying a current i to the left.



15P

(a) The contribution to $\mathbf{B}_a$ due to the straight sections of the wire is

$$\mathbf{B}_{a1} = \frac{\mu_0 i\mathbf{e}}{2\pi R},$$

where $\mathbf{e}$ is a unit vector which points out of the page. For the bent section (see 12P)

$$\mathbf{B}_{a2} = \frac{\mu_0 i\mathbf{e}}{4R}.$$

Thus

$$\begin{aligned}\mathbf{B}_a &= \mathbf{B}_{a1} + \mathbf{B}_{a2} = \frac{\mu_0 i}{2R}\left(\frac{1}{\pi} + \frac{1}{2}\right)\mathbf{e} \\ &= \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(10\,\mathrm{A})}{2(5.0\times10^{-3}\,\mathrm{m})}\left(\frac{1}{\pi} + \frac{1}{2}\right)\mathbf{e} = (1.0\times10^{-3}\,\mathrm{T})\,\mathbf{e}.\end{aligned}$$

(b) Now we only need to consider the two straight wires:

$$\mathbf{B}_b = \frac{2\mu_0 i\mathbf{e}}{2\pi R} = \frac{2(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(10\,\mathrm{A})\mathbf{e}}{2\pi(5.0\times10^{-3}\,\mathrm{m})} = (8.0\times10^{-4}\,\mathrm{T})\,\mathbf{e}.$$

Here the factor of 2 in the numerator is due to the fact that two wires are involved.

16P

Sum the fields of the two straight wires and the circular arc. Look at the derivation of the expression for the field of a long straight wire, leading to Eq. 30–6. Since the wires we are considering are infinite in only one direction the field of either of them is half the field of an infinite wire. That is, the magnitude is $\mu_0 i/4\pi r$, where r is the distance from the end of the wire to the center of the arc. It is the radius of the arc. The fields of both wires are out of the page at the center of the arc.

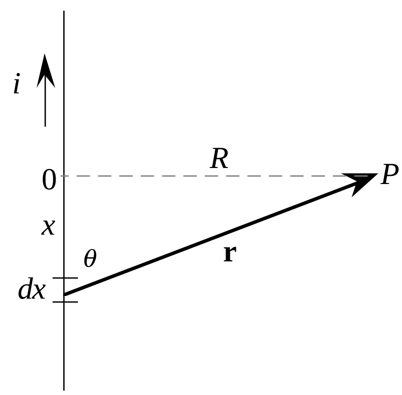
Now find an expression for the field of the arc, at its center. Divide the arc into infinitesimal segments. Each segment produces a field in the same direction. If ds is the length of a segment the magnitude of the field it produces at the arc center is $(\mu_0 i/4\pi r^2)\,ds$. If θ is the angle subtended by the arc in radians, then $r\theta$ is the length of the arc and the total field of the arc is $\mu_0 i\theta/4\pi r$. For the arc of the diagram the field is into the page. The total field at the center, due to the wires and arc together, is

$$B = \frac{\mu_0 i}{4\pi r} + \frac{\mu_0 i}{4\pi r} - \frac{\mu_0 i\theta}{4\pi r} = \frac{\mu_0 i}{4\pi r}\,(2-\theta)\,.$$

For this to vanish θ must be 2 radians.

17P

Put the x axis along the wire with the origin at the midpoint and the current in the positive x direction. All segments of the wire produce magnetic fields at P that are into the page so we simply divide the wire into infinitesimal segments and sum the fields due to all the segments. The diagram shows one infinitesimal segment, with width dx. According to the Biot-Savart law the magnitude of the field it produces at P is given by



$$dB = \frac{\mu_0 i}{4\pi}\frac{\sin\theta}{r^2}\,dx\,.$$

θ and r are functions of x. Replace r with $\sqrt{x^2+R^2}$ and $\sin\theta$ with $R/r = R/\sqrt{x^2+R^2}$, then integrate from $x=-L/2$ to $x=L/2$. The total field is

$$B = \frac{\mu_0 iR}{4\pi}\int_{-L/2}^{L/2}\frac{dx}{(x^2+R^2)^{3/2}} = \frac{\mu_0 iR}{4\pi}\,\frac{1}{R^2}\,\frac{x}{(x^2+R^2)^{1/2}}\Bigg|_{-L/2}^{L/2} = \frac{\mu_0 i}{2\pi R}\,\frac{L}{\sqrt{L^2+4R^2}}\,.$$

If $L \gg R$ then R^2 in the denominator can be ignored and

$$B = \frac{\mu_0 i}{2\pi R}$$

is obtained. This is the field of a long straight wire. For points close to a finite wire the field is quite similar to that of an infinitely long wire.

18P

Follow the same steps as in the solution of 17P above but replace R with D, change the lower limit of integration to $-L$, and change the upper limit to 0. The magnitude of the total field is

$$B = \frac{\mu_0 i D}{4\pi}\int_{-L}^{0}\frac{dx}{(x^2+D^2)^{3/2}} = \frac{\mu_0 i D}{4\pi}\frac{1}{D^2}\frac{x}{(x^2+D^2)^{1/2}}\bigg|_{-L}^{0} = \frac{\mu_0 i}{4\pi D}\frac{L}{\sqrt{L^2+D^2}}\,.$$

19P

You can easily check that each of the four sides produces the same magnetic field B_1 at the center of the square. Apply the result of 17P for B_1 and let $R = a/2$ and $L = a$ to obtain

$$B = 4B_1 = \frac{4\mu_0 i}{2\pi(a/2)}\frac{a}{(a^2+a^2)^{1/2}} = \frac{2\sqrt{2}\mu_0 i}{\pi a}\,.$$

20P

The **B**-field produced by the four sides of the rectangle have the same direction. For each of the two longer sides (see 17P)

$$B_L = \frac{\mu_0 i}{2\pi(W/2)}\frac{L}{(L^2+W^2)^{1/2}}\,.$$

Similarly for each of the shorter sides

$$B_W = \frac{\mu_0 i}{2\pi(L/2)}\frac{W}{(L^2+W^2)^{1/2}}\,.$$

Thus

$$\begin{aligned} B = 2B_L + 2B_W &= \frac{2\mu_0 i L}{\pi W(L^2+W^2)^{1/2}} + \frac{2\mu_0 W}{\pi L(L^2+W^2)^{1/2}} \\ &= \frac{2\mu_0 i(L^2+W^2)^{1/2}}{\pi L W}\,. \end{aligned}$$

For $L \gg W$

$$B \to \frac{2\mu_0 i L}{\pi L W} = \frac{2\mu_0 i}{\pi W}\,,$$

which is consistent with Eq. 30-15 for $W = 2d$ and $x = 0$.

21P

When the fields of the four sides are summed vectorially the horizontal components add to zero. The vertical components are all the same, so the total field is given by

$$B_{\text{total}} = 4B\cos\theta = \frac{4Ba}{2R} = \frac{4Ba}{\sqrt{4x^2+a^2}}\,.$$

Thus

$$B_{\text{total}} = \frac{4\mu_0 i a^2}{\pi(4x^2+a^2)\sqrt{4x^2+2a^2}}\,.$$

For $x=0$ the expression reduces to

$$B_{\text{total}} = \frac{4\mu_0 i a^2}{\pi a^2\sqrt{2}a} = \frac{2\sqrt{2}\mu_0 i}{\pi a}\,,$$

in agreement with the result of 19P.

22P

The square has sides of length $L/4$. The magnetic field at the center of the square is given by the result of 19P, with $a = L/4$ and $x=0$. It is

$$B_{\text{sq}} = \frac{8\sqrt{2}\mu_0 i}{\pi L} = 11.31\frac{\mu_0 i}{\pi L}\,.$$

The radius of the circle is $R = L/2\pi$. Use Eq. 30–28 of the text, with $R = L/2\pi$ and $z=0$. The field is

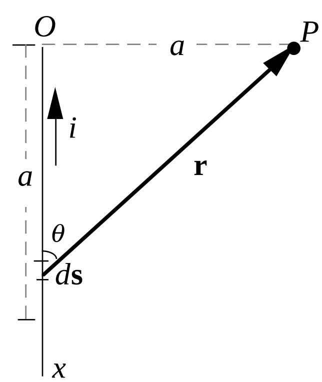
$$B_{\text{circ}} = \frac{\pi^2\mu_0 i}{\pi L} = 9.87\frac{\mu_0 i}{\pi L}\,.$$

The square produces the larger magnetic field.

