Chapter 29

Answer to Checkpoint Questions

1. (a) $+z$; (b) $-x$; (c) $\mathbf{F}_B = 0$

2. (a) 2, then tie of 1 and 3 (zero); (b) 4

3. (a) $+z$ and $-z$ tie, then $+y$ and $-y$ tie, then $+x$ and $-x$ tie (zero); (b) $+y$

4. (a) electron; (b) clockwise

5. $-y$

6. (a) all tie; (b) 1 and 4 tie , then 2 and 3 tie

Answer to Questions

1. (a) all tie; (b) 1 and 2 (charge is negative)

2. tie of (a), (b), and (c), and then (d) (zero)

3. (a) no, $\mathbf{v}$ and $\mathbf{F}_B$ must be perpendicular; (b) yes; (c) no, $\mathbf{B}$ and $\mathbf{F}_B$ must be perpendicular

4. (a) upper plate; (b) lower plate; (c) out of the page

5. (a) $\mathbf{F}_E$; (b) $\mathbf{F}_B$

6. 2, 5, 6, 9, 10

7. (a) right; (b) right

8. (a) a and c, a at higher potential; (b) b and d, b at higher potential; (c) no Hall voltage

9. (a) negative; (b) equal; (c) equal; (d) half a circle

10. into page: a, d, e; out of page: b, c, f (the particle is negatively charged)

11. (a) B_1; (b) $\mathbf{B}_1$ into page, $\mathbf{B}_2$ out of page; (c) less

12. $1i$, $2e$, $3c$, $4a$, $5g$, $6j$, $7d$, $8b$, $9h$, $10f$, $11k$

13. all

14. (b)

15. all tie

16. downward

17. (*a*) positive; (*b*) (1) and (2) tie, then (3), which is zero

Solutions to Exercises & Problems

1E

One way to do this is through Eq. 29-2: $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$, which gives

$$[B] = \frac{[F]}{[q][v]} = \frac{\mathrm{ML/T^2}}{(\mathrm{Q})(\mathrm{L/T})} = \frac{\mathrm{M}}{\mathrm{QT}} \,.$$

2E

(*a*) Use eq. 29-3: $F_B = |q|vB\sin\phi = (+3.2 \times 10^{-19}\,\mathrm{C})(550\,\mathrm{m/s})(0.045\,\mathrm{T})(\sin 52^\circ) = 6.2 \times 10^{-18}\,\mathrm{N}$.
(*b*) $a = F_B/m = (6.2 \times 10^{-18}\,\mathrm{N})/(6.6 \times 10^{-27}\,\mathrm{kg}) = 9.5 \times 10^8\,\mathrm{m/s^2}$.
(*c*) Since it is perpendicular to $\mathbf{v}$, $\mathbf{F}_B$ does not do any work on the particle. Thus from the work-energy theorem both the kinetic energy and the speed of the particle remain unchanged.

3E

(*a*) $F_{B,\,\mathrm{max}} = q|\mathbf{v} \times \mathbf{B}|_{\mathrm{max}} = qvB\sin(90^\circ) = qvB = (1.60 \times 10^{-19}\,\mathrm{C})(7.20 \times 10^6\,\mathrm{m/s}) \times (83.0 \times 10^{-3}\,\mathrm{T}) = 9.56 \times 10^{-14}\,\mathrm{N}$; $F_{B,\,\mathrm{min}} = q|\mathbf{v} \times \mathbf{B}|_{\mathrm{min}} = qvB\sin(0) = 0$.
(*b*) $a = F_B/m_e = qvB\sin\theta/m_e$, so the angle θ between $\mathbf{v}$ and $\mathbf{B}$ is

$$\theta = \sin^{-1}\left(\frac{m_e a}{qvB}\right) = \sin^{-1}\left[\frac{(9.11 \times 10^{-31}\,\mathrm{kg})(4.90 \times 10^{14}\,\mathrm{m/s^2})}{(1.60 \times 10^{-16}\,\mathrm{C})(7.20 \times 10^6\,\mathrm{m/s})(83.0 \times 10^{-3}\,\mathrm{T})}\right] = 0.267^\circ \,.$$

4E

(*a*) The magnitude of the magnetic force on the proton is given by $F_B = evB\sin\phi$, where v is the speed of the proton, B is the magnitude of the magnetic field, and ϕ is the angle between the particle velocity and the field when they are drawn with their tails at the same point. Thus

$$v = \frac{F_B}{eB\sin\phi} = \frac{6.50 \times 10^{-17}\,\mathrm{N}}{(1.60 \times 10^{-19}\,\mathrm{C})(2.60 \times 10^{-3}\,\mathrm{T})\sin 23.0^\circ} = 4.00 \times 10^5\ \mathrm{m/s} \,.$$

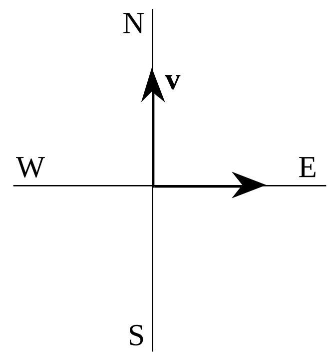
(b) The kinetic energy of the proton is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67 \times 10^{-27}\,\mathrm{k}g)(4.00 \times 10^5\ \mathrm{m/s})^2 = 1.34 \times 10^{-16}\,\mathrm{J}\,.$$

This is $(1.34 \times 10^{-16}\,\mathrm{J})/(1.60 \times 10^{-19}\ \mathrm{J/eV}) = 835\ \mathrm{eV}$.

5P

(a) The diagram shows the electron traveling to the north. The magnetic field is into the page. The right-hand rule tells us that $\mathbf{v}\times\mathbf{B}$ is to the west, but since the electron is negatively charged the magnetic force on it is to the east.



(b) Use $F = m_e a$, with $F = evB\sin\phi$. Here v is the speed of the electron, B is the magnitude of the magnetic field, and ϕ is the angle between the electron velocity and the magnetic field. The velocity and field are perpendicular to each other so $\phi = 90°$ and $\sin\phi = 1$. Thus $a = evB/m_e$.

The electron speed can be found from its kinetic energy. Since $K = \frac{1}{2}mv^2$,

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(12.0 \times 10^3\,\mathrm{eV})(1.60 \times 10^{-19}\,\mathrm{J/eV})}{9.11 \times 10^{-31}\,\mathrm{kg}}} = 6.49 \times 10^7\,\mathrm{m/s}\,.$$

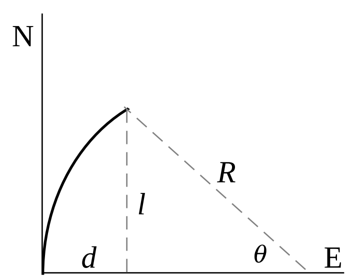
Thus

$$a = \frac{evB}{m_e} = \frac{(1.60 \times 10^{-19}\,\mathrm{C})(6.49 \times 10^7\,\mathrm{m/s})(55.0 \times 10^{-6}\,\mathrm{T})}{9.11 \times 10^{-31}\,\mathrm{kg}} = 6.27 \times 10^{14}\,\mathrm{m/s^2}\,.$$

(c) The electron follows a circular path. Its acceleration is given by v^2/R, where R is the radius of the path. Thus

$$R = \frac{v^2}{a} = \frac{(6.49 \times 10^7\,\mathrm{m/s})^2}{6.27 \times 10^{14}\,\mathrm{m/s^2}} = 6.72\,\mathrm{m}\,.$$

The solid curve on the diagram is the path. Suppose it subtends the angle θ at the center. l $(= 0.200\,\mathrm{m})$ is the length of the tube and d is the deflection. The right triangle yields $d = R - R\cos\theta$ and $l = R\sin\theta$. Thus $R\cos\theta = R - d$ and $R\sin\theta = l$. Square both these equations and add the results to obtain $R^2 = (R-d)^2 + l^2$, or $d^2 - 2Rd + l^2 = 0$. The solution is $d = R \pm \sqrt{R^2 - l^2}$. The plus sign corresponds to an angle of $180° - \theta$; the minus sign corresponds to the correct solution.



Since l is much less than R, use the binomial theorem to expand $\sqrt{R^2 - l^2}$. The first two terms are $R - \frac{1}{2}l^2/R$, so $d = \frac{1}{2}l^2/R$ and when numerical values are substituted, $d = 0.00298\,\mathrm{m}$ is obtained.

6P

(*a*) $\mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = q(v_x\mathbf{i}+v_y\mathbf{j})\times(B_x\mathbf{i}+B_y\mathbf{j}) = q(v_xB_y - v_yB_x)\mathbf{k} = (-1.6\times10^{-19}\,\mathrm{C})[(2.0\times 10^6\,\mathrm{m/s})(-0.15\,\mathrm{T}) - (3.0\times10^6\,\mathrm{m/s})(0.030\,\mathrm{T})] = (6.2\times10^{-14}\,\mathrm{N})\mathbf{k}$. So the magnitude of $\mathbf{F}_B$ is $6.2\times10^{14}\,\mathrm{N}$, and $\mathbf{F}_B$ points in the positive z direction.

