CHAPTER 31

Answer to Checkpoint Questions

1. b, then d and e tie, and then a and c tie (zero)
2. (a) and (b) tie, then (c) (zero)
3. c and d tie, then a and b tie
4. b, out; c, out; d, into; e, into
5. (d) and (e)
6. (a) 2, 3, 1 (zero); (b) 2, 3, 1
7. a and b tie, then c

Answer to Questions

1. (a) all tie (zero); (b) all tie (nonzero); (c) 3, then tie of 1 and 2 (zero)
2. (a) all tie (zero; (b) 2, then tie of 1 and 3 (zero)
3. out
4. (a) 2, 6; (b) 4; (c) 1, 3, 5, 7
5. (a) into; (b) counterclockwise; (c) larger
6. (a) clockwise; (b) increasing and then decreasing
7. (a) leftward; (b) rightward
8. clockwise
9. c, a, b
10. d and c tie, then b, a
11. (a) 1, 3, 2; (b) 1 and 3 tie, then 2
12. c, b, a
13. (a), then tie of (b) and (c)
14. (c), (a), (b)
15. (a) more; (b) same; (c) same; (d) same (zero)

16. (a) all tie (zero); (b) 1 and 2 tie, then 3; (c) all tie (zero)

17. a 2, b 4, c 1, d 3

18. (a) all tie (independent of the current); (b) 1, then 2 and 3 tie

Solutions to Exercises & Problems

1E

$$\Phi_B = \int \mathbf{B}\cdot d\mathbf{A} = BA\cos 57° = (4.2\times 10^{-6}\,\mathrm{T})(2.5\,\mathrm{m}^2)(\cos 57°) = 5.7\times 10^{-5}\,\mathrm{Wb}\,.$$

2E

$$\begin{aligned}\mathcal{E} &= -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -A\frac{dB}{dt} = -A\frac{d}{dt}(\mu_0 in) = -A\mu_0 n\frac{d}{dt}(i_0\sin\omega t)\\ &= -A\mu_0 n i_0\omega\cos\omega t\,.\end{aligned}$$

3E

The magnetic field is normal to the plane of the loop and is uniform over the loop. Thus at any instant the magnetic flux through the loop is given by $\Phi_B = AB = \pi r^2 B$, where A $(= \pi r^2)$ is the area of the loop. According to Faraday's law the magnitude of the emf in the loop is

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \pi r^2\frac{dB}{dt} = \pi(0.055\,\mathrm{m})^2(0.16\,\mathrm{T/s}) = 1.5\times 10^{-3}\,\mathrm{V}\,.$$

4E

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -A\frac{dB}{dt} = -\pi r^2\frac{d}{dt}(B_0 e^{-t/\tau}) = \frac{\pi r^2 B_0 e^{-t/\tau}}{\tau}\,.$$

5E

(a)

$$|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right| = \frac{d}{dt}(6.0t^2 + 7.0t) = 12t + 7.0 = 12(2.0) + 7.0 = 31\,\mathrm{mV}\,.$$

(b) Use Lenz's law to determine the direction of the current flow. It is from right to left.

6E

Use $\mathcal{E} = -d\Phi_B/dt = -\pi r^2 dB/dt$.

(a) For $0 < t < 2.0\,\mathrm{s}$:

$$\mathcal{E} = -\pi r^2 \frac{dB}{dt} = -\pi(0.12\,\mathrm{m})^2 \left(\frac{0.5\,\mathrm{T}}{2.0\,\mathrm{s}}\right) = -1.1 \times 10^{-2}\,\mathrm{V}\,.$$

(b) $2.0\,\mathrm{s} < t < 4.0\,\mathrm{s}$: $\mathcal{E} \propto dB/dt = 0$.