23P

Since $d\mathbf{s}$ is parallel to $\mathbf{r}$,

$$\mathbf{B}_Q \propto \int \frac{i d\mathbf{s}\times\mathbf{r}}{r^3} = 0\,.$$



Let $\mathbf{e}$ be a unit vector unit vector pointing into the page. At point P

$$\begin{aligned}\mathbf{B}_P &= \frac{\mu_0}{4\pi}\int\frac{i d\mathbf{s}\times\mathbf{r}}{r^3} = \frac{\mu_0 i\mathbf{e}}{4\pi}\int\frac{\sin\theta\, ds}{r^2}\\ &= \frac{\mu_0 i\mathbf{e}}{4\pi}\int_0^a\frac{a\,dx}{r^3} = \frac{\mu_0 i\mathbf{e}}{4\pi}\int_0^a\frac{a\,dx}{(a^2+x^2)^{3/2}}\\ &= \frac{\sqrt{2}\mu_0 i\mathbf{e}}{8\pi a}\,.\end{aligned}$$

24P

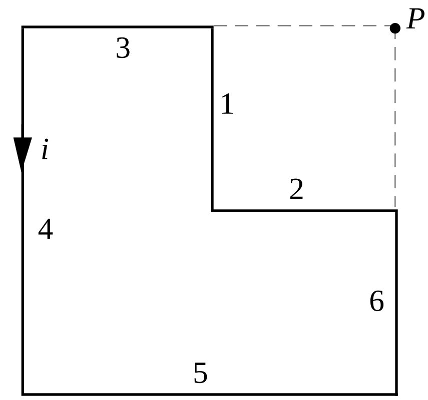
Obviously, $B_{P3} = B_{P6} = 0$,

$$B_{P1} = B_{P2} = \frac{\sqrt{2}\mu_0 i}{8\pi a},$$

and

$$B_{P4} = B_{P5} = \frac{\sqrt{2}\mu_0 i}{8\pi(2a)}$$

(see 23P). Thus

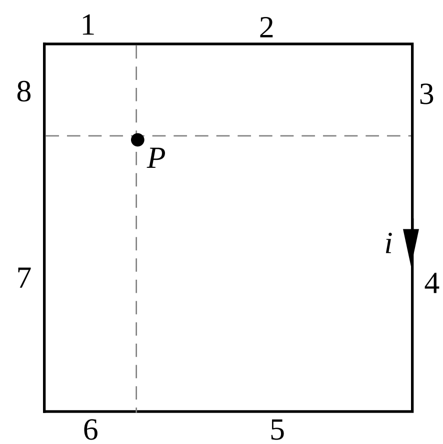


$$\mathbf{B}_P = \sum_{n=1}^{6} \mathbf{B}_{Pn} = (B_{P1} + B_{P2})\mathbf{e} - (B_{P4} + B_{P5})\mathbf{e}$$
$$= 2\left(\frac{\sqrt{2}\mu_0 i}{8\pi a} - \frac{\sqrt{2}\mu_0 i}{16\pi a}\right)\mathbf{e} = \frac{\sqrt{2}\mu_0 i\mathbf{e}}{8\pi a},$$

where $\mathbf{e}$ is a unit vector pointing into the page.

25P

Let $\mathbf{e}$ be a unit vector pointing into the page. Use the results of 18P and 23P to calculate B_{P1} through B_{P8}:



$$B_{P1} = B_{P8} = \frac{\sqrt{2}\mu_0 i}{8\pi(a/4)} = \frac{\sqrt{2}\mu_0 i}{2\pi a},$$

$$B_{P4} = B_{P5} = \frac{\sqrt{2}\mu_0 i}{8\pi(3a/4)} = \frac{\sqrt{2}\mu_0 i}{6\pi a},$$

$$B_{P2} = B_{P7} = \frac{\mu_0 i}{4\pi(a/4)} = \frac{3a/4}{[(3a/4)^2 + (a/4)^2]^{1/2}}$$
$$= \frac{3\mu_0 i}{\sqrt{10}\pi a},$$

and

$$B_{P3} = B_{P6} = \frac{\mu_0 i}{4\pi(3a/4)} = \frac{a/4}{[(a/4)^2 + (3a/4)^2]^{1/2}} = \frac{\mu_0 i}{3\sqrt{10}\pi a}.$$

Finally,

$$\mathbf{B}_P = \sum_{n=1}^{8} B_{Pn}\mathbf{e}$$
$$= 2\frac{\mu_0 i}{\pi a}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{6} + \frac{3}{\sqrt{10}} + \frac{1}{3\sqrt{10}}\right)\mathbf{e}$$
$$= \frac{2(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(10\,\mathrm{A})}{\pi(8.0 \times 10^{-2}\,\mathrm{m})}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{6} + \frac{3}{\sqrt{10}} + \frac{1}{3\sqrt{10}}\right)\mathbf{e}$$
$$= (2.0 \times 10^{-4}\,\mathrm{T})\,\mathbf{e},$$

where $\mathbf{e}$ is a unit vector pointing into the page.

26P

Consider a section of the ribbon of thickness dx located a distance x away from point P. The current it carries is $di = i\,dx/w$, and its contribution to B_P is

$$dB_P = \frac{\mu_0 di}{2\pi x} = \frac{\mu_0 i dx}{2\pi x w}\,.$$

Thus

$$B_P = \int d\,B_P = \frac{\mu_0 i}{2\pi w}\int_d^{d+w}\frac{dx}{x} = \frac{\mu_0 i}{2\pi w}\ln\left(1+\frac{w}{d}\right),$$

and $\mathbf{B}_P$ points upward.

27E

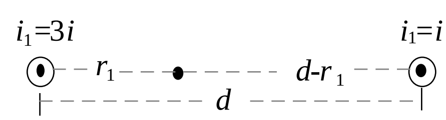
(a) If the currents are parallel, the two fields are in opposite directions in the region between the wires. Since the currents are the same the total field is zero along the line that runs halfway between the wires. There is no possible current for which the field does not vanish.

(b) If the currents are antiparallel, the fields are in the same direction in the region between the wires. At a point halfway between they have the same magnitude, $\mu_0 i/2\pi r$. Thus the total field at the midpoint has magnitude $B = \mu_0 i/\pi r$ and

$$i = \frac{\pi r B}{\mu_0} = \frac{\pi(0.040\,\mathrm{m})(300\times10^{-6}\,\mathrm{T})}{4\pi\times10^{-7}\,\mathrm{T{\cdot}m/A}} = 30\,\mathrm{A}\,.$$

28E

The point P at which $B_P = 0$ form a line parallel to both currents passing through the line joining the two wires, as shown. Note that $B_{P1}\propto i_1/r_1$, and $B_{P2}\propto i_2/(d-r_1)$. Let $B_{P1} = B_{P2}$ to obtain $i_1/r_1 = i_2/(d-r_1)$. Solve for r_1:



$$r_1 = \frac{i_1 d}{i_1+i_2} = \frac{3id}{3i+i} = \frac{3d}{4}\,.$$

29E

The current i_2 carried by wire 2 must be out of the page. Since $B_{P1}\propto i_1/r_1$ where $i_1 = 6.5\,\mathrm{A}$ and $r_1 = 0.75\,\mathrm{cm}+1.5\,\mathrm{cm} = 2.25\,\mathrm{cm}$, and $B_{P2}\propto i_2/r_2$ where $r_2 = 1.5\,\mathrm{cm}$, from $B_{P1} = B_{P2}$ we get

$$i_2 = i_1\left(\frac{r_2}{r_1}\right) = (6.5\,\mathrm{A})\left(\frac{1.5\,\mathrm{cm}}{2.25\,\mathrm{cm}}\right) = 4.3\,\mathrm{A}\,.$$

30E
Lable these wires 1 through 5, left to right. Then

$$\begin{aligned}\mathbf{F}_1 &= \frac{\mu_0 i^2}{2\pi}\left(\frac{1}{d}+\frac{1}{2d}+\frac{1}{3d}+\frac{1}{4d}\right)\mathbf{j} = \left(\frac{25\mu_0 i^2}{24\pi d}\right)\mathbf{j} \\ &= \frac{(13)(4\pi\times 10^{-7}\,\mathrm{T\cdot m/A})(3.00\,\mathrm{A})^2(1.00\,\mathrm{m})\mathbf{j}}{24\pi(8.00\times 10^{-2}\,\mathrm{m})} \\ &= (4.69\times 10^{-5}\,\mathrm{T})\,\mathbf{j}\,;\end{aligned}$$

$$\mathbf{F}_2 = \frac{\mu_0 i^2}{2\pi}\left(\frac{1}{2d}+\frac{1}{3d}\right)\mathbf{j} = \left(\frac{5\mu_0 i^2}{12\pi d}\right)\mathbf{j} = (1.88\times 10^{-5}\,\mathrm{T})\,\mathbf{j}\,;$$

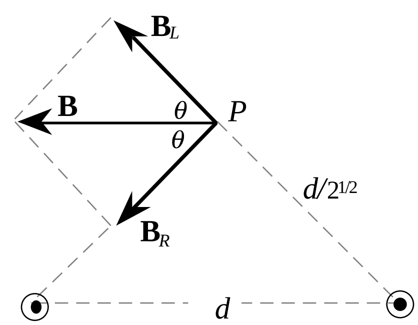
$F_3 = 0$ (because of symmetry); $\mathbf{F}_4 = -\mathbf{F}_2$; and $\mathbf{F}_5 = -\mathbf{F}_1$.