(*b*) All you need to do is to change in sign in q. So now $\mathbf{F}_B$ still has the same magnitude but points in the negative z diretion.

7P

Let $\mathbf{B} = B_y\,\mathbf{j}+B_z\mathbf{k}$, then $\mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = q(v_x\,\mathbf{i}+v_y\,\mathbf{j})\times(B_y\,\mathbf{j}+B_z\mathbf{k}) = q(v_yB_z\,\mathbf{i} - v_xB_z\,\mathbf{j} + v_xB_y\mathbf{k}) = F_x\,\mathbf{i}+F_y\,\mathbf{j}$. Thus $qv_yB_z = F_x$, $-qv_xB_z = F_y$ and $qv_xB_y = 0$. The last equation gives $B_y = 0$. So

$$\mathbf{B} = B_z\mathbf{k} = \frac{F_x\mathbf{k}}{qv_y} = \frac{(-4.2\times10^{-15}\mathbf{k})\,\mathrm{N}}{(-1.60\times10^{-19}\,\mathrm{C})(35\times10^3\,\mathrm{m/s})} = (0.75\mathbf{k})\,\mathrm{T}\,.$$

8E

(*a*) The net force on the proton is given by

$$\begin{aligned}\mathbf{F} &= \mathbf{F}_E + \mathbf{F}_B = q\mathbf{E} + q\mathbf{v}\times\mathbf{B}\\ &= (1.6\times10^{-19}\,\mathrm{C})[(4.0\,\mathrm{V/m})\,\mathbf{k} + (2000\,\mathrm{m/s})\,\mathbf{j}\times(-2.5\,\mathrm{mT})\,\mathbf{i}]\\ &= (1.4\times10^{-18}\,\mathrm{N})\,\mathbf{k}\,.\end{aligned}$$

(*b*) In this case

$$\begin{aligned}\mathbf{F} &= \mathbf{F}_E + \mathbf{F}_B = q\mathbf{E} + q\mathbf{v}\times\mathbf{B}\\ &= (1.6\times10^{-19}\,\mathrm{C})[(-4.0\,\mathrm{V/m})\,\mathbf{k} + (2000\,\mathrm{m/s})\,\mathbf{j}\times(-2.5\,\mathrm{mT})\,\mathbf{i}]\\ &= (1.6\times10^{-19}\,\mathrm{N})\,\mathbf{k}\,.\end{aligned}$$

(*a*) In this case

$$\begin{aligned}\mathbf{F} &= \mathbf{F}_E + \mathbf{F}_B = q\mathbf{E} + q\mathbf{v}\times\mathbf{B}\\ &= (1.6\times10^{-19}\,\mathrm{C})[(4.0\,\mathrm{V/m})\,\mathbf{i} + (2000\,\mathrm{m/s})\,\mathbf{j}\times(-2.5\,\mathrm{mT})\,\mathbf{i}]\\ &= (6.4\times10^{-19}\,\mathrm{N})\,\mathbf{i} + (8.0\times10^{-19}\,\mathrm{N})\,\mathbf{k}\,.\end{aligned}$$

The magnitude of $\mathbf{F}_B$ is now

$$F_B = \sqrt{F_{Bx}^2 + F_{By}^2 + F_{Bz}^2} = \sqrt{(6.4\times10^{-19}\,\mathrm{N})^2 + (8.0\times10^{-19}\,\mathrm{N})^2} = 1.0\times10^{-18}\,\mathrm{N}\,.$$

9E

(*a*) The total force on the electron is $\mathbf{F} = -e(\mathbf{E}+\mathbf{v}\times\mathbf{B})$, where $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic field and $\mathbf{v}$ is the electron velocity. The magnitude of the magnetic force is $evB\sin\phi$, where ϕ is the angle between the velocity and the field. Since the total force must vanish, $B = E/v\sin\phi$. The force is the smallest it can be when the field is perpendicular to the velocity and $\phi = 90°$. Then $B = E/v$. Use $K = \frac{1}{2}mv^2$ to find the speed:

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(2.5\times10^3\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}{9.11\times10^{-31}\,\text{kg}}} = 2.96\times10^7\,\text{m/s}\,.$$

Thus

$$B = \frac{E}{v} = \frac{10\times10^3\,\text{V/m}}{2.96\times10^7\,\text{m/s}} = 3.4\times10^{-4}\,\text{T}\,.$$

The magnetic field must be perpendicular to both the electric field and the velocity of the electron.

(b) A proton will pass undeflected if its velocity is the same as that of the electron. Both the electric and magnetic forces reverse direction, but they still cancel.

10E

(a) Let $\mathbf{F} = q(\mathbf{E}+\mathbf{v}\times\mathbf{B}) = 0$, we get $vB\sin(\mathbf{v},\mathbf{B}) = E$. So $v_{\min} = E/B = (1.50\times10^3\,\text{V/m})/(0.400\,\text{T}) = 3.75\times10^3\,\text{m/s}$.

11P*

Apply $\mathbf{F} = q(\mathbf{E}+\mathbf{v}\times\mathbf{B}) = m_e\mathbf{a}$ to solve for $\mathbf{E}$:

$$\begin{aligned}\mathbf{E} &= \frac{m_e\mathbf{a}}{q} + \mathbf{B}\times\mathbf{v}\\ &= \frac{(9.11\times10^{-31}\,\text{kg})(2.00\times10^{12}\,\text{m/s}^2)\,\mathbf{i}}{-1.60\times10^{-19}\,\text{C}} + (400\,\mu\text{T})\,\mathbf{i}\times[(12.0\,\text{km/s})\,\mathbf{j} + (15.0\,\text{km/s})\mathbf{k}]\\ &= (-11.4\,\mathbf{i} - 6.00\,\mathbf{j} + 4.80\,\mathbf{k})(\text{V/m})\,.\end{aligned}$$

12P

(a) Let $\mathbf{F} = q(\mathbf{E}+\mathbf{v}\times\mathbf{B}) = 0$. Note that $\mathbf{v}\perp\mathbf{B}$ so $|\mathbf{v}\times\mathbf{B}| = vB$. Thus

$$\begin{aligned}B &= \frac{E}{v} = \frac{E}{\sqrt{2m_eK}} = \frac{100\,\text{V}/(20\times10^{-3}\,\text{m})}{\sqrt{2(9.11\times10^{-31}\,\text{kg})(1.0\times10^3\,\text{V})(1.60\times10^{-19}\,\text{C})}}\\ &= 2.7\times10^{-4}\,\text{T}\,.\end{aligned}$$

13P

Since the total force, given by $\mathbf{F} = e(\mathbf{E}+\mathbf{v}\times\mathbf{B})$, vanishes, the electric field $\mathbf{E}$ must be perpendicular to both the particle velocity $\mathbf{v}$ and the magnetic field $\mathbf{B}$. The magnetic field is perpendicular to the velocity so $\mathbf{v} \times \mathbf{B}$ has magnitude vB and the magnitude of the electric field is given by $E = vB$. Since the particle has charge e and is accelerated through a potential difference V, $\frac{1}{2}mv^2 = eV$ and $v = \sqrt{2eV/m}$. Thus

$$E = B\sqrt{\frac{2eV}{m}} = (1.2\,\mathrm{T})\sqrt{\frac{2(1.60\times10^{-19}\,\mathrm{C})(10\times10^3\,\mathrm{V})}{(6.0\,\mathrm{u})(1.661\times10^{-27}\,\mathrm{kg/u})}} = 6.8\times10^5\,\mathrm{V/m}\,.$$

14P

Use Eq. 29-12 to solve for V:

$$V = \frac{iB}{nle} = \frac{(23\,\mathrm{A})(0.65\,\mathrm{T})}{(8.47\times10^{28}/\mathrm{m}^3)(150\,\mu\mathrm{m})(1.6\times10^{-19}\,\mathrm{C})} = 7.4\times10^{-6}\,\mathrm{V}\,.$$

15E

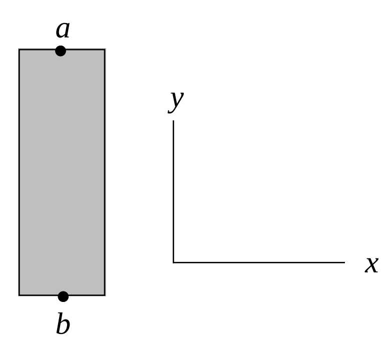
From $\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) = 0$ we get $v = E/B$. Also note that $J = nev$, so $J = ne(E/B)$, or $n = JB/eE$.

16P

(a) Use Eq. 29-10: $v_d = E/B = (10\times10^{-6}\,\mathrm{V}/1.0\times10^{-2}\,\mathrm{m})/(1.5\,\mathrm{T}) = 6.7\times10^{-4}\,\mathrm{m/s}$.
(b) From the result of 15E

$$\begin{aligned} n &= \frac{JB}{eE} = \frac{iB}{AeE} = \frac{i}{Aev} \\ &= \frac{3.0\,\mathrm{A}}{(1.0\times10^{-2}\,\mathrm{m})(10\times10^{-6}\,\mathrm{m})(1.60\times10^{-19}\,\mathrm{C})(6.7\times10^{-4}\,\mathrm{m/s})} \\ &= 2.8\times10^{29}/\,\mathrm{m}^3\,. \end{aligned}$$

(c) Refer to the diagram to the right. If the electrons move out of the page and the magnetic field $\mathbf{B}$ is in the positive x direction, then $V_a > V_b$.