(c) $4.0\,\mathrm{s} < t < 6.0\,\mathrm{s}$:

$$\mathcal{E} = -\pi r^2 \frac{dB}{dt} = -\pi(0.12\,\mathrm{m})^2 \left(\frac{-0.5\,\mathrm{T}}{6.0\,\mathrm{s} - 4.0\,\mathrm{s}}\right) = 1.1 \times 10^{-2}\,\mathrm{V}\,.$$

7E

(a) The magnitude of the average induced emf is

$$\mathcal{E}_{\mathrm{av}} = \left|\frac{-d\Phi_B}{dt}\right| = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{BA_i}{\Delta t} = \frac{(2.0\,\mathrm{T})(0.20\,\mathrm{m})^2}{0.20\,\mathrm{s}} = -0.40\,\mathrm{V}\,.$$

(b)

$$i_{\mathrm{av}} = \frac{\mathcal{E}_{\mathrm{av}}}{R} = \frac{0.40\,\mathrm{V}}{20 \times 10^{-3}\Omega} = 20\,\mathrm{A}\,.$$

8E

(a)

$$R = \rho\frac{L}{A} = (1.68 \times 10^{-8}\Omega\cdot\mathrm{m})\left[\frac{\pi(0.10\,\mathrm{m})}{\pi(2.5 \times 10^{-3})^2/4}\right] = 1.1 \times 10^{-3}\Omega\,.$$

(b) Use $i = |\mathcal{E}|/R = |d\Phi_B/dt|/R = (\pi r^2/R)|dB/dt|$. Thus

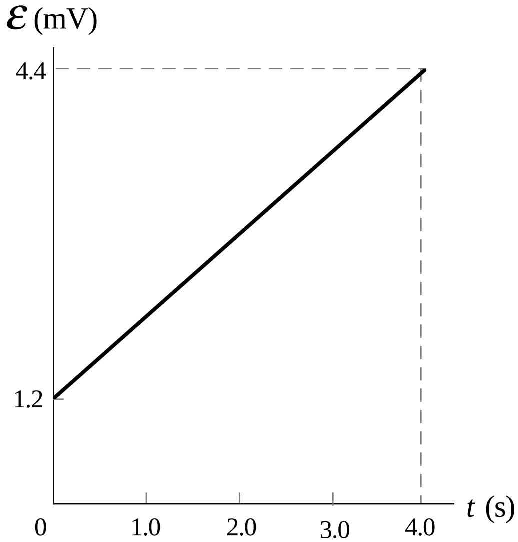
$$\left|\frac{dB}{dt}\right| = \frac{iR}{\pi r^2} = \frac{(10\,\mathrm{A})(1.1 \times 10^{-3}\Omega)}{\pi(0.10\,\mathrm{m})^2/4} = 1.4\,\mathrm{T/s}\,.$$

9P

(a) The emf as a function of time is given by

$$\begin{aligned}\mathcal{E} &= N\frac{d\Phi_B}{dt} = N\frac{d(BA)}{dt} = NA\frac{d}{dt}(\mu_0 ni) = N\left(\frac{\pi d^2}{4}\right)\mu_0 n\frac{di}{dt} \\ &= N\left(\frac{\pi d^2}{4}\right)\mu_0 n\frac{d}{dt}(3.0t + 1.0t^2) = \frac{1}{4}\pi d^2 N\mu_0 n(3.0 + 2.0t)\,.\end{aligned}$$

The plot is shown in the next page.



(b)

$$\begin{aligned} i|_{t=2.0\,\mathrm{s}} &= \frac{\mathcal{E}|_{t=2.0\,\mathrm{s}}}{R} = \frac{\pi d^2 N \mu_0 n (3.0 + 2.0t)}{4R} \\ &= \frac{\pi(2.1\times10^{-2}\,\mathrm{m})^2(130)(1.26\times10^{-6}\,\mathrm{T\cdot m/A})(2.2\,\mathrm{turns/m})[3.0+(2.0)(2.0)](\,\mathrm{A/s})}{4(0.15\Omega)} \\ &= 5.8\times10^{-2}\,\mathrm{A}\,. \end{aligned}$$