31E
Consider, for example, $x > d$. Then in Fig. 30-10 the direction of $\mathbf{B}_b$ is reversed, and

$$B(x) = \left|B_a(x) - B_b(x)\right| = \left|\frac{\mu_0 i}{2\pi(d+x)} - \frac{\mu_0 i}{2\pi(x-d)}\right| = \frac{\mu_0 i d}{\pi(d^2-x^2)}\,.$$

32E
(a) Refer to the figure to the right. We have

$$\begin{aligned}B_P &= |B_L + B_R| \\ &= 2B_L\cos\theta \\ &= 2\left(\frac{\mu_0 i}{2\pi d\sqrt{2}}\right)\cos\theta \\ &= \frac{(4\pi\times 10^{-7}\,\mathrm{T\cdot m/A})(100\,\mathrm{A})(\cos 45^\circ)\sqrt{2}}{\pi(10\times 10^{-2}\,\mathrm{m})} \\ &= 4.0\times 10^{-4}\,\mathrm{T}\,.\end{aligned}$$



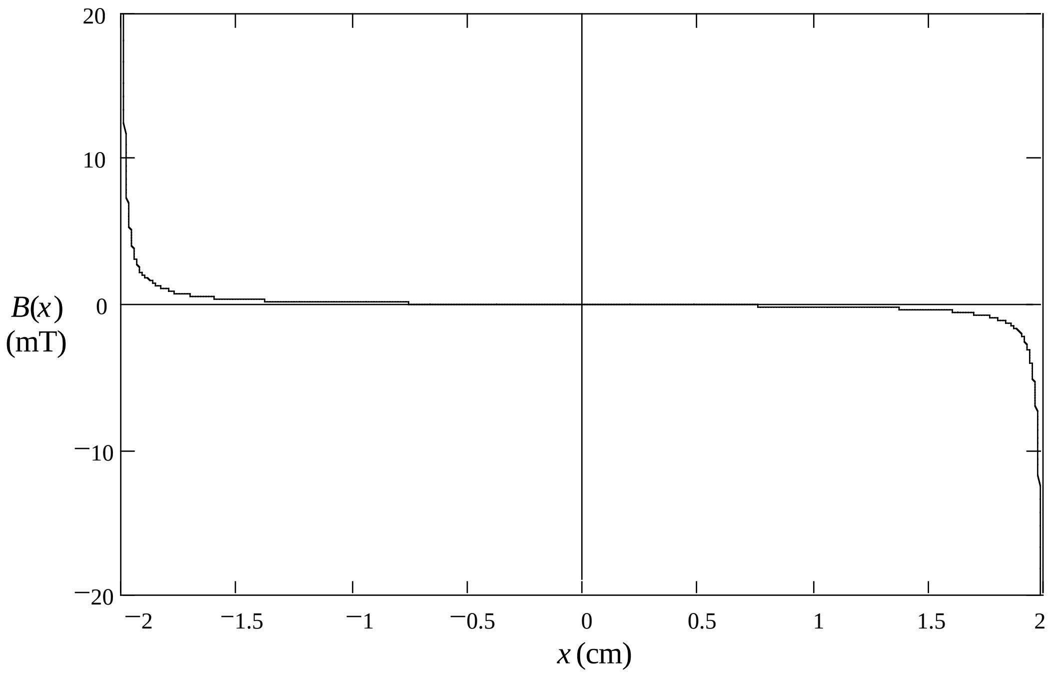
$\mathbf{B}_P$ points to the left.
(b) Reverse the direction of $\mathbf{B}_R$ in the figure above but keep its magnitude unchanged. Obviously $\mathbf{B}_P$ still has a magnitude of $4.0\times 10^{-4}\,\mathrm{T}$ but now points up the page.

33P
For $0 < x < d$

$$B(x) = B_a(x) - B_b(x) = \frac{\mu_0 i}{2\pi(d+x)} - \frac{\mu_0 i}{2\pi(d-x)} = \frac{\mu_0 i x}{\pi(x^2-d^2)}\,;$$

and for $x > d$

$$B(x) = B_a(x) + B_b(x) = \frac{\mu_0 i}{2\pi(x+d)} + \frac{\mu_0 i}{2\pi(x-d)} = \frac{\mu_0 i x}{\pi(x^2 - d^2)}\,.$$



The same expression for $B(x)$ is valid for $x < 0$ due to symmetry.

34P

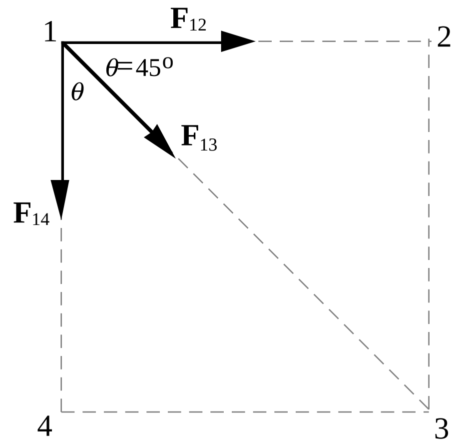
Each wire produces a field with magnitude given by $B = \mu_0 i/2\pi r$, where r is the distance from the corner of the square to the center. According to the Pythagorean theorem the diagonal of the square has length $\sqrt{2}a$, so $r = a/\sqrt{2}$ and $B = \mu_0 i/\sqrt{2}\pi a$. The fields due to the wires at the upper left and lower right corners both point toward the upper right corner of the square. The fields due to the wires at the upper right and lower left corners both point toward the upper left corner. The horizontal components cancel and the vertical components sum to

$$\begin{aligned} B_{\text{total}} &= 4\frac{\mu_0 i}{\sqrt{2}\pi a}\cos 45^\circ = \frac{2\mu_0 i}{\pi a} \\ &= \frac{2(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(20\,\text{A})}{\pi(0.20\,\text{m})} = 8.0 \times 10^{-5}\,\text{T}\,. \end{aligned}$$

In the calculation $\cos 45^\circ$ was replaced with $1/\sqrt{2}$. The total field points up the page.

35P

Refer to the figure as shown. For example, the force on wire 1 is

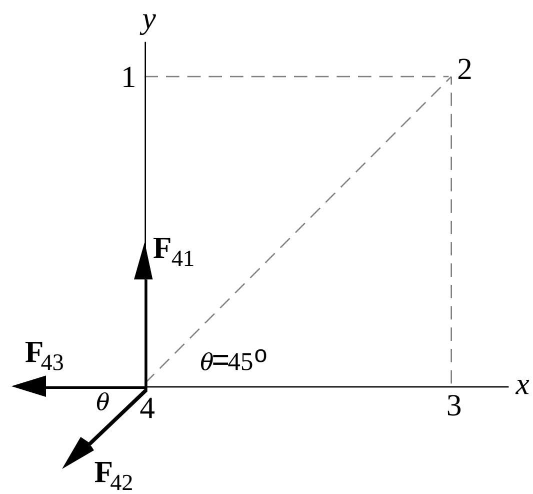


$$\begin{aligned} F_1 &= |\mathbf{F}_{12} + \mathbf{F}_{13} + \mathbf{F}_{14}| \\ &= 2F_{12}\cos\theta + F_{13} \\ &= 2\left(\frac{\mu_0 i^2}{2\pi a}\right)\cos 45^\circ + \frac{\mu_0 i^2}{2\sqrt{2}\pi a} \\ &= 0.338\left(\frac{\mu_0 i^2}{a}\right). \end{aligned}$$

$\mathbf{F}_1$ points from wire 1 to the center of the square.

36P

Use $\mathbf{F}_4 = \mathbf{F}_{14} + \mathbf{F}_{24} + \mathbf{F}_{34}$. The components of $\mathbf{F}_4$ are then given by



$$\begin{aligned} \mathbf{F}_{4x} &= -\mathbf{F}_{43} - \mathbf{F}_{42}\cos\theta \\ &= -\frac{\mu_0 i^2}{2\pi a} - \frac{\mu_0 i^2 \cos 45^\circ}{2\sqrt{2}\pi a} \\ &= -\frac{3\mu_0 i^2}{4\pi a} \end{aligned}$$

and

$$\begin{aligned} F_{4y} &= F_{41} - F_{42}\sin\theta \\ &= \frac{\mu_0 i^2}{2\pi a} - \frac{\mu_0 i^2 \sin 45^\circ}{2\sqrt{2}\pi a} \\ &= \frac{\mu_0 i^2}{4\pi a}. \end{aligned}$$

Thus

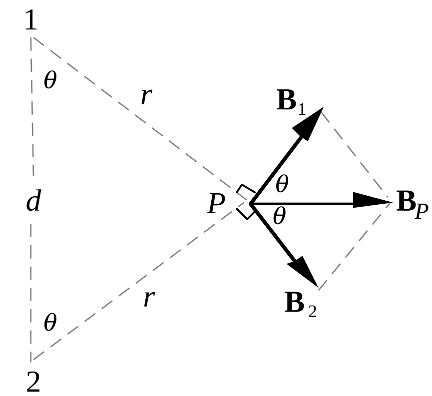
$$F_4 = (F_{4x}^2 + F_{4y}^2)^{1/2} = \left[\left(-\frac{3\mu_0 i^2}{4\pi a}\right)^2 + \left(\frac{\mu_0 i^2}{4\pi a}\right)^2\right]^{1/2} = \frac{\sqrt{10}\mu_0 i^2}{4\pi a},$$

and $\mathbf{F}_4$ makes an angle ϕ with the positive x axis, where

$$\phi = \tan^{-1}\left(\frac{F_{4y}}{F_{4x}}\right) = \tan^{-1}\left(-\frac{1}{3}\right) = 162^\circ.$$

37P

(a) From the figure to the right



$$\begin{aligned} B_P &= |\mathbf{B}_1 + \mathbf{B}_2| = 2B_1\cos\theta \\ &= 2\left(\frac{\mu_0 i}{2\pi r}\right)\left(\frac{d/2}{r}\right) \\ &= \frac{\mu_0 i d}{2\pi[R^2 + (d/2)^2]} \\ &= \frac{2\mu_0 i d}{\pi(4R^2 + d^2)}\,. \end{aligned}$$

(b) $\mathbf{B}_P$ points to the right, as shown.