17P

Since $J = \sigma E_c = E_c/\rho$ and $J = neE/B$ (see 15E), we get $E_c/\rho = neE/B$, i.e., $E/E_c = B/ne\rho$.

(*b*)

$$\frac{E}{E_c} = \frac{B}{ne\rho} = \frac{0.65\,\mathrm{T}}{(8.47\times10^{28}/\mathrm{m}^3)(1.60\times10^{-19}\,\mathrm{C})(1.69\times10^{-8}\,\Omega\cdot\mathrm{m})} = 2.84\times10^{-3}\,.$$

18P

For a free charge q inside the metal strip with velocity $\mathbf{v}$ we have $\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})$, so

$$v = \frac{E}{B} = \frac{|V_x - V_y|/d_{xy}}{B} = \frac{(3.90\times10^{-9}\,\mathrm{V})}{(1.20\times10^{-3}\,\mathrm{T})(0.850\times10^{-2}\,\mathrm{m})} = 0.382\,\mathrm{m/s}\,.$$

19E

(*a*) In the accelerating process the electron loses potential energy eV and gains the same amount of kinetic energy. Since it starts from rest, $\frac{1}{2}m_e v^2 = eV$ and

$$v = \sqrt{\frac{2eV}{m_e}} = \sqrt{\frac{2(1.60\times10^{-19}\ \mathrm{C})(350\ \mathrm{V})}{9.11\times10^{-31}\ \mathrm{kg}}} = 1.11\times10^{7}\ \mathrm{m/s}\,.$$

(*b*) The electron travels with constant speed around a circle. The magnetic force on it has magnitude $F_B = evB$ and its acceleration is v^2/R, where R is the radius of the circle. Newton's second law yields $evB = m_e v^2/R$, so

$$R = \frac{m_e v}{eB} = \frac{(9.11\times10^{-31}\ \mathrm{kg})(1.11\times10^{7}\ \mathrm{m/s})}{(1.60\times10^{-19}\ \mathrm{C})(200\times10^{-3}\ \mathrm{T})} = 3.16\times10^{-4}\ \mathrm{m}\,.$$

20E

Since the magnetic field is perpendicular to the particle velocity the magnitude of the magnetic force is given by $F_B = evB$ and the acceleration of the electron has magnitude $a = F_B/m_e$, where v is the speed of the electron and m_e is its mass. B is the magnitude of the magnetic field. Since the electron is traveling with uniform speed in a circle, its acceleration is $a = v^2/r$, where r is the radius of the circle. Thus $evB/m_e = v^2/r$ and

$$B = \frac{m_e v}{er} = \frac{(9.11\times10^{-31}\,\mathrm{kg})(1.3\times10^{6}\,\mathrm{m/s})}{(1.60\times10^{-19}\,\mathrm{C})(0.35\,\mathrm{m})} = 2.1\times10^{-5}\,\mathrm{T}\,.$$

21P

(*a*) Use Eq. 29-16 to calculate r:

$$r = \frac{m_e v}{qB} = \frac{(9.11\times10^{-31}\,\mathrm{kg})(10\%)(3.00\times10^{8}\,\mathrm{m/s})}{(1.60\times10^{-19}\,\mathrm{C})(0.50\,\mathrm{T})} = 3.4\times10^{-4}\,\mathrm{m}\,.$$

(b)

$$K = \frac{1}{2}m_e v^2 = \frac{(9.11 \times 10^{-31}\,\text{kg})(3.0 \times 10^7\,\text{m/s})^2}{2(1.6 \times 10^{-19}\,\text{J/eV})} = 2.6 \times 10^3\,\text{eV}\,.$$

22E

Use Eq. 29-16 to calculate B:

$$B = \frac{m_e v}{qr} = \frac{(1.67 \times 10^{-27}\,\text{kg})(1.0 \times 10^7\,\text{m/s})}{(1.60 \times 10^{-19}\,\text{C})(6.37 \times 10^6\,\text{m})} = 1.6 \times 10^{-8}\,\text{T}\,.$$

23E

(a) From $K = \frac{1}{2}m_e v^2$ we get

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(1.20 \times 10^3\,\text{eV})(1.60 \times 10^{-19}\,\text{eV/J})}{9.11 \times 10^{-31}\,\text{kg}}} = 2.05 \times 10^7\,\text{m/s}\,.$$

(b) From $r = m_e v/qB$ we get

$$B = \frac{m_e v}{qr} = \frac{(9.11 \times 10^{-31}\,\text{kg})(2.05 \times 10^7\,\text{m/s})}{(1.60 \times 10^{-19}\,\text{C})(25.0 \times 10^{-2}\,\text{m})} = 4.67 \times 10^{-4}\,\text{T}\,.$$

(c)

$$f = \frac{v}{2\pi r} = \frac{2.07 \times 10^7\,\text{m/s}}{2\pi(25.0 \times 10^{-2}\,\text{m})} = 1.31 \times 10^7\,\text{Hz}\,.$$

(d) $T = 1/f = (1.31 \times 10^7\,\text{Hz})^{-1} = 7.63 \times 10^{-8}\,\text{s}$.

24E

The period of revolution for the iodine ion is $T = 2\pi r/v = 2\pi m/Bq$, which gives

$$m = \frac{BqT}{2\pi} = \frac{(45.0 \times 10^{-3}\,\text{T})(1.60 \times 10^{-19}\,\text{C})(1.29 \times 10^{-3}\,\text{s})}{(7)(2\pi)(1.66 \times 10^{-27}\,\text{kg/u})} = 127\,\text{u}\,.$$

25E

(a)

$$v = \frac{rqB}{m_\alpha} = \frac{2eB}{4.00\,\text{u}} = \frac{2(4.50 \times 10^{-2}\,\text{m})(1.60 \times 10^{-19}\,\text{C})(1.20\,\text{T})}{(4.00\,\text{u})(1.66 \times 10^{-27}\,\text{kg/u})} = 2.60 \times 10^6\,\text{m/s}\,.$$

(b) $T = 2\pi r/v = 2\pi(4.50 \times 10^{-2}\,\text{m})/(2.60 \times 10^6\,\text{m/s}) = 1.09 \times 10^{-7}\,\text{s}$.

(c)

$$K = \frac{1}{2} m_\alpha v^2 = \frac{(4.00\,\mathrm{u})(1.66 \times 10^{-27}\,\mathrm{kg/u})(2.60 \times 10^6\,\mathrm{m/s})^2}{2(1.60 \times 10^{-19}\,\mathrm{J/eV})} = 1.40 \times 10^5\,\mathrm{eV}\,.$$

(d) $\Delta V = K/q = 1.40 \times 10^5\,\mathrm{eV}/2e = 7.00 \times 10^4\,\mathrm{V}$.

26E

(a)

$$f = \frac{Bq}{2\pi m_e} = \frac{(35.0 \times 10^{-6}\,\mathrm{T})(1.60 \times 10^{-19}\,\mathrm{C})}{2\pi(9.11 \times 10^{-31}\,\mathrm{kg})} = 9.78 \times 10^5\,\mathrm{Hz}\,.$$

(b)

$$r = \frac{m_e v}{qB} = \frac{\sqrt{2m_e K}}{qB} = \frac{\sqrt{2(9.11 \times 10^{-31}\,\mathrm{kg})(100\,\mathrm{eV})(1.60 \times 10^{-19}\,\mathrm{J/eV})}}{(1.60 \times 10^{-19}\,\mathrm{C})(35.0 \times 10^{-6}\,\mathrm{T})} = 0.964\,\mathrm{m}\,.$$

27E

Orient the magnetic field so it is perpendicular to the plane of page. Then the electron will travel with constant speed around a circle in the plane of the page. The magnetic force on an electron has magnitude $F_B = evB$, where v is the speed of the electron and B is the magnitude of the magnetic field. If r is the radius of the circle, the acceleration of the electron has magnitude $a = v^2/r$. Newton's second law yields $evB = m_e v^2/r$, so the radius of the circle is given by $r = m_e v/eB$. The kinetic energy of the electron is $K = \frac{1}{2} m_e v^2$, so $v = \sqrt{2K/m_e}$. Thus

$$r = \frac{m_e}{eB}\sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2m_e K}{e^2 B^2}}\,.$$

This must be less than d, so

$$\sqrt{\frac{2m_e K}{e^2 B^2}} \le d$$

or

$$B \ge \sqrt{\frac{2m_e K}{e^2 d^2}}\,.$$

If the electrons are to travel as shown in Fig. 29-40 the magnetic field must be out of the page. Then the magnetic force is toward the center of the circular path.