10P

The magnitude of the magnetic field inside the solenoid is $B = \mu_0 n i_s$, where n is the number of turns per unit length and i_s is the current. The field is parallel to the solenoid axis, so the flux through a cross section of the solenoid is $\Phi_B = A_s B = \mu_0 \pi r_s^2 n i_s$, where A_s $(= \pi r_s^2)$ is the cross-sectional area of the solenoid. Since the magnetic field is zero outside the solenoid this is also the flux through the coil. The emf in the coil has magnitude

$$\mathcal{E} = \frac{N d\Phi}{dt} = \mu_0 \pi r_s^2 N n \,\frac{di_s}{dt}$$

and the current in the coil is

$$i_c = \frac{\mathcal{E}}{R} = \frac{\mu_0 \pi r_s^2 N n}{R}\,\frac{di_s}{dt}\,,$$

where N is the number of turns in the coil and R is the resistance of the coil.
According to Sample Problem 31–1 the current changes linearly by 3.0 A in 50 ms, so $di_s/dt = (3.0\,\mathrm{A})/(50\times10^{-3}\,\mathrm{s}) = 60\,\mathrm{A/s}$. Thus

$$i_c = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})\pi(0.016\,\mathrm{m})^2(120)(220\times10^2\,\mathrm{m}^{-1})}{5.3\,\Omega} = 3.0\times10^{-2}\,\mathrm{A}\,.$$

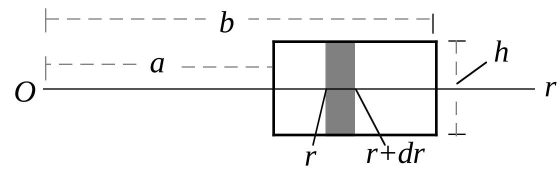
11P

$$\begin{aligned}\mathcal{E} &= -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -A\frac{d}{dt}(\mu_0 ni) = -\mu_0 n\pi r^2\frac{di}{dt}\\ &= -(1.26\times10^{-6}\,\mathrm{T\cdot m/A})(1.00\,\mathrm{turns/m})(\pi)(25\times10^{-3}\,\mathrm{m})^2\left(\frac{0.50\,\mathrm{A}-1.0\,\mathrm{A}}{10\times10^{-3}\,\mathrm{s}}\right)\\ &= 1.2\times10^{-3}\,\mathrm{V}\,.\end{aligned}$$

Note that since **B** only appears inside the solenoid the area A should be the cross-sectional area of the solenoid, not the (larger) loops.

12P

Consider the cross-section of the toroid. The magnetic flux through the shaded area as shown is $d\Phi_B = B(r)dA = B(r)hdr$. Thus from Eq. 31-35



cross-sectional view

$$\begin{aligned}\Phi_B &= \int_a^b B(r)hdr = \int_a^b \frac{\mu_0 i_0 Nh}{2\pi r}\,dr\\ &= \frac{\mu_0 iNh}{2\pi}\ln\frac{b}{a}\,.\end{aligned}$$

13P

$$\begin{aligned}\Phi_B &= BA = \left(\frac{\mu_0 i_0 N}{2\pi r}\right)A\\ &= \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(0.800\,\mathrm{A})(500)(5.00\times10^{-2}\,\mathrm{m})^2}{2\pi(0.150\,\mathrm{m}+0.0500\,\mathrm{m}/2)}\\ &= 1.15\times10^{-6}\,\mathrm{Wb}\,.\end{aligned}$$

14P

$$\begin{aligned}\mathcal{E} &= -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -B\frac{dA}{dt} = -B\frac{d(\pi r^2)}{dt} = -2\pi rB\frac{dr}{dt}\\ &= -2\pi(0.12\,\mathrm{m})(0.800\,\mathrm{T})(-0.750\,\mathrm{m/s}) = 0.452\,\mathrm{V}\,.\end{aligned}$$

15P

The magnetic flux Φ_B through the loop is given by $\Phi_B = 2B(\pi r^2/2)(\cos 45^\circ) = \pi r^2 B/\sqrt{2}$. Thus

$$\begin{aligned}\mathcal{E} &= -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(\frac{\pi r^2 B}{\sqrt{2}}\right) = -\frac{\pi r^2}{\sqrt{2}}\left(\frac{\Delta B}{\Delta t}\right)\\ &= -\frac{\pi(3.7\times10^{-2}\,\mathrm{m})^2}{\sqrt{2}}\left(\frac{0-76\times10^{-3}\,\mathrm{T}}{4.5\times10^{-3}\,\mathrm{s}}\right) = 5.1\times10^{-2}\,\mathrm{V}\,.\end{aligned}$$

The direction of the induced current is clockwise when viewed along the direction of **B**.