38P

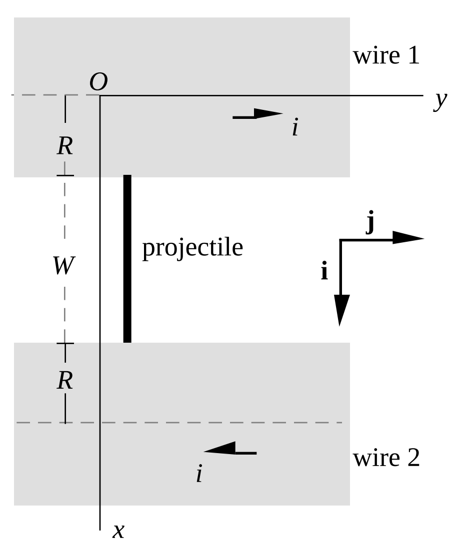
The forces on the two sides of length b cancel out. For the remaining two sides

$$\begin{aligned} F &= \frac{\mu_0 i_1 i_2 L}{2\pi}\left(\frac{1}{a} - \frac{1}{a+d}\right) = \frac{\mu i_1 i_2 b}{2\pi a(a+b)} \\ &= \frac{(1.26\times 10^{-6}\,\mathrm{T\cdot m/A})(30\,\mathrm{A})(20\,\mathrm{A})(8.0\,\mathrm{cm})(30\times 10^{-2}\,\mathrm{m})}{2\pi(1.0\,\mathrm{cm} + 8.0\,\mathrm{cm})} \\ &= 3.2\times 10^{-3}\,\mathrm{N}\,, \end{aligned}$$

and $\mathbf{F}$ points toward the wire.

39P

Consider a segment of the projectile between x and $x + dx$. Use Eq. 30-13 to find the magnetic force on the segment:



$$\begin{aligned} d\mathbf{F} &= d\mathbf{F}_1 + d\mathbf{F}_2 \\ &= i(dx\,\mathbf{i})\times\mathbf{B}_1(x) + i(dx\,\mathbf{i})\times\mathbf{B}_2(x) \\ &= i[B_1(x) + B_2(x)]\,\mathbf{j}\,dx \\ &= i\left[\frac{\mu_0 i}{4\pi x} + \frac{\mu_0 i}{4\pi(2R + w - x)}\right]\mathbf{j}\,dx\,, \end{aligned}$$

where $\mathbf{j}$ is a unit vector pointing to the right. Here we used the expression for the magnetic field of a semi-infinite wire. Thus

$$\mathbf{F} = \int d\mathbf{F} = \frac{i^2\mu}{4\pi}\int_R^{R+w}\left(\frac{1}{x} + \frac{1}{2R+w-x}\right)\mathbf{j}\,dx = \frac{\mu_0 i^2}{2\pi}\ln\left(1+\frac{w}{R}\right)\mathbf{j}\,.$$

(b) Use $\Delta K = \frac{1}{2}mv_f^2 = W_{\text{ext}} = \int \mathbf{F}\cdot d\mathbf{s}$ for the projectile:

$$\begin{aligned} v_f &= \left(\frac{2W_{\text{ext}}}{m}\right)^{1/2} = \left[\frac{2}{m}\int_0^L \frac{\mu_0 i^2}{2\pi}\ln\left(1+\frac{w}{R}\right)dy\right]^{1/2} \\ &= \left[\frac{2(4\pi\times 10^{-7}\,\text{T}\cdot\text{m/A})(450\times 10^3\,\text{A})^2(4.0\,\text{m})\ln(1+1.2\,\text{cm}/6.7\,\text{cm})}{2\pi(10\times 10^{-3}\,\text{kg})}\right]^{1/2} \\ &= 2.3\times 10^3\,\text{m/s}\,. \end{aligned}$$

40E

(a) Two of the currents are out of the page and one is into the page, so the net current enclosed by the path is 2.0 A, out of the page. Since the path is traversed in the clockwise sense a current into the page is positive and a current out of the page is negative, as indicated by the right-hand rule associated with Ampere's law. Thus

$$\oint \mathbf{B}\cdot d\mathbf{s} = -\mu_0 i = -(2.0\,\text{A})(4\pi\times 10^{-7}\,\text{T}\cdot\text{m/A}) = -2.5\times 10^{-6}\,\text{T}\cdot\text{m}\,.$$

(b) The net current enclosed by the path is zero (two currents are out of the page and two are into the page), so $\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 i = 0$.

41E

A close look at the path reveals that only currents no.1 and no.6 are enclosed. Thus

$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0(6i_0 - i_0) = 5\mu_0 i\,.$$

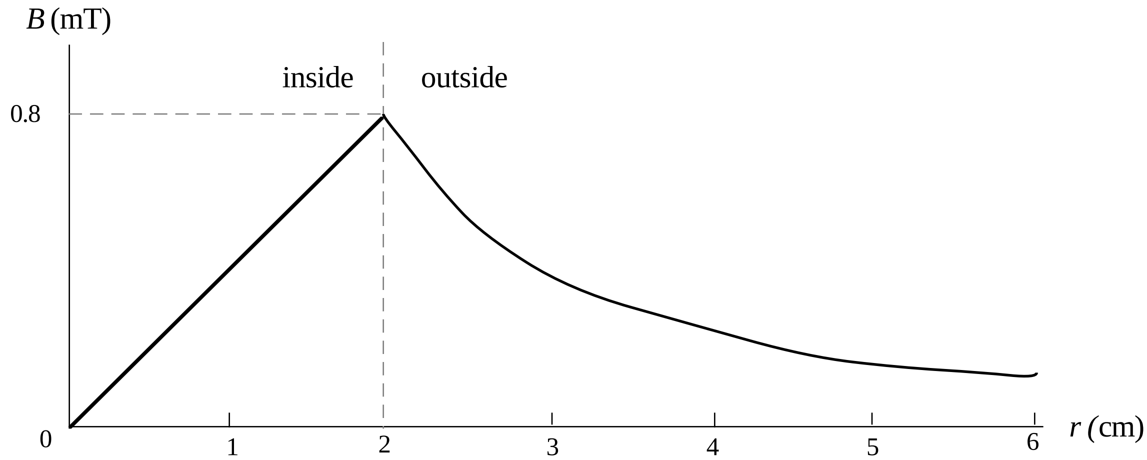
42E

The area enclosed by the loop L is $A = \frac{1}{2}(4d)(3d) = 6d^2$. Thus

$$\begin{aligned} \oint_c \mathbf{B}\cdot d\mathbf{s} &= \mu_0 i = \mu_0 j A \\ &= (4\pi\times 10^{-7}\,\text{T}\cdot\text{m/A})(15\,\text{A/m}^2)(6)(0.20\,\text{m})^2 = 4.5\times 10^{-6}\,\text{T}\cdot\text{m}\,. \end{aligned}$$

43E

Use Eq. 30-22 for the B-field inside the wire and 30-19 for that ouside the wire. The plot is shown in the next page.



44P

Use Ampere's law: $\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 i$, where the integral is around a closed loop and i is the net current through the loop. For path 1

$$\oint_1 \mathbf{B}\cdot d\mathbf{s} = \mu_0(-5.0\,\mathrm{A} + 3.0\,\mathrm{A}) = (-2.0\,\mathrm{A})(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A})$$
$$= -2.5 \times 10^{-6}\,\mathrm{T{\cdot}m}\,;$$

and for path 2

$$\oint_2 \mathbf{B}\cdot d\mathbf{s} = \mu_0(-5.0\,\mathrm{A} - 5.0\,\mathrm{A} - 3.0\,\mathrm{A}) = (-13.0\,\mathrm{A})(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A})$$
$$= -1.6 \times 10^{-5}\,\mathrm{T{\cdot}m}\,.$$

45P

Use Ampere's law. For the dotted loop shown on the diagram $i = 0$. The integral $\int \mathbf{B}\cdot d\mathbf{s}$ is zero along the bottom, right, and top sides of the loop. Along the right side the field is zero, along the top and bottom sides the field is perpendicular to d**s**. If ℓ is the length of the left edge then direct integration yields $\oint \mathbf{B}\cdot d\mathbf{s} = B\ell$, where B is the magnitude of the field at the left side of the loop. Since neither B nor ℓ is zero, Ampere's law is contradicted. We conclude that the geometry shown for the magnetic field lines is in error. The lines actually bulge outward and their density decreases gradually, not precipitously as shown.