28P

Let $v_\parallel = v\cos\theta$. The electron will proceed with a uniform speed $v_\parallel$ in the direction of $\mathbf{B}$ while undergoing uniform cirular motion with frequency f in the direction perpendicular to B: $f = eB/2\pi m_e$. The distance d is then

$$\begin{aligned} d = v_\parallel T = \frac{v_\parallel}{f} &= \frac{(v\cos\theta)2\pi m_e}{eB} \\ &= \frac{2\pi(1.5 \times 10^7\,\mathrm{m/s})(9.11 \times 10^{-31}\,\mathrm{kg})(\cos 10^\circ)}{(1.60 \times 10^{-19}\,\mathrm{C})(1.0 \times 10^{-3}\,\mathrm{T})} = 0.53\,\mathrm{m}\,. \end{aligned}$$

29P

From 26E, part (b) we see that $r = \sqrt{2mK}/qB$, or $K = (rqB)^2/2m \propto q^2m^{-1}$. Thus
(a) $K_\alpha = (q_\alpha/q_p)^2(m_p/m_\alpha)K_p = (2)^2(1/4)K_p = K_p = 1.0\,\mathrm{MeV}$;
(b) $K_d = (q_d/q_p)^2(m_p/m_d)K_p = (1)^2(1/2)K_p = 1.0\,\mathrm{MeV}/2 = 0.50\,\mathrm{MeV}$.

30P

(a) Since $K = qV$ we have $K_p = K_d = \frac{1}{2}K_\alpha$ (as $q_\alpha = 2K_d = 2K_p$).
(b) and (c) Since $r = \sqrt{2mK}/qB \propto \sqrt{mK}/q$, we have

$$r_d = \sqrt{\frac{m_dK_d}{m_pK_p}}\,\frac{q_pr_p}{q_d} = \sqrt{\frac{(2.00\,\mathrm{u})K_p}{(1.00\,\mathrm{u})K_p}}\,r_p = 10\sqrt{2}\,\mathrm{cm} = 14\,\mathrm{cm}\,,$$

$$r_\alpha = \sqrt{\frac{m_\alpha K_\alpha}{m_pK_p}}\,\frac{q_pr_p}{q_\alpha} = \sqrt{\frac{(4.00\,\mathrm{u})K_\alpha}{(1.00\,\mathrm{u})(K_\alpha/2)}}\,\frac{er_p}{2e} = 10\sqrt{2}\,\mathrm{cm} = 14\,\mathrm{cm}\,.$$

31P

Use the result of the previous problem: $r \propto \sqrt{mK}/qB$. Thus

$$r_\alpha = \sqrt{\frac{m_\alpha K_\alpha}{m_pK_p}}\,\frac{q_p}{q_\alpha}r_p = \sqrt{\frac{4.0\,\mathrm{u}}{1.0\,\mathrm{u}}}\,\frac{er_p}{2e} = r_p\,;$$

$$r_d = \sqrt{\frac{m_dK_d}{m_pK_p}}\,\frac{q_p}{q_d}r_d = \sqrt{\frac{2.0\,\mathrm{u}}{1.0\,\mathrm{u}}}\,\frac{er_d}{e} = \sqrt{2}r_p\,.$$

32

The equation of motion for the proton is

$$\begin{aligned}\mathbf{F} &= q\mathbf{v}\times\mathbf{B} = q(v_x\,\mathbf{i} + v_y\,\mathbf{j} + v_z\mathbf{k})\times B\mathbf{i} = qB(v_z\,\mathbf{j} - v_y\mathbf{k})\\ &= m_p\mathbf{a} = m_p\left[\left(\frac{dv_x}{dt}\right)\mathbf{i} + \left(\frac{dv_y}{dt}\right)\mathbf{j} + \left(\frac{dv_z}{dt}\right)\mathbf{k}\right].\end{aligned}$$

Thus

$$\begin{cases}\dfrac{dv_x}{dt} = 0\\[2mm] \dfrac{dv_y}{dt} = \omega v_z\\[2mm] \dfrac{dv_z}{dt} = -\omega v_y\,,\end{cases}$$

where $\omega = eB/m_p$. The solution is $v_x = v_{0x}$, $v_y = v_{0y}\cos\omega t$ and $v_z = -v_{0y}\sin\omega t$; i.e., $\mathbf{v}(t) = v_{0x}\,\mathbf{i} + v_{0y}\cos(\omega t)\,\mathbf{j} - v_{0y}(\sin\omega t)\mathbf{k}$.

33P

(a) From $m = B^2qx^2/8V$ we have $\Delta m = (B^2q/8V)(2x\Delta x)$. Here $x = \sqrt{8Vm/B^2q}$, which we sbustitute into the experssion for Δm to obatin

$$\Delta m = \left(\frac{B^2q}{8V}\right)2\sqrt{\frac{8mV}{B^2q}}\Delta x = B\sqrt{\frac{mq}{2V}}\,\Delta x\,.$$

(b)

$$\begin{aligned}\Delta x &= \frac{\Delta m}{B}\sqrt{\frac{2V}{mq}}\\ &= \frac{(37\,\mathrm{u} - 35\,\mathrm{u})(1.66\times10^{-27}\,\mathrm{kg/u})}{0.50\,\mathrm{T}}\sqrt{\frac{2(7.3\times10^{3}\,\mathrm{V})}{(36\,\mathrm{u})(1.66\times10^{-27}\,\mathrm{kg/u})(1.60\times10^{-19}\,\mathrm{C})}}\\ &= 8.2\times10^{-3}\,\mathrm{m}\,.\end{aligned}$$

34P

(a) Solve B from $m = B^2qx^2/8V$:

$$B = \sqrt{\frac{8Vm}{qx^2}}\,.$$

Evaluate this expression using $x = 2.00\,\mathrm{m}$:

$$B = \sqrt{\frac{8(100\times10^{3}\,\mathrm{V})(3.92\times10^{-25}\,\mathrm{kg})}{(3.20\times10^{-19}\,\mathrm{C})(2.00\,\mathrm{m})^2}} = 0.495\,\mathrm{T}\,.$$

(b) Let N be the number of ions that are separated by the machine per unit time. The current is $i = qN$ and the mass that is separated per unit time is $M = mN$, where m is the mass of a single ion. M has the value

$$M = \frac{100\times10^{-6}\,\mathrm{kg}}{3600\,\mathrm{s}} = 2.78\times10^{-8}\,\mathrm{kg/s}\,.$$

Since $N = M/m$ we have

$$i = \frac{qM}{m} = \frac{(3.20\times10^{-19}\,\mathrm{C})(2.78\times10^{-8}\,\mathrm{kg/s})}{3.92\times10^{-25}\,\mathrm{kg}} = 2.27\times10^{-2}\,\mathrm{A}\,.$$

(c) Each ion deposits an energy of qV in the cup, so the energy deposited in time Δt is given by

$$E = NqV\,\Delta t = \frac{iqV}{q}\,\Delta t = iV\,\Delta t\,.$$

For $\Delta t = 1.0\,\text{h}$,

$$E = (2.27 \times 10^{-2}\,\text{A})(100 \times 10^3\,\text{V})(3600\,\text{s}) = 8.17 \times 10^6\,\text{J}\,.$$

To obtain the second expression, i/q was substituted for N.

35P

The condition for the ion beam to be undeviated is $v = E/B$. Then as the ion beam enters the magnetic field $\mathbf{B}'$ we have $r = mv/(qB') = mE/(qBB')$, or $q/m = E/(rBB')$.

36P

(a) If v is the speed of the positron then $v \sin\phi$ is the component of its velocity in the plane that is perpendicular to the magnetic field. Here ϕ is the angle between the velocity and the field (89°). Newton's second law yields $eBv\sin\phi = m_e(v\sin\phi)^2/r$, where r is the radius of the orbit. Thus $r = (m_e v/eB)\sin\phi$. The period is given by

$$T = \frac{2\pi r}{v\sin\phi} = \frac{2\pi m_e}{eB} = \frac{2\pi(9.11 \times 10^{-31}\,\text{kg})}{(1.60 \times 10^{-19}\,\text{C})(0.10\,\text{T})} = 3.6 \times 10^{-10}\,\text{s}\,.$$

The equation for r was substituted to obtain the second expression for T.

(b) The pitch is the distance traveled along the line of the magnetic field in a time interval of one period. Thus $p = vT\cos\phi$. Use the kinetic energy to find the speed: $K = \frac{1}{2}m_e v^2$ means

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(2.0 \times 10^3\,\text{eV})(1.60 \times 10^{-19}\,\text{J/eV})}{9.11 \times 10^{-31}\,\text{kg}}} = 2.651 \times 10^7\,\text{m/s}\,.$$

Thus

$$p = (2.651 \times 10^7\,\text{m/s})(3.58 \times 10^{-10}\,\text{s})\cos 89° = 1.7 \times 10^{-4}\,\text{m}\,.$$

(c) The orbit radius is

$$R = \frac{m_e v\sin\phi}{eB} = \frac{(9.11 \times 10^{-31}\,\text{kg})(2.651 \times 10^7\,\text{m/s})\sin 89°}{(1.60 \times 10^{-19}\,\text{C})(0.10\,\text{T})} = 1.5 \times 10^{-3}\,\text{m}\,.$$

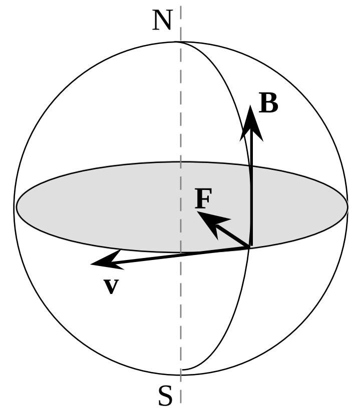
37P

(a) $-q$, from conservation of charges.