16P

Since Φ_B does not change, $\mathcal{E} = -d\Phi_B/dt = 0$.

17P

(a) Use $B \simeq \mu_0 i/2R$, where R is the radius of the large loop. Here $i(t) = i_0 + kt$, where $i_0 = 200\,\mathrm{A}$ and $k = (-200\,\mathrm{A} - 200\,\mathrm{A})/1.00\,\mathrm{s} = -400\,\mathrm{A/s}$. Thus

$$B|_{t=0} = \frac{\mu_0 i_0}{2R} = \frac{(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A})(200\,\mathrm{A})}{2(1.00\,\mathrm{m})} = 1.26 \times 10^{-4}\,\mathrm{T}\,;$$

$$B|_{t=0.500\,\mathrm{s}} = \frac{\mu_0(i_0 + kt)}{2R} = \frac{(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A})[200\,\mathrm{A} - (400\,\mathrm{A/s})(0.500\,\mathrm{s})]}{2(1.00\,\mathrm{m})} = 0\,;$$

and

$$\begin{aligned} B|_{t=1.00\,\mathrm{s}} &= \frac{\mu_0(i_0 + kt)}{2R} = \frac{(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A})[200\,\mathrm{A} - (400\,\mathrm{A/s})(1.00\,\mathrm{s})]}{2(1.00\,\mathrm{m})} \\ &= -1.26 \times 10^{-4}\,\mathrm{T}\,. \end{aligned}$$

(b) Let the area of the small loop be a. Then $\Phi_B = Ba$, and

$$\begin{aligned} \mathcal{E}(t) &= \frac{d\Phi_B}{dt} = -\frac{d(Ba)}{dt} = -a\frac{dB}{dt} = -a\left(\frac{\Delta B}{\Delta t}\right) \\ &= -(2.00 \times 10^{-4}\,\mathrm{m}^2)\left(\frac{-1.26 \times 10^{-4}\,\mathrm{T} - 1.26 \times 10^{-4}\,\mathrm{T}}{1.00\,\mathrm{s}}\right) = 5.04 \times 10^{-8}\,\mathrm{V}\,. \end{aligned}$$

18P

(a) In the region of the smaller loop the magnetic field produced by the larger loop may be taken to be uniform and equal to its value at the center of the smaller loop, on the axis. Eq. 30–29, with $z = x$ and much greater than R, gives

$$B = \frac{\mu_0 i R^2}{2x^3}$$

for the magnitude. The field is upward in the diagram. The magnetic flux through the smaller loop is the product of this field and the area (πr^2) of the smaller loop:

$$\Phi_B = \frac{\pi \mu_0 i r^2 R^2}{2x^3}\,.$$

(b) The emf is given by Faraday's law:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\left(\frac{\pi\mu_0 i r^2 R^2}{2}\right)\frac{d}{dt}\left(\frac{1}{x^3}\right) = -\left(\frac{\pi\mu_0 i r^2 R^2}{2}\right)\left(-\frac{3}{x^4}\frac{dx}{dt}\right) = \frac{3\pi\mu_0 i r^2 R^2 v}{2x^4}.$$

(c) The field of the larger loop is upward and decreases with distance away from the loop. As the smaller loop moves away the flux through it decreases. The induced current will be directed so as to produce a magnetic field that is upward through the smaller loop, in the same direction as the field of the larger loop. It will be counterclockwise as viewed from above, in the same direction as the current in the larger loop.