46P

(a) For the circular path L of radius r concentric with the conductor

$$\oint_L \mathbf{B}\cdot d\mathbf{s} = 2\pi r B(r) = \mu_0 i_{\,\mathrm{enclosed}} = \mu_0 i \frac{\pi(r^2 - b^2)}{\pi(a^2 - b^2)}\,.$$

Thus

$$B(r) = \frac{\mu_0 i}{2\pi(a^2 - b^2)}\left(\frac{r^2 - b^2}{r}\right).$$

(b) At $r = a$,

$$B(a) = \frac{\mu_0 i}{2\pi(a^2 - b^2)}\left(\frac{a^2 - b^2}{a}\right) = \frac{\mu_0 i}{2\pi a}.$$

At $r = b$, $B(b) \propto r^2 - b^2 = 0$. For $b = 0$

$$B(r) = \frac{\mu_0 i}{2\pi a^2}\frac{r^2}{r} = \frac{\mu_0 i r}{2\pi a^2}.$$

47P

Use $\oint \mathbf{s} \cdot d\mathbf{B} = 2\pi r B = \mu_0 i_{\text{enclosed}}$, or $B = \mu_0 i_{\text{enclosed}}/2\pi r$.

(a) $r < c$:

$$B(r) = \frac{\mu_0 i_{\text{encl}}}{2\pi r} = \frac{\mu_0 i}{2\pi r}\left(\frac{\pi r^2}{\pi c^2}\right) = \frac{\mu_0 i r}{2\pi c^2}.$$

(b) $c < r < b$:

$$B(r) = \frac{\mu_0 i_{\text{encl}}}{2\pi r} = \frac{\mu_0 i}{2\pi r}.$$

(c) $b < r < a$:

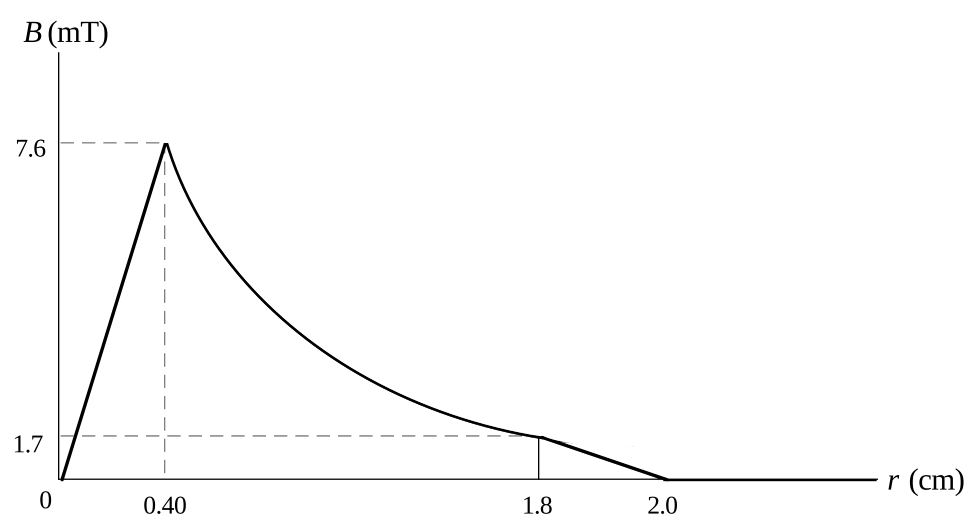
$$B(r) = \frac{\mu_0 i_{\text{encl}}}{2\pi r} = \frac{\mu_0 i}{2\pi r}\left[1 - \frac{\pi(r^2 - b^2)}{\pi(a^2 - b^2)}\right] = \frac{\mu_0 i(a^2 - r^2)}{2\pi r(a^2 - b^2)}.$$

(d) $r > a$:

$$B(r) = \frac{\mu_0 i_{\text{encl}}}{2\pi r} = 0.$$

(e) For example, check what happens if $b = c$. In this case the expressions in (a), (b) and (c) above should yield the same result at $r = b = c$. This is indeed the case.

(f)



48P

For $r < a$,

$$B(r) = \frac{\mu_0 i_{\text{enclosed}}}{2\pi r} = \frac{\mu_0}{2\pi r}\int_0^r J(r) 2\pi r\,dr = \frac{\mu_0}{2\pi}\int_0^r J_0\left(\frac{r}{a}\right) 2\pi r\,dr = \frac{\mu_0 J_0 r^2}{3a}\,.$$

49P

The field at the center of the pipe (point C) is due to the wire alone, with a magnitude of

$$B_C = \frac{\mu_0 i_{\text{wire}}}{2\pi(3R)} = \frac{\mu_0 i_{\text{wire}}}{6\pi R}\,.$$

For the wire we have $B_{P,\,\text{wire}} > B_{C,\,\text{wire}}$, so for $B_P = B_C = B_{C,\,\text{wire}}$, i_{wire} must be into the page:

$$B_P = B_{P,\,\text{wire}} - B_{P,\,\text{pipe}} = \frac{\mu_0 i_{\text{wire}}}{2\pi R} - \frac{\mu_0 i}{2\pi(2R)}\,.$$

Let $B_C = -B_P$ to obtain $i_{\text{wire}} = 3i/8$.

50P

(a) Take the magnetic field at a point within the hole to be the sum of the fields due to two current distributions. The first is the solid cylinder obtained by filling the hole and has a current density that is the same as that in the original cylinder with the hole. The second is the solid cylinder that fills the hole. It has a current density with the same magnitude as that of the original cylinder but it is in the opposite direction. Notice that if these two situations are superposed the total current in the region of the hole is zero.

Recall that a solid cylinder carrying current i, uniformly distributed over a cross section, produces a magnetic field with magnitude $B = \mu_0 i r/2\pi R^2$ a distance r from its axis, inside the cylinder. Here R is the radius of the cylinder.

For the cylinder of this problem the current density is

$$J = \frac{i}{A} = \frac{i}{\pi(a^2 - b^2)}\,,$$

where $A\ [= \pi(a^2 - b^2)]$ is the cross-sectional area of the cylinder with the hole. The current in the cylinder without the hole is

$$I_1 = JA = \pi J a^2 = \frac{ia^2}{a^2 - b^2}$$

and the magnetic field it produces at a point inside, a distance r_1 from its axis, has magnitude

$$B_1 = \frac{\mu_0 I_1 r_1}{2\pi a^2} = \frac{\mu_0 i r_1 a^2}{2\pi a^2(a^2 - b^2)} = \frac{\mu_0 i r_1}{2\pi(a^2 - b^2)}\,.$$

The current in the cylinder that fills the hole is

$$I_2 = \pi J b^2 = \frac{ib^2}{a^2 - b^2}$$

and the field it produces at a point inside, a distance r_2 from the its axis, has magnitude

$$B_2 = \frac{\mu_0 I_2 r_2}{2\pi b^2} = \frac{\mu_0 i r_2 b^2}{2\pi b^2(a^2 - b^2)} = \frac{\mu_0 i r_2}{2\pi(a^2 - b^2)}\,.$$

At the center of the hole this field is zero and the field there is exactly the same as it would be if the hole were filled. Place $r_1 = d$ in the expression for B_1 and obtain

$$B = \frac{\mu_0 i d}{2\pi(a^2 - b^2)}\,.$$

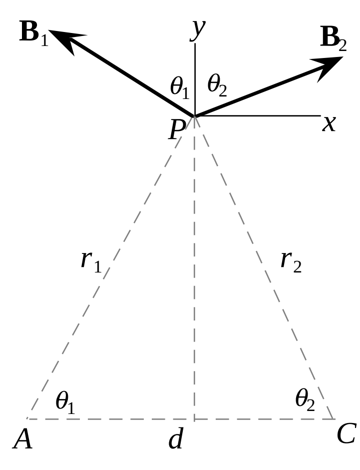
for the field at the center of the hole. The field points upward in the diagram if the current is out of the page.

(b) If $b = 0$ the formula for the field becomes

$$B = \frac{\mu_0 i d}{2\pi a^2}\,.$$

This correctly gives the field of a solid cylinder carrying a uniform current i, at a point inside the cylinder a distance d from the axis. If $d = 0$ the formula gives $B = 0$. This is correct for the field on the axis of a cylindrical shell carrying a uniform current.