(b) Each of the two particles will move in the same circular path, initially going in the opposite direction. After traveling half of the circular path they will collide. So the time is given by $t = T/2 = \pi m/Bq$.

38P

(*a*) The radius r of the circular orbit is given by $r = p/eB$, where B is the magnitude of the magnetic field. The relativisitic expression $p = m_p v/\sqrt{1 - v^2/c^2}$ must be used for the magnitude of the momentum. Here v is the speed of the proton, m_p is its mass, and c is the speed of light. Hence



$$r = \frac{m_p v}{eB\sqrt{1 - v^2/c^2}} \,.$$

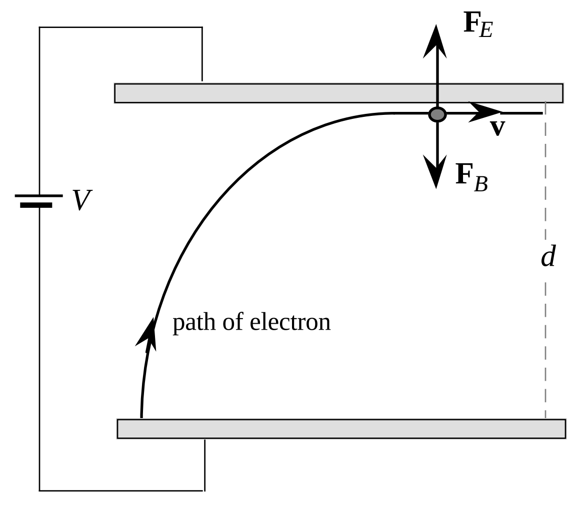
Square both sides and solve for v. The result is

$$v = \frac{reBc}{\sqrt{m_p^2 c^2 + r^2 e^2 B^2}} \,.$$

Substitute $r = 6.37 \times 10^6$ m (the radius of the Earth), $e = 1.6022 \times 10^{-19}$ C (the charge on a proton), $B = 41 \times 10^{-6}$ T, $m_p = 1.6726 \times 10^{-27}$ kg (the mass of a proton), and $c = 2.9979 \times 10^8$ m/s to obtain $v = 2.9977 \times 10^8$ m/s.

39P

The figure to the right depicts the path of the electron in the case of minumum B. When the electron is just about to strike the top plate its velocity $\mathbf{v}$ must be parallel to the plate or it will hit the plate. The magnetic force F_B exerted on the plate is now directed away from the top plate, with magnitude $F_B = eBv$ (Note that $\mathbf{B} \perp \mathbf{v}$). Here v satisfies $K = \frac{1}{2}m_e v^2 = eV$, or $v = \sqrt{2eV/m_e}$. Now, F_B must be greater or equal to the electric force $F_E = eE = eV/d$ exerted on the electron, which tends to push the electron toward the top plate. Thus



$$F_B = eBv = eB\sqrt{\frac{2eV}{m_e}} \geq F_E = eE = \frac{eV}{d} \,,$$

or

$$B \geq \sqrt{\frac{m_e V}{2ed^2}} \,.$$

40E

(*a*)

$$f_{\text{osc}} = \frac{qB}{2\pi m_p} = \frac{(1.60 \times 10^{-19}\,\text{C})(1.2\,\text{T})}{2\pi(1.67 \times 10^{-27}\,\text{kg})} = 1.8 \times 10^7\,\text{Hz} \,.$$

(b) From $r = m_p v/qB = \sqrt{2m_p K}/qB$ we have

$$K = \frac{(rqB)^2}{2m_p} = \frac{[(0.50\,\mathrm{m})(1.60\times10^{-19}\,\mathrm{C})(1.2\,\mathrm{T})]^2}{2(1.67\times10^{-27}\,\mathrm{kg})(1.60\times10^{-19}\,\mathrm{J/eV})} = 1.7\times10^7\,\mathrm{eV}\,.$$

41E

(a)

$$r = \frac{m_p v}{qB} = \frac{(1.67\times10^{-27}\,\mathrm{kg})(3.00\times10^8\,\mathrm{m/s})/10}{(1.60\times10^{-19}\,\mathrm{C})(1.4\,\mathrm{T})} = 0.22\,\mathrm{m}\,.$$

(b)

$$f_{\mathrm{osc}} = \frac{v}{2\pi r} = \frac{3.00\times10^7\,\mathrm{m/s}}{2\pi(0.22\,\mathrm{m})} = 2.1\times10^7\,\mathrm{Hz}\,.$$

42P

(a) Since $K = \frac{1}{2}mv^2 = \frac{1}{2}m(2\pi R f_{\mathrm{osc}})^2 \propto m$,

$$K_p = \left(\frac{m_p}{m_d}\right)K_d = \frac{1}{2}K_d = \frac{1}{2}(17\,\mathrm{MeV}) = 8.5\,\mathrm{MeV}\,.$$

(b)

$$B_p = \frac{1}{2}B_d = \frac{1}{2}(1.6\,\mathrm{T}) = 0.80\,\mathrm{T}\,.$$

(c) Since $K \propto B^2/m$,

$$K_p' = \left(\frac{m_d}{m_p}\right)K_d = 2K_d = 2(17\,\mathrm{MeV}) = 34\,\mathrm{MeV}\,.$$

(d) Since $f_{\mathrm{osc}} = Bq/(2\pi m) \propto m^{-1}$,

$$f_{\mathrm{osc},\,d} = \left(\frac{m_d}{m_P}\right)f_{\mathrm{osc},\,p} = 2(12\times10^6\,\mathrm{s}^{-1}) = 2.4\times10^7\,\mathrm{Hz}.$$

(e) Now

$$\begin{aligned}
K_\alpha &= \left(\frac{m_\alpha}{m_d}\right)K_d = 2K_d = 2(17\,\mathrm{MeV}) = 34\,\mathrm{MeV}\,,\\
B_\alpha &= \left(\frac{m_\alpha}{m_d}\right)\left(\frac{q_d}{q_\alpha}\right)B_d = 2\left(\frac{1}{2}\right)(1.6\,\mathrm{T}) = 1.6\,\mathrm{T}\,,\\
K_\alpha' &= K_\alpha = 34\,\mathrm{MeV} \qquad\qquad (\text{Since } B_\alpha = B_d = 1.6\,\mathrm{T})\,,
\end{aligned}$$

and

$$f_{\mathrm{osc},\,\alpha} = \left(\frac{q_\alpha}{a_d}\right)\left(\frac{m_d}{m_\alpha}\right) f_{\mathrm{osc,\,d}} = 2\left(\frac{2}{4}\right)(12 \times 10^6\,\mathrm{s}^{-1}) = 1.2 \times 10^7\,\mathrm{Hz}\,.$$

43P

The speed of the deuteron before it breaks up is

$$v_d = \frac{rqB}{m_d} = \frac{(50 \times 10^{-2}\,\mathrm{m}))(1.60 \times 10^{-19}\,\mathrm{C})(1.5\,\mathrm{T})}{2(1.66 \times 10^{-27}\,\mathrm{kg})} = 3.6 \times 10^5\,\mathrm{m/s}\,.$$

Since $K_d = \frac{1}{2}m_d v_d^2 = K_n + K_p = \frac{1}{2}m_n v_n^2 + \frac{1}{2}m_p v_p^2$ and $m_d v_d = m_n v_n + m_p v_p$ (conservation of linear momentum), we find $v_p \approx v_n \approx v_d$, where we noted that $m_n \approx m_p \approx m_d/2$. So the neutron will proceed with speed $v_n \approx v_d = 3.6 \times 10^5$ m/s, moving in a straight line tangent to the original path of the deuteron (since $q_n = 0$); while the proton will move in a circle with radius

$$r_p = \left(\frac{m_p}{m_d}\right)\left(\frac{q_d}{q_p}\right) r_d = \left(\frac{1}{2}\right)(1)(50\,\mathrm{cm}) = 25\,\mathrm{cm}\,.$$

44P

Approximate the total distance by the number of revolutions times the circumference of the orbit corresponding to the average energy. This should be a good approximation since the deuteron receives the same energy each revolution and its period does not depend on its energy. The deuteron accelerates twice in each cycle and each time it receives an energy of $qV = 80 \times 10^3$ eV. Since its final energy is 16.6 MeV the number of revolutions it makes is

$$n = \frac{16.6 \times 10^6\,\mathrm{eV}}{2(80 \times 10^3\,\mathrm{eV})} = 104\,.$$

Its average energy during the accelerating process is 8.3 MeV. The radius of the orbit is given by $r = mv/qB$, where v is the deuteron's speed. Since this is given by $v = \sqrt{2K/m}$, the radius is

$$r = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{1}{qB}\sqrt{2Km}\,.$$

For the average energy

$$r = \frac{\sqrt{2(8.3 \times 10^6\,\mathrm{eV})(1.60 \times 10^{-19}\,\mathrm{J/eV})(3.34 \times 10^{-27}\,\mathrm{kg})}}{(1.60 \times 10^{-19}\,\mathrm{C})(1.57\,\mathrm{T})} = 0.375\,\mathrm{m}\,.$$

The total distance traveled is about $n2\pi r = (104)(2\pi)(0.375) = 2.4 \times 10^2$ m.