19P

(a) The emf induced around the loop is given by Faraday's law: $\mathcal{E} = -d\Phi_B/dt$ and the current in the loop is given by $i = \mathcal{E}/R = -(1/R)(d\Phi_B/dt)$. The charge that passes through the resistor from time zero to time t is given by the integral

$$q = \int_0^t i\,dt = -\frac{1}{R}\int_0^t \frac{d\Phi_B}{dt}\,dt = -\frac{1}{R}\int_{\Phi_B(0)}^{\Phi_B(t)} d\Phi_B = \frac{1}{R}\left[\Phi_B(0) - \Phi_B(t)\right].$$

All that matters is the change in the flux, not how it was changed.

(b) If $\Phi_B(t) = \Phi_B(0)$ then $q = 0$. This does not mean that the current was zero for any extended time during the interval. If Φ_B increases and then decreases back to its original value there is current in the resistor while Φ_B is changing. It is in one direction at first, then in the opposite direction. When equal charge has passed through the resistor in opposite directions the net charge is zero.

20P

From the result of the last problem

$$\begin{aligned} q &= \frac{1}{R}[\Phi_B(0) - \Phi_B(t)] = \frac{A}{R}[B(0) - B(t)] \\ &= \frac{1.20\times10^{-3}\,\mathrm{m}^2}{13.0\,\Omega}[1.60\,\mathrm{T} - (-1.60\,\mathrm{T})] = 2.95\times10^{-2}\,\mathrm{C}. \end{aligned}$$

21P

$$\begin{aligned} q(t) &= \frac{N}{R}\Delta\Phi_B = \frac{N}{R}[BA\cos 70^\circ - (-BA\cos 70^\circ)] = \frac{2WBA\cos 30^\circ}{R} \\ &= \frac{2(1000)(0.590\times10^{-4}\,\mathrm{T})\pi(0.100\,\mathrm{m})^2(\cos 30^\circ)}{85.0\,\Omega + 140\,\Omega} = 1.55\times10^{-5}\,\mathrm{C}. \end{aligned}$$

Note that the axis of the coil is at 30°, not 70°, from the magnetic field of the Earth.

22P

(a) Let L be the length of a side of the square circuit. Then the magnetic flux through the circuit is $\Phi_B = L^2B/2$ and the induced emf has magnitude

$$\mathcal{E}_i = -\frac{\mathrm{d}\Phi_B}{dt} = -\frac{L^2}{2}\frac{dB}{dt}\,.$$

Now $B = 0.042 - 0.870t$ and $dB/dt = -0.870\,\mathrm{T/s}$. Thus

$$\mathcal{E}_i = \frac{(2.00\,\mathrm{m})^2}{2}(0.870\,\mathrm{T/s}) = 1.74\,\mathrm{V}\,.$$

The magnetic field is out of the page and decreasing so the induced emf is counterclockwise around the circuit, in the same direction as the emf of the battery. The total emf is $\mathcal{E} + \mathcal{E}_i = 20.0\,\mathrm{V} + 1.74\,\mathrm{V} = 21.7\,\mathrm{V}$.

(b) The current is in the sense of the total emf, counterclockwise.

23P

(a) The magnetic flux Φ_B through the loop is

$$\Phi = \int \mathbf{B}\cdot d\mathbf{A} = \frac{1}{4}\pi r^2 B\,.$$

Thus

$$\begin{aligned}|\mathcal{E}| &= \left|\frac{d\Phi_B}{dt}\right| = \left|\frac{d}{dt}\left(\frac{1}{4}\pi r^2 B\right)\right| = \frac{\pi r^2}{4}\left|\frac{dB}{dt}\right| \\ &= \frac{1}{4}\pi(0.10\,\mathrm{m})^2(3.0\times10^{-3}\,\mathrm{T/s}) = 2.4\times10^{-5}\,\mathrm{V}\,.\end{aligned}$$

(b) From c to b (following Lenz's law).