(c) The diagram shows the situation in a cross-sectional plane of the cylinder. P is a point within the hole, A is on the axis of the cylinder, and C is on the axis of the hole. The magnetic field due to the cylinder without the hole, carrying a uniform current out of the page, is labeled $\mathbf{B}_1$ and the magnetic field of the cylinder that fills the hole, carrying a uniform current into the page, is labeled $\mathbf{B}_2$. The line from A to P makes the angle θ_1 with the line that joins the centers of the cylinders and the line from C to P makes the angle θ_2 with that line, as shown. $\mathbf{B}_1$ is perpendicular to the line from A to P and so makes the angle θ_1 with the vertical. Similarly, $\mathbf{B}_2$ is perpendicular to the line from C to P and so makes the angle θ_2 with the vertical.



The x component of the total field is

$$B_x = B_2 \sin\theta_2 - B_1 \sin\theta_1 = \frac{\mu_0 i r_2}{2\pi(a^2 - b^2)}\,\sin\theta_2 - \frac{\mu_0 i r_1}{2\pi(a^2 - b^2)}\,\sin\theta_1$$
$$= \frac{\mu_0 i}{2\pi(a^2 - b^2)}\left[r_2 \sin\theta_2 - r_1 \sin\theta_1\right]\,.$$

As the diagram shows $r_2 \sin\theta_2 = r_1 \sin\theta_1$, so $B_x = 0$. The y component is given by

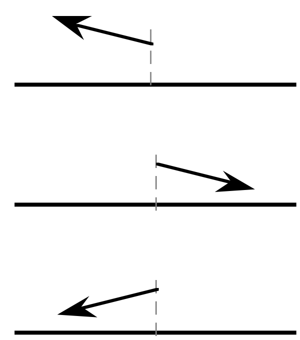
$$\begin{aligned} B_y = B_2 \cos\theta_2 + B_1 \cos\theta_1 &= \frac{\mu_0 i r_2}{2\pi(a^2 - b^2)} \cos\theta_2 + \frac{\mu_0 i r_1}{2\pi(a^2 - b^2)} \cos\theta_1 \\ &= \frac{\mu_0 i}{2\pi(a^2 - b^2)} \left[r_2 \cos\theta_2 + r_1 \cos\theta_1\right] . \end{aligned}$$

The diagram shows that $r_2 \cos\theta_2 + r_1 \cos\theta_1 = d$, so

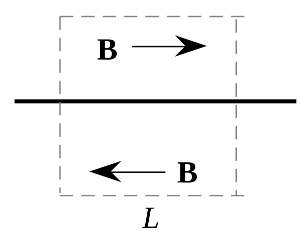
$$B_y = \frac{\mu_0 i d}{2\pi(a^2 - b^2)} .$$

This is identical to the result found in part (a) for the field on the axis of the hole. It is independent of r_1, r_2, θ_1, and θ_2, showing that the field is uniform in the hole.

51P



(a) Suppose the field is not parallel to the sheet, as shown in the upper diagram. Reverse the direction of the current. According to the Biot-Savart law the field reverses, so it will be as in the second diagram. Now rotate the sheet by 180° about a line that perpendicular to the sheet. The field, of course, will rotate with it and end up in the direction shown in the bottom diagram. The current distribution is now exactly as it was originally, so the field must also be as it was originally. But it is not. Only if the field is parallel to the sheet will be final direction of the field be the same as the original direction. If the current is out of the page any infinitesimal portion of the sheet, in the form of a long straight wire, produces a field that is to the left above the sheet and to the right below the sheet. The field must be as drawn in Fig. 30–65.



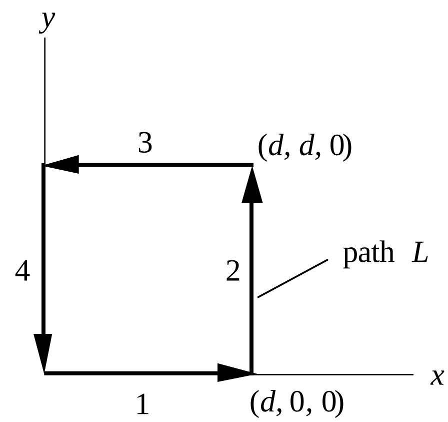
Integrate the tangential component of the magnetic field around the rectangular loop shown with dotted lines. The upper and lower edges are the same distance from the current sheet and each has length L. This means the field has the same magnitude along these edges. It points to the left along the upper edge and to the right along the lower.

If the integration is carried out in the counterclockwise sense the contribution of the upper edge is BL, the contribution of the lower edge is also BL, and the contribution of each of the sides is zero because the field is perpendicular to the sides. Thus $\oint \mathbf{B}\cdot d\mathbf{s} = 2BL$. The total current through the loop is λL. Ampere's law yields $2BL = \mu_0 \lambda L$, so $B = \mu_0 \lambda/2$.

52P*

(a)

$$
\begin{aligned}
\oint_L \mathbf{B}\cdot d\mathbf{s} &= \left(\int_1 + \int_2 + \int_3 + \int_4\right)\mathbf{B}\cdot d\mathbf{s} \\
&= \int_0^d [3.0\,\mathbf{i} + 8.0(x^2/d^2)\,\mathbf{j}]\cdot\,\mathbf{i}\,dx|_{y=0} \\
&\quad + \int_0^d [3.0\,\mathbf{i} + 8.0(x^2/d^2)\,\mathbf{j}]\cdot\,\mathbf{j}\,dy|_{x=d} \\
&\quad + \int_d^0 [3.0\,\mathbf{i} + 8.0(x^2/d^2)\,\mathbf{j}]\cdot\,\mathbf{i}\,dx|_{y=d} \\
&\quad + \int_d^0 [3.0\,\mathbf{i} + 8.0(x^2/d^2)\,\mathbf{j}]\cdot\,\mathbf{j}\,dy|_{x=0} \\
&= (3.0d + 8.0d - 3.0d)\,\mathrm{mT} \\
&= (8.0d)\,\mathrm{mT}\,,
\end{aligned}
$$



where d is in meters.

(b) From $\oint_L \mathbf{B}\cdot d\mathbf{s} = (8.0d)\,\mathrm{mT} = \mu_0 i_{\,\mathrm{enclosed}}$, we get

$$
i_{\,\mathrm{enclosed}} = \frac{(8.0d)\,\mathrm{mT}}{\mu_0} = \frac{(8.0)(0.50)(10^{-3}\,\mathrm{T})}{4\pi\times10^{-7}\,\mathrm{T\cdot m/A}} = 3.2\times10^3\,\mathrm{A}.
$$

(c) Since $\oint_L \mathbf{B}\cdot d\mathbf{s} > 0$, $i_{\,\mathrm{enclosed}} > 0$, so the current is in the positive $\mathbf{k}$ direction.

53E

The magnetic field inside an ideal solenoid is given by Eq. 30–25. The number of turns per unit length is $n = (200\,\mathrm{turns})/(0.25\,\mathrm{m}) = 800\,\mathrm{turns/m}$. Thus

$$
B = \mu_0 n i_0 = (4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(800\,\mathrm{m}^{-1})(0.30\,\mathrm{A}) = 3.0\times10^{-4}\,\mathrm{T}\,.
$$

54E

$$
B = \mu_0 i_0 n = (4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(3.60\,\mathrm{A})(1200/0.950\,\mathrm{m}) = 5.71\times10^{-3}\,\mathrm{T}\,.
$$

55E

Solve N, the number of turns of the solenoid, from $B = \mu_0 i n = \mu_0 i N/L$: $N = BL/\mu_0 i$. Thus the length of wire is

$$
\begin{aligned}
l = 2\pi r N &= \frac{2\pi r B L}{\mu_0 i} \\
&= \frac{2\pi(2.60\times10^{-2}\,\mathrm{m})(23.0\times10^{-3}\,\mathrm{T})(1.30\,\mathrm{m})}{2(1.26\times10^{-6}\,\mathrm{T\cdot m/A})(18.0\,\mathrm{A})} = 108\,\mathrm{m}\,.
\end{aligned}
$$

56E

(a) Use Eq. 30-26. The inner radius is $r = 15.0\,\text{cm}$ so the field there is

$$B = \frac{\mu_0 i N}{2\pi r} = \frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(0.800\,\text{A})(500)}{2\pi(0.150\,\text{m})} = 5.33 \times 10^{-4}\,\text{T}\,.$$

(b) The outer radius is $r = 20.0\,\text{cm}$. The field there is

$$B = \frac{\mu_0 i N}{2\pi r} = \frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(0.800\,\text{A})(500)}{2\pi(0.200\,\text{m})} = 4.00 \times 10^{-4}\,\text{T}\,.$$

57E

In this case $L = 2\pi r$ is roughly the length of the toroid so

$$B = \mu_0 i_0 \left(\frac{N}{2\pi r}\right) = \mu_0 n i_0\,.$$

This result is expected, since from the perspective of a point inside the toroid the portion of the toroid in the vicinity of the point resembles part of a long solenoid.