45E

The magnitude of the magnetic force on the wire is given by $F_B = iLB\sin\phi$, where i is the current in the wire, L is the length of the wire, B is the magnitude of the magnetic field, and ϕ is the angle between the current and the field. In this case $\phi = 70°$. Thus

$$F_B = (5000\,\mathrm{A})(100\,\mathrm{m})(60.0\times10^{-6}\,\mathrm{T})\sin 70° = 28.2\,\mathrm{N}\,.$$

Apply the right-hand rule to the vector product $\mathbf{F}_B = i\mathbf{L}\times\mathbf{B}$ to show that the force is to the west.

46P

The magnetic force on the wire must be upward and have a magnitude equal to the gravitational force mg on the wire. Apply the right-hand rule to show that the current must be from left to right. Since the field and the current are perpendicular to each other the magnitude of the magnetic force is given by $F_B = iLB$, where L is the length of the wire. The condition that the tension in the supports vanish is $iLB = mg$, which yields

$$i = \frac{mg}{LB} = \frac{(0.0130\,\mathrm{kg})(9.8\,\mathrm{m/s^2})}{(0.620\,\mathrm{m})(0.440\,\mathrm{T})} = 0.467\,\mathrm{A}\,.$$

47E

$$F_B = iBL\sin\theta = (13.0\,\mathrm{A})(1.50\,\mathrm{T})(1.80\,\mathrm{m})(\sin 35.0°) = 20.1\,\mathrm{N}\,.$$

48P

$$\begin{aligned}\mathbf{F}_B &= i\mathbf{L}\times\mathbf{B} = iL\,\mathbf{i}\times(B_y\,\mathbf{j}+B_z\mathbf{k}) = iL(-B_z\,\mathbf{j}+B_y\mathbf{k})\\ &= (0.50\,\mathrm{A})(0.50\,\mathrm{m})[-(0.010\,\mathrm{T})\,\mathbf{j}+(0.0030\,\mathrm{T})\mathbf{k}]\\ &= (-2.5\times10^{-3}\,\mathbf{j}+0.75\times10^{-3}\mathbf{k})\,\mathrm{N}\,.\end{aligned}$$

49P

The magnetic force on the wire is $F_B = idB$, pointing to the left. Thus $v = at = (F_B/m)t = idBt/m$, to the left (i.e., away from the generator).

50P

(a) Since $\mathbf{B}$ is uniform,

$$\mathbf{F}_B = \int_{\text{wire}} i d\mathbf{L}\times\mathbf{B} = i\left(\int_{\text{wire}} d\mathbf{L}\right)\times\mathbf{B} = i\mathbf{L}_{ab}\times\mathbf{B}\,,$$

where we noted that $\int_{\text{wire}} d\mathbf{L} = \mathbf{L}_{ab}$, $\mathbf{L}_{ab}$ being the displacement vector from a to b.
(b) Now $\mathbf{L}_{ab} = 0$, so $\mathbf{F}_B = i\mathbf{L}_{ab} \times \mathbf{B} = 0$.

51P

Use $d\mathbf{F}_B = id\mathbf{L} \times \mathbf{B}$, where $d\mathbf{L} = dx\,\mathbf{i}$ and $\mathbf{B} = B_x\,\mathbf{i} + B_y\,\mathbf{j}$. Thus

$$\mathbf{F}_B = \int id\mathbf{L} \times \mathbf{B} = \int_{x_i}^{x_f} idx\,\mathbf{i} \times (B_x\,\mathbf{i} + B_y\,\mathbf{j}) = i\int_{x_i}^{x_f} B_y\,dx\mathbf{k}$$
$$= (-5.0\,\mathrm{A})\left[\int_{1.0}^{3.0} (8.0x^2\,dx)\,(\mathrm{m}\cdot\mathrm{mT})\right]\mathbf{k} = (-0.35\,\mathrm{N})\,\mathbf{k}\,.$$

52P

(a) From $F_B = iLB$ we get

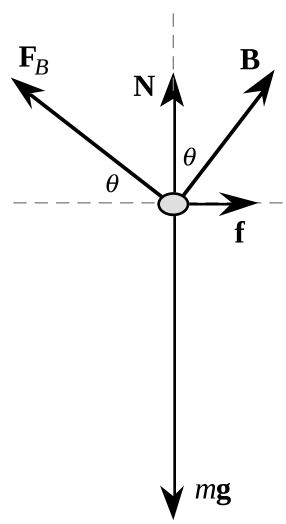
$$i = \frac{F_B}{LB} = \frac{10 \times 10^3\,\mathrm{N}}{(3.0\,\mathrm{m})(10 \times 10^{-6}\,\mathrm{T})} = 3.3 \times 10^8\,\mathrm{A}\,.$$

(b) $P = i^2R = (3.3 \times 10^8\,\mathrm{A})^2(1.0\,\Omega) = 1.0 \times 10^{17}\,\mathrm{W}$.
(c) It is totally unrealistic because of the huge current and the consequent high power loss.

53P

The magnetic force must push horizontally on the rod to overcome the force of friction. But it can be oriented so it also pulls up on the rod and thereby reduces both the normal force and the force of friction.

Suppose the magnetic field makes the angle θ with the vertical. The diagram to the right shows the view from the end of the sliding rod. The forces are also shown: F_B is the force of the magnetic field if the current is out of the page, mg is the force of gravity, N is the normal force of the stationary rails on the rod, and f is the force of friction. Notice that the magnetic force makes the angle θ with the *horizontal*. When the rod is on the verge of sliding, the net force acting on it is zero and the magnitude of the frictional force is given by $f = \mu_s N$, where μ_s is the coefficient of static friction. The magnetic field is perpendicular to the wire so the magnitude of the magnetic force is given by $F_B = iLB$, where i is the current in the rod and L is the length of the rod.



The vertical component of Newton's second law yields

$$N + iLB\sin\theta - mg = 0$$

and the horizontal component yields

$$iLB\cos\theta - \mu_s N = 0\,.$$

Solve the second equation for N and substitute the resulting expression into the first equation, then solve for B. You should get

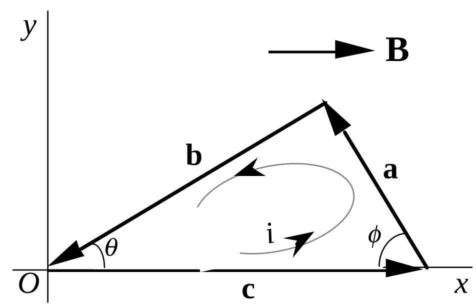
$$B = \frac{\mu_s mg}{iL(\cos\theta + \mu_s\sin\theta)}\,.$$

The minimum value of B occurs when $\cos\theta + \mu_s\sin\theta$ is a maximum. Set the derivative of $\cos\theta + \mu_s\sin\theta$ equal to zero and solve for θ. You should get $\theta = \tan^{-1}\mu_s = \tan^{-1}(0.60) = 31^\circ$. Now evaluate the expression for the minimum value of B:

$$B_{\min} = \frac{0.60(1.0\,\mathrm{kg})(9.8\,\mathrm{m/s^2})}{(50\,\mathrm{A})(1.0\,\mathrm{m})(\cos 31^\circ + 0.60\sin 31^\circ)} = 0.10\,\mathrm{T}\,.$$

54P

(a) Denote the vectors representing the three sids of the triangle as **a**, **b**, and **c**. The direction of each vector is so chosen that it is in the same direction as the current flow. Choose a coordinate system as shown. Then $\mathbf{B} = B\mathbf{i}$. We have