58P

Use $B = \mu_0 i_0 n$ and note that $n i_0 = \lambda$. Thus $B = \mu_0 \lambda$. Also from 51P we have $\Delta B = \frac{1}{2}\mu_0\lambda - (-\frac{1}{2}\mu_0\lambda) = \mu_0\lambda$ as we move through an infinite plane current sheet of current density λ.

59P

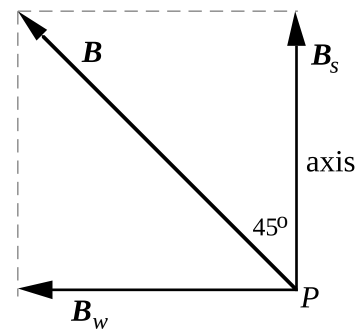
Consider a circle of radius r, inside the toroid and concentric with it. The current that passes through the region between the circle and the outer rim of the toroid is Ni, where N is the number of turns and i is the current. The current per unit length of circle is $\lambda = Ni/2\pi r$ and $\mu_0\lambda$ is $\mu_0 Ni/2\pi r$, the magnitude of the magnetic field at the circle. Since the field is zero outside a toroid, this is also the change in the magnitude of the field encountered as you move from the circle to the outside.

The equality is not really surprising in light of Ampere's law. You are moving perpendicularly to the magnetic field lines. Consider an extremely narrow loop, with the narrow sides along field lines and the two long sides perpendicular to the field lines. If B_1 is the field at one end and B_2 is the field at the other end then $\oint \mathbf{B}\cdot d\mathbf{s} = (B_2 - B_1)w$, where w is the width of the loop. The current through the loop is $w\lambda$, so Ampere's law yields $(B_2 - B_1)w = \mu_0 w\lambda$ and $B_2 - B_1 = \mu_0\lambda$.

60P

(a) Denote the **B**-fields at point P on the axis due to the solenoid and the wire as $\mathbf{B}_s$

and $\mathbf{B}_w$, respectively. Since $\mathbf{B}_s$ is along the axis of the solenoid and $\mathbf{B}_w$ is perpendicular to it, $\mathbf{B}_s \perp \mathbf{B}_w$, respectively. For the net field $\mathbf{B}$ to be at 45° with the axis we then must have $B_s = B_w$. Thus



$$B_s = \mu_0 i_s n = B_w = \frac{\mu_0 i_w}{2\pi d}\,,$$

which gives the separation d to point P on the axis:

$$d = \frac{i_w}{2\pi i_s n} = \frac{6.00\,\mathrm{A}}{2\pi(20.0\times10^{-3}\,\mathrm{A})(10\,\mathrm{turns/cm})} = 4.77\,\mathrm{cm}\,.$$

(b)

$$\begin{aligned} B &= \sqrt{2}B_s \\ &= \sqrt{2}(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(20.0\times10^{-3}\,\mathrm{A})(10\,\mathrm{turns}/0.0100\,\mathrm{m}) \\ &= 3.55\times10^{-5}\,\mathrm{T}m\,. \end{aligned}$$

61P

Let

$$r = \frac{mv}{eB} = \frac{mv}{e\mu_0 n i}$$

and solve for i:

$$\begin{aligned} i &= \frac{mv}{e\mu_0 n r} \\ &= \frac{(9.11\times10^{-31}\,\mathrm{kg})(0.0460)(3.00\times10^{8}\,\mathrm{m/s})}{(1.60\times10^{-19}\,\mathrm{C})(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(100/0.0100\,\mathrm{m})(2.30\times10^{-2}\,\mathrm{m})} \\ &= 0.272\,\mathrm{A}\,. \end{aligned}$$

62E

The magnitude of the dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current, and A is the area. Use $A = \pi R^2$, where R is the radius. Thus

$$\mu = (200)(0.30\,\mathrm{A})\pi(0.050\,\mathrm{m})^2 = 0.47\,\mathrm{A\cdot m^2}\,.$$

63E

(a) Set $z = 0$ in Eq. 30-28: $B(0) \propto i/R$. Since case b has two loops,

$$\frac{B_b}{B_a} = \frac{2i/R_b}{i/R_a} = \frac{2R_a}{R_b} = 4\,.$$

(b)

$$\frac{\mu_b}{\mu_a} = \frac{2iA_b}{iA_a} = \frac{2R_b^2}{R_a^2} = 2\left(\frac{1}{2}\right)^2 = \frac{1}{2}\,.$$

64E

Use Eq. 30-28 and note that the contribute to $\mathbf{B}_P$ from the two coils are the same. Thus

$$B_P = \frac{2\mu_0 i R^2 N}{2[R^2 + (R/2)^2]^{3/2}} = \frac{8\mu_0 N i}{5\sqrt{5}R}\,.$$

$\mathbf{B}_P$ is in the positive x direction.

65E

(a) Similar to 62E, the magnitude of the dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current, and A is the area. Use $A = \pi R^2$, where R is the radius. Thus

$$\mu = Ni\pi R^2 = (300)(4.0\,\mathrm{A})\pi(0.025\,\mathrm{m})^2 = 2.4\,\mathrm{A\cdot m^2}\,.$$

(b) The magnetic field on the axis of a magnetic dipole, a distance z away, is given by Eq. 30–29:

$$B = \frac{\mu_0}{2\pi}\frac{\mu}{z^3}\,.$$

Solve for z:

$$z = \left(\frac{\mu_0}{2\pi}\frac{\mu}{B}\right)^{1/3} = \left[\frac{(4\pi\times 10^{-7}\,\mathrm{T\cdot m/A})(2.36\,\mathrm{A\cdot m^2})}{2\pi(5.0\times 10^{-6}\,\mathrm{T})}\right]^{1/3} = 46\,\mathrm{cm}\,.$$

66E

(a) For $x \gg a$ the result of 21P reduces to

$$B(x) \approx \frac{4\mu_0 i a^2}{\pi(4x^2)(4x^2)^{1/2}} = \frac{\mu_0(ia^2)}{4\pi x^3}\,,$$

indeed the B-field of a magnetic diploe (see Eq. 30-29).

(b) $\mu = ia^2$, by comparison between Eq. 30-29 and the result above.

67P

(a) The two straight segments of the wire do not contribute to $\mathbf{B}_P$. For the larger semicircular loop $B_{P1} = \mu_0 i/4b$ (see 12P), and for the smaller one $B_{P2} = \mu_0 i/4a$. Thus

$$B_p = B_{P1} + B_{P2} = \frac{\mu_0 i}{4}\left(\frac{1}{a} + \frac{1}{b}\right)\,.$$

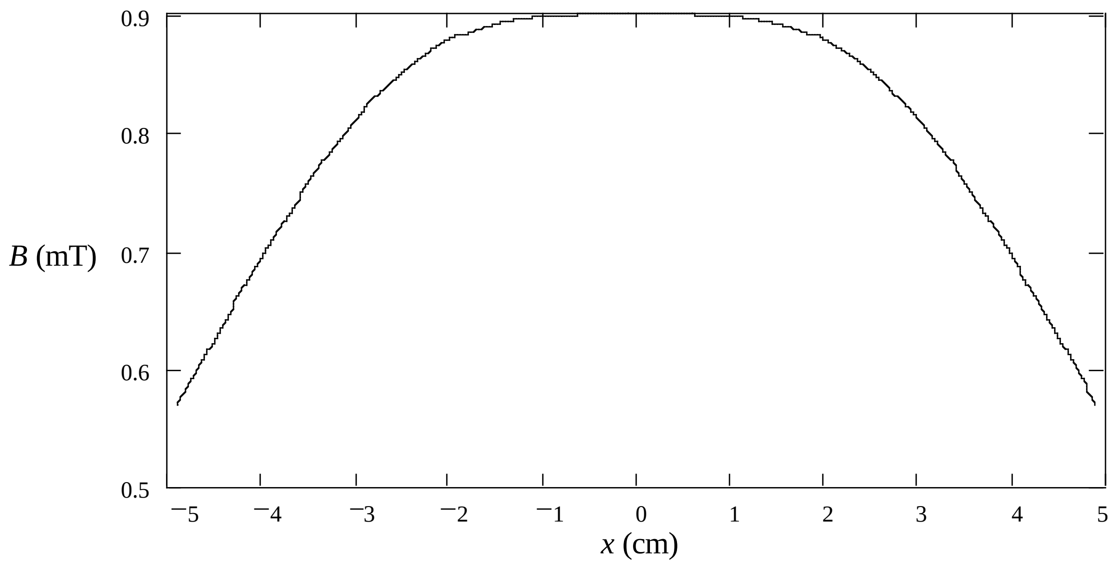
$\mathbf{B}_P$ points into the page.
(b) $\mu = A_{\text{loop}} i = \frac{1}{2}(\pi a^2 + \pi b^2) i$, pointing into the page.

68P

Use Eq. 30-28 to obtain

$$B(x) = \frac{\mu_0 i R^2}{2} \left[\left(\frac{1}{R^2 + (x - R/2)^2} \right)^{3/2} + \left(\frac{1}{R^2 + (x + R/2)^2} \right)^{3/2} \right].$$

The plot of $B(x)$ is as follows. Note that $B(x)$ is almost a constant in the vicinity of $x = 0$.



69P

Denote the large and small loops with subscripts 1 and 2, respectively.
(a)

$$B_1 = \frac{\mu_0 i_1}{2R_1} = \frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(15\,\text{A})}{2(0.12\,\text{m})} = 7.9 \times 10^{-5}\,\text{T}.$$

(b)

$$\begin{aligned}\tau &= |\boldsymbol{\mu}_2 \times \mathbf{B}_1| = \mu B_1 \sin 90^\circ = N_2 i_2 A_2 B_1 = \pi N_2 i_2 r_2^2 B_1 \\ &= \pi(50)(1.3\,\text{A})(0.82 \times 10^{-2}\,\text{m})^2 (7.9 \times 10^{-5}\,\text{T}) = 1.1 \times 10^{-6}\,\text{N}\cdot\text{m}.\end{aligned}$$

70P

(a) Contribution to B_C from the straight segment of the wire:

$$B_{C1} = \frac{\mu_0 i}{2\pi R}.$$

Contribution from the circular loop:

$$B_{C2} = \frac{\mu_0 i}{2R}\,.$$

Thus

$$B_C = B_{C1} + B_{C2} = \frac{\mu_0 i}{2R}\left(1 + \frac{1}{\pi}\right).$$

$\mathbf{B}_C$ points out of the page.
(b) Now $\mathbf{B}_{C1} \perp \mathbf{B}_{C2}$ so

$$B_C = \sqrt{B_{C1}^2 + B_{C2}^2} = \frac{\mu_0 i}{2R}\sqrt{1 + \frac{1}{\pi^2}}\,.$$

and $\mathbf{B}_C$ points at an angle θ out of the page, where

$$\theta = \tan^{-1}\left(\frac{B_{C1}}{B_{C2}}\right) = \tan^{-1}\left(\frac{1}{\pi}\right) = 18^\circ\,.$$

71P

(a) By imagining that both bg and cf have a pair of currents of the same magnitude ($= i$) and opposite direction, you can justify the superposition.
(b)

$$\begin{aligned}\boldsymbol{\mu} &= \boldsymbol{\mu}_{bcfgb} + \boldsymbol{\mu}_{abgha} + \boldsymbol{\mu}_{cdefc} = (ia^2)(\mathbf{j} - \mathbf{i} + \mathbf{i}) = ia^2\,\mathbf{j} \\ &= (6.0\,\mathrm{A})(0.10\,\mathrm{m})^2\,\mathbf{j} = (6.0 \times 10^{-2}\,\mathrm{m}^2\!\cdot\!\mathrm{A})\,\mathbf{j}\,.\end{aligned}$$

(c) Since both points are far from the cube we can use the dipole approximation. For $(x,\, y,\, z) = (0,\, 5.0\,\mathrm{m},\, 0)$

$$\begin{aligned}\mathbf{B}(0,\, 5.0\,\mathrm{m},\, 0) &\approx \frac{\mu_0}{2\pi}\frac{\boldsymbol{\mu}}{y^3} \\ &= \frac{(1.26 \times 10^{-6}\,\mathrm{T\!\cdot\!m/A})(6.0 \times 10^{-2}\,\mathrm{m}^2 \cdot \mathrm{A})\,\mathbf{j}}{2\pi(5.0\,\mathrm{m})^3} \\ &= (9.6 \times 10^{-11}\,\mathrm{T})\,\mathbf{j}\,.\end{aligned}$$

For $(x,\, y,\, z) = (5.0\,\mathrm{m},\, 0,\, 0)$, note that the line joining the end point of interest and the location of the dipole is perpendicular to the axis of the dipole. You can check easily that if an electric diploe is used, the field wold be $E \approx (1/4\pi\epsilon_0)(p/x^3)$, which is half of the magnitude of E for a point on the y axis the same distance from the dipole. By analogy, in our case B is also half the value or $B(0,\, 5.0\,\mathrm{m},\, 0)$, i.e.,

$$B(5.0\,\mathrm{m},\, 0,\, 0) = \frac{1}{2}B(0,\, 5.0\,\mathrm{m},\, 0) = \frac{1}{2}(9.6 \times 10^{-11}\,\mathrm{T}) = 4.8 \times 10^{-11}\,\mathrm{T}\,.$$

Just like the electric dipole case, $\mathbf{B}(5.0\,\mathrm{m},\,0,\,0)$ points in the negative y direction.

72P

(a) The magnitude of the magnetic field on the axis of a circular loop, a distance z from the loop center, is given by Eq. 30–28:

$$B = \frac{N\mu_0 i R^2}{2(R^2+z^2)^{3/2}}\,,$$

where R is the radius of the loop, N is the number of turns, and i is the current. Both of the loops in the problem have the same radius, the same number of turns, and carry the same current. The currents are in the same sense and the fields they produce are in the same direction in the region between them. Place the origin at the center of the left-hand loop and let x be the coordinate of a point on the axis between the loops. To calculate the field of the left-hand loop set $z = x$ in the equation above. The chosen point on the axis is a distance $s - x$ from the center of the right-hand loop. To calculate the field it produces put $z = s - x$ in the equation above. The total field at the point is therefore

$$B = \frac{N\mu_0 i R^2}{2}\left[\frac{1}{(R^2+x^2)^{3/2}} + \frac{1}{(R^2+x^2-2sx+s^2)^{3/2}}\right].$$

Its derivative with respect to x is

$$\frac{dB}{dx} = -\frac{N\mu_0 i R^2}{2}\left[\frac{3x}{(R^2+x^2)^{5/2}} + \frac{3(x-s)}{(R^2+x^2-2sx+s^2)^{5/2}}\right].$$

When this is evaluated for $x = s/2$ (the midpoint between the loops) the result is

$$\left.\frac{dB}{dx}\right|_{s/2} = -\frac{N\mu_0 i R^2}{2}\left[\frac{3s/2}{(R^2+s^2/4)^{5/2}} - \frac{3s/2}{(R^2+s^2/4-s^2+s^2)^{5/2}}\right] = 0\,,$$

independently of the value of s.

(b) The second derivative is

$$\frac{d^2B}{dx^2} = \frac{N\mu_0 i R^2}{2}\left[-\frac{3}{(R^2+x^2)^{5/2}} + \frac{15x^2}{(R^2+x^2)^{7/2}} \right.$$
$$\left. - \frac{3}{(R^2+x^2-2sx+s^2)^{5/2}} + \frac{15(x-s)^2}{(R^2+x^2-2sx+s^2)^{7/2}}\right].$$

At $x = s/2$,

$$\left.\frac{d^2B}{dx^2}\right|_{s/2} = \frac{N\mu_0 i R^2}{2}\left[-\frac{6}{(R^2+s^2/4)^{5/2}} + \frac{30s^2/4}{(R^2+s^2/4)^{7/2}}\right]$$
$$= \frac{N\mu_0 R^2}{2}\left[\frac{-6(R^2+s^2/4)+30s^2/4}{(R^2+s^2/4)^{7/2}}\right] = 3N\mu_0 i R^2\,\frac{s^2-R^2}{(R^2+s^2/4)^{7/2}}\,.$$

Clearly, this is zero if $s = R$.

73
(a) B from sum: 7.069×10^{-5} T; $\mu_0 in = 5.027 \times 10^{-5}$ T; 40% difference;
(b) B from sum: 1.043×10^{-4} T; $\mu_0 in = 1.005 \times 10^{-4}$ T; 4% difference;
(c) B from sum: 2.506×10^{-4} T; $\mu_0 in = 2.513 \times 10^{-4}$ T; 0.3% difference

74
(a) each side contributes 3.14×10^{-7} T·m; the total is 1.26×10^{-6} T·m; $\mu_0 i = 1.26 \times 10^{-6}$ T·m;
(b) the sides contribute 2.98×10^{-7} T·m, 2.48×10^{-7} T·m, 2.98×10^{-7} T·m, and 4.12×10^{-7} T·m; the total is 1.26×10^{-6} T·m;
(c) the sides contribute 2.52×10^{-7} T·m, 2.03×10^{-7} T·m, 2.52×10^{-7} T·m, and 5.49×10^{-7} T·m; the total is 1.26×10^{-6} T·m;
(d) the sides contribute 1.89×10^{-7} T·m, 1.71×10^{-7} T·m, 1.89×10^{-7} T·m, and -5.49×10^{-7} T·m; the total is essentially zero

75
(a) $\mathbf{B} = (\mu_0/2\pi)[i_1/(x-a) + i_2/x]\,\mathbf{j}$;
(b) $\mathbf{B} = (\mu_0/2\pi)(i_1/a)(1 + b/2)\,\mathbf{j}$