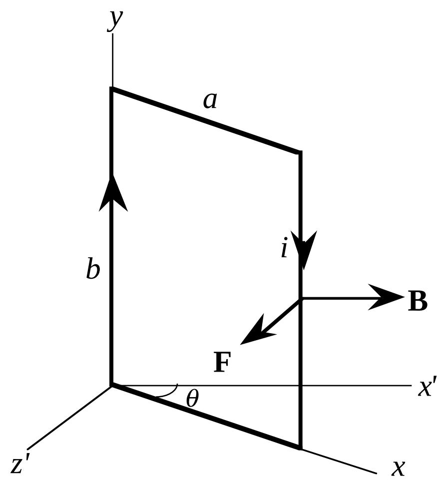


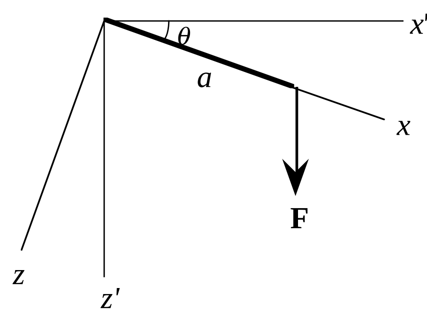
$$\begin{cases} \mathbf{F}_c = i\mathbf{c}\times\mathbf{B} = (ic)\,\mathbf{i}\times B\,\mathbf{i} = 0, \\ \mathbf{F}_b = i\mathbf{b}\times\mathbf{B} = ibB\sin\theta\mathbf{k} \\ \quad = (4.00\,\mathrm{A})(1.20\,\mathrm{m})(0.0750\,\mathrm{T})(50.0/130)\mathbf{k} \\ \quad = (0.138\,\mathrm{N})\mathbf{k}\,, \\ \mathbf{F}_a = i\mathbf{a}\times\mathbf{B} = (-iaB\sin\phi)\mathbf{k} = (-ibB\sin\theta)\mathbf{k} \\ \quad = -\mathbf{F}B = -(0.138\,\mathrm{N})\mathbf{k}\,. \end{cases}$$

(b) The sum of forces on the three sides is $\mathbf{F}_a + \mathbf{F}_b + \mathbf{F}_c = (-iaB\sin\phi)\mathbf{k} + (ibB\sin\theta)\mathbf{k} = 0$, where we noted that $a\sin\phi = b\sin\theta = ab/c$. This is in fact a special case of 50P, part (b).

55P

The situation is shown in the left diagram in the next page. The y axis is along the hinge and the magnetic field is in the positive x' direction. A torque around the hinge is associated with the wire opposite the hinge and not with the other wires. The force on this wire is in the positive z' direction and has magnitude $F = NibB$, where N is the number of turns.





The right diagram shows the view from above. The magnitude of the torque is given by

$$\begin{aligned}\tau &= Fa\cos\theta = NibBa\cos\theta\\ &= 20(0.10\,\mathrm{A})(0.10\,\mathrm{m})(0.50\times10^{-3}\,\mathrm{T})(0.050\,\mathrm{m})\cos 30^\circ\\ &= 4.33\times10^{-3}\,\mathrm{N\cdot m}\,.\end{aligned}$$

Use the right-hand rule to show that the torque is directed downward, in the negative y direction.

56P

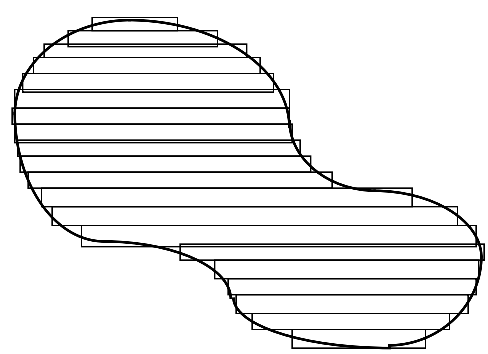
If N closed loops are formed from the wire of length L, the circumference of each loop is L/N, the radius of each loop is $R = L/2\pi N$, and the area of each loop is $A = \pi R^2 = \pi(L/2\pi N)^2 = L^2/4\pi N^2$. For maximum torque, orient the plane of the loops parallel to the magnetic field, so the dipole moment is perpendicular to the field. The magnitude of the torque is then

$$\tau = NiAB = (Ni)\left(\frac{L^2}{4\pi N^2}\right)B = \frac{iL^2B}{4\pi N}\,.$$

To maximize the torque take N to have the smallest possible value, 1. Then $\tau = iL^2B/4\pi$.

57P

Replace the current loop of arbitrary shape with an assembly of adjacent long, thin, rectangular loops, each with N turns and carrying a current i flowing in the same sense as the original loop of arbitrary shape. See the figure to the right. As the widths of these rectangles shrink to infinitesimally small values the assembly gives a current distribution equivalent to that of the original loop. The magnitude of the torque $\Delta\boldsymbol{\tau}$ exerted by $\mathbf{B}$ on the nth rectangular loop of area ΔA_n is given by $\Delta\tau_n = NiB\sin\theta\Delta A_n$.



Thus for the whole assembly

$$\tau = \sum_n \Delta\tau_n = NiB \sum_n \Delta A_n = NiAB\sin\theta\,.$$

58P

The total magnetic force on the loop L is

$$\mathbf{F}_B = i\oint_L (d\mathbf{L}\times\mathbf{B}) = i(\oint_L d\mathbf{L})\times\mathbf{B} = 0\,,$$

where we noted that $\oint_L d\mathbf{L} = 0$. If $\mathbf{B}$ is not a constant, however, then the equality

$$\oint_L (d\mathbf{L}\times\mathbf{B}) = (\oint_L d\mathbf{L})\times\mathbf{B}$$

is not necessarily valid so $\mathbf{F}_B$ is not always zero.

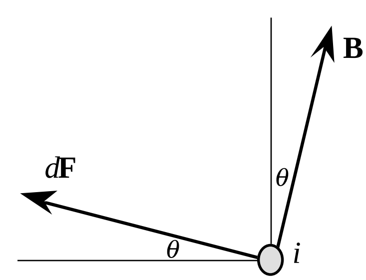
59P

Use $\tau_{\max} = |\boldsymbol{\mu}\times\mathbf{B}|_{\max} = \mu B = i\pi a^2 B$, and note that $i = qf = qv/2\pi a$. So

$$\tau_{\max} = \left(\frac{qv}{2\pi a}\right)\pi a^2 B = \frac{1}{2}qvaB\,.$$

60P

Consider an infinitesimal segment of the loop, of length ds. The magnetic field is perpendicular to the segment so the magnetic force on it is has magnitude $dF = iB\,ds$. The diagram shows the direction of the force for the segment on the far right of the loop. The horizontal component of the force has magnitude $dF_h = (iB\cos\theta)\,ds$ and points inward toward the center of the loop. The vertical component has magnitude $dF_v = (iB\sin\theta)\,ds$ and points upward.



Now sum the forces on all the segments of the loop. The horizontal component of the total force vanishes since each segment of wire can be paired with another, diametrically opposite, segment. The horizontal components of these forces are both toward the center of the loop and thus in opposite directions. The vertical component of the total force is

$$F_v = iB\sin\theta\int ds = (iB\sin\theta)2\pi a\,.$$

Notice the i, B, and θ have the same value for every segment and so can be factored from the integral.

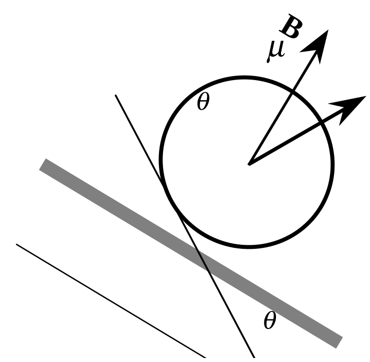
61P

(a) The current in the galvanometer should be $1.62\,\mathrm{mA}$ when the potential difference across the resistor-galvanometer combination is $1.00\,\mathrm{V}$. The potential difference across the galvanometer alone is $iR_g = (1.62 \times 10^{-3}\,\mathrm{A})(75.3\,\Omega) = 0.122\,\mathrm{V}$ so the resistor must be in series with the galvanometer and the potential difference across it must be $1.00\,\mathrm{V} - 0.122\,\mathrm{V} = 0.878\,\mathrm{V}$. The resistance should be $R = (0.878\,\mathrm{V})/(1.62 \times 10^{-3}\,\mathrm{A}) = 542\,\Omega$.

(b) The current in the galvanometer should be $1.62\,\mathrm{mA}$ when the total current in the resistor-galvanometer combination is $50.0\,\mathrm{mA}$. The resistor should be in parallel with the galvanometer and the current through it should be $50\,\mathrm{mA} - 1.62\,\mathrm{mA} = 48.38\,\mathrm{mA}$. The potential difference across the resistor is the same as that across the galvanometer, $0.122\,\mathrm{V}$, so the resistance should be $R = (0.122\,\mathrm{V})/(48.38 \times 10^{-3}\,\mathrm{A}) = 2.52\,\Omega$.

62P

In the diagram to the right $\boldsymbol{\mu}$ is the magnetic dipole moment of the wire loop and $\mathbf{B}$ is the magnetic field. Since the plane of the loop is parallel to the incline the dipole moment is normal to the incline. The forces acting on the cylinder are the force of gravity mg, acting downward from the center of mass, the normal force of the incline N, acting perpendicularly to the incline through the center of mass, and the force of friction f, acting up the incline at the point of contact.



Take the x axis to be positive down the incline. Then the x component of Newton's second law for the center of mass yields

$$mg\sin\theta - f = ma\,.$$

For purposes of calculating the torque take the axis of the cylinder to be the axis of rotation. The magnetic field produces a torque with magnitude $\mu B\sin\theta$ and the force of friction produces a torque with magnitude fr, where r is the radius of the cylinder. The first tends to produce an angular acceleration in the counterclockwise direction and the second tends to produce an angular acceleration in the clockwise direction. Newton's second law for rotation about the center of the cylinder, $\tau = I\alpha$, gives

$$fr - \mu B\sin\theta = I\alpha\,.$$

Since you want the current that holds the cylinder in place, set $a = 0$ and $\alpha = 0$, then use one equation to eliminate f from the other. You should obtain $mgr = \mu B$. The loop is rectangular with two sides of length L and two of length $2r$, so its area is $A = 2rL$ and the dipole moment is $\mu = NiA = 2NirL$. Thus $mgr = 2NirLB$ and

$$i = \frac{mg}{2NLB} = \frac{(0.250\,\mathrm{kg})(9.8\,\mathrm{m/s^2})}{2(10.0)(0.100\,\mathrm{m})(0.500\,\mathrm{T})} = 2.45\,\mathrm{A}\,.$$

63E

(*a*) The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current in each turn, and A is the area of a loop. In this case the loops are circular, so $A = \pi r^2$, where r is the radius of a turn. Thus

$$i = \frac{\mu}{N\pi r^2} = \frac{2.30\,\mathrm{A\cdot m^2}}{(160)(\pi)(0.0190\,\mathrm{m})^2} = 12.7\,\mathrm{A}\,.$$

(*b*) The maximum torque occurs when the dipole moment is perpendicular to the field (or the plane of the loop is parallel to the field). It is given by $\tau_{\mathrm{max}} = \mu B = (2.30\,\mathrm{A{\cdot}m^2})(35.0 \times 10^{-3}\,\mathrm{T}) = 8.05 \times 10^{-2}\,\mathrm{N{\cdot}m}$.

64E

(*a*) From $\mu = NiA = i\pi r^2$ we get

$$i = \frac{\mu}{\pi r^2} = \frac{8.00 \times 10^{22}\,\mathrm{J/T}}{\pi(3500 \times 10^3\,\mathrm{m})^2} = 2.08 \times 10^9\,\mathrm{A}.$$

65E

(*a*) $\mu = NAi = \pi r^2 i = \pi(0.150\,\mathrm{m})^2(2.60\,\mathrm{A}) = 0.184\,\mathrm{A\cdot m^2}$.
(*b*) $\tau = |\boldsymbol{\mu} \times \mathbf{B}| = \mu B \sin\theta = (0.184\,\mathrm{A\cdot m^2})(12.0\,\mathrm{T})(\sin 41.0°) = 1.45\,\mathrm{N\cdot m}$.

66E

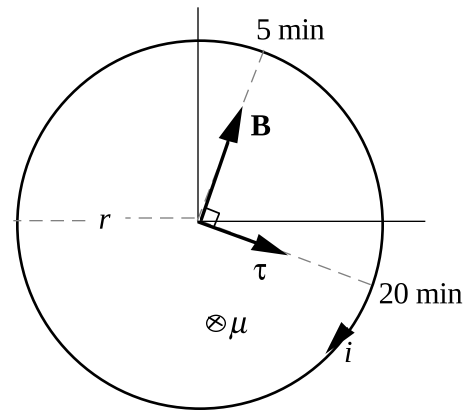
(*a*) The area A of the current loop is $A = \frac{1}{2}(30\,\mathrm{cm})(40\,\mathrm{cm}) = 6.0 \times 10^2\,\mathrm{cm}^2$, so

$$\mu = iA = (5.0\,\mathrm{A})(6.0 \times 10^{-2}\,\mathrm{m^2}) = 0.30\,\mathrm{A\cdot m^2}\,.$$

(*b*)

$$\tau = \mu B \sin\theta = (0.30\,\mathrm{A\cdot m^2})(80 \times 10^3\,\mathrm{T})(\sin 90°) = 2.4 \times 10^{-2}\,\mathrm{N\cdot m}\,.$$

67E



(*a*) Use $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$. From the diagram to the right it is clear that $\boldsymbol{\tau}$ points at the 20-minute mark. So the time interval is 20 min.
(*b*) The torque is given by

$$\begin{aligned}\tau &= |\boldsymbol{\mu} \times \mathbf{B}| = \mu B \sin(\boldsymbol{\mu}, \mathbf{B}) \\ &= NiAB \sin 90^\circ \\ &= \pi N i r^2 B \\ &= 6\pi(2.0\,\mathrm{A})(0.15\,\mathrm{m})^2(70 \times 10^{-3}\,\mathrm{T}) \\ &= 5.9 \times 10^{-2}\,\mathrm{N{\cdot}m}\,.\end{aligned}$$

68E

(*a*)

$$\mu = \sum_n i_n A_n = \pi r_1^2 i_1 + \pi r_2^2 i_2 = \pi(7.00\,\mathrm{A})[(0.300\,\mathrm{m})^2 + (0.200\,\mathrm{m})^2] = 2.86\,\mathrm{A{\cdot}m^2}\,.$$

(*b*) Now

$$\mu = \pi r_1^2 i_1 - \pi r_2^2 i_2 = \pi(7.00\,\mathrm{A})[(0.300\,\mathrm{m})^2 - (0.200\,\mathrm{m})^2] = 1.10\,\mathrm{A{\cdot}m^2}\,.$$

69P

The magnetic dipole moment is $\boldsymbol{\mu} = \mu(0.60\,\mathbf{i} - 0.80\,\mathbf{j})$, where $\mu = NiA = Ni\pi r^2 = 1(0.20\,\mathrm{A})\pi(0.080\,\mathrm{m})^2 = 4.02 \times 10^{-4}\,\mathrm{A\cdot m^2}$. Here i is the current in the loop, N is the number of turns, A is the area of the loop, and r is its radius.
(*a*) The torque is

$$\begin{aligned}\boldsymbol{\tau} &= \boldsymbol{\mu} \times \mathbf{B} = \mu(0.60\,\mathbf{i} - 0.80\,\mathbf{j}) \times (0.25\,\mathbf{i} + 0.30\,\mathbf{k}) \\ &= \mu\,[(0.60)(0.30)(\,\mathbf{i} \times \mathbf{k}) - (0.80)(0.25)(\,\mathbf{j} \times \mathbf{i}) - (0.80)(0.30)(\,\mathbf{j} \times \mathbf{k})] \\ &= \mu[-0.18\,\mathbf{j} + 0.20\,\mathbf{k} - 0.24\,\mathbf{i}]\,.\end{aligned}$$

Here $\mathbf{i} \times \mathbf{k} = -\mathbf{j}$, $\mathbf{j} \times \mathbf{i} = -\mathbf{k}$, and $\mathbf{j} \times \mathbf{k} = \mathbf{i}$ were used. We also used $\mathbf{i} \times \mathbf{i} = 0$. Substitute the value for μ to obtain

$$\boldsymbol{\tau} = [-0.97 \times 10^{-4}\,\mathbf{i} - 7.2 \times 10^{-4}\,\mathbf{j} + 8.0 \times 10^{-4}\,\mathbf{k}]\,\mathrm{N{\cdot}m}\,.$$

(*b*) The potential energy of the dipole is given by

$$\begin{aligned}U &= -\boldsymbol{\mu}\cdot\mathbf{B} = -\mu(0.60\,\mathbf{i} - 0.80\,\mathbf{j})\cdot(0.25\,\mathbf{i} + 0.30\,\mathbf{k}) \\ &= -\mu(0.60)(0.25) = -0.15\mu = -6.0 \times 10^{-4}\,\mathrm{J}\,.\end{aligned}$$

Here $\mathbf{i}\cdot\mathbf{i} = 1$, $\mathbf{i}\cdot\mathbf{k} = 0$, $\mathbf{j}\cdot\mathbf{i} = 0$, and $\mathbf{j}\cdot\mathbf{k} = 0$ were used.

70P

Let $a = 30.0\,\mathrm{cm}$, $b = 20.0\,\mathrm{cm}$, and $c = 10.0\,\mathrm{cm}$. From the hint given we write

$$\begin{aligned}\boldsymbol{\mu} &= \boldsymbol{\mu}_1 + \boldsymbol{\mu}_2 = iab(-\mathbf{k}) + iab(\mathbf{j}) = ia(c\mathbf{j} - b\mathbf{k})\\ &= (5.00\,\mathrm{A})(0.300\,\mathrm{m})[(0.100\,\mathrm{m})\mathbf{j} - (0.200\,\mathrm{m})\mathbf{k}]\\ &= (0.150\,\mathbf{j} - 0.300\mathbf{k})\,\mathrm{A}\cdot\mathrm{m}^2.\end{aligned}$$

Thus $\mu = \sqrt{(0.150)^2 + (0.300)^2}\,\mathrm{A}\cdot\mathrm{m}^2 = 0.335\,\mathrm{A}\cdot\mathrm{m}^2$, and $\vec{\mu}$ is in the yz plane at angle θ to the $+y$ direction, where

$$\theta = \tan^{-1}\left(\frac{\mu_y}{\mu_x}\right) = \tan^{-1}\left(\frac{-0.300}{0.150}\right) = -63.4^\circ\,.$$