24P

(a) and (b) Let $A = \pi r^2$. The emf as a function of time is given by

$$\begin{aligned}\mathcal{E} &= -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(\mathbf{B}\cdot\mathbf{A}) = -BA\frac{d}{dt}\cos(\mathbf{B},\mathbf{A}) = -BA\frac{d}{dt}\cos(2\pi ft+\phi_0) \\ &= -92\pi f)BA\sin(2\pi ft+\phi_0)\,.\end{aligned}$$

Thus the frequency of $\mathcal{E}$ is f and the amplitude is $\mathcal{E}_\mathrm{m} = 2\pi fBA = (2\pi f)B(\pi r^2/2) = \pi^2r^2Bf$.

25P

(a) The area of the coil is $A = ab$. Suppose that at some instant of time the normal to the loop makes the angle θ with the magnetic field. The magnetic flux through the loop is then $\Phi_B = NabB\cos\theta$ and the emf induced around the coil is

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(NabB\cos\theta) = (NabB\sin\theta)\frac{d\theta}{dt}.$$

In terms of the frequency of rotation f and the time t, θ is given by $\theta = 2\pi ft$ and $d\theta/dt = 2\pi f$. The emf is therefore

$$\mathcal{E} = 2\pi fNabB\sin(2\pi ft).$$

This can be written $\mathcal{E} = \mathcal{E}_0\sin(2\pi ft)$, where $\mathcal{E}_0 = 2\pi fNabB$.

(b) You want $2\pi fNabB = 150\,\mathrm{V}$. This means

$$Nab = \frac{\mathcal{E}_0}{2\pi fB} = \frac{150\,\mathrm{V}}{2\pi(60.0\,\mathrm{rev/s})(0.500\,\mathrm{T})} = 0.796\,\mathrm{m}^2.$$

Any loop for which $Nab = 0.796\,\mathrm{m}^2$ will do the job. An example is $N = 100$ turns, $a = b = 8.92\,\mathrm{cm}$.

26P

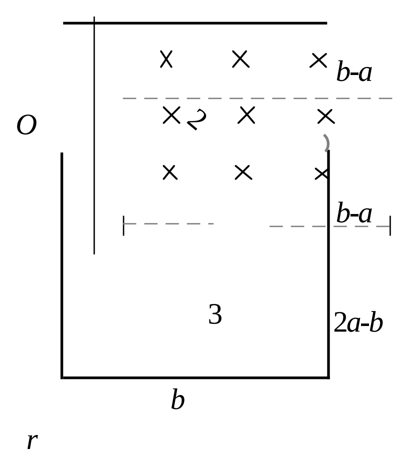
Use the result obtained in 24P, part(b):

$$\mathcal{E}_\mathrm{m} = (2\pi fBA)N = 2\pi(1000/60\,\mathrm{s})(3.50\,\mathrm{T})(0.500\,\mathrm{m})(0.300\,\mathrm{m})(100) = 5.50\times10^3\,\mathrm{V}.$$

27P

(a) It is clear that the magnetic flux through areas 1 and 2 as shown cancel out. Thus

$$\begin{aligned}\Phi_B = \Phi_{B3} &= \int_{b-a}^{a} B(r)(bdr)\\ &= \int_{b-a}^{a}\left(\frac{\mu_0 i}{2\pi r}\right)bdr\\ &= \frac{\mu_0 ib}{2\pi}\ln\left(\frac{a}{b-a}\right),\end{aligned}$$



and

$$\begin{aligned}
\mathcal{E} &= -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 ib}{2\pi}\ln\left(\frac{a}{b-a}\right)\right] = -\frac{\mu_0 b}{2\pi}\ln\left(\frac{a}{b-a}\right)\frac{di}{dt} \\
&= -\frac{\mu_0 b}{2\pi}\ln\left(\frac{a}{b-a}\right)\frac{d}{dt}(4.50t^2 - 10.0t) = \frac{-4.50\mu_0 bt}{\pi}\ln\left(\frac{a}{b-a}\right) \\
&= -\frac{(4.50)(1.26\times 10^{-6}\,\mathrm{T\cdot m/A})(0.160\,\mathrm{m})(3.00\,\mathrm{s})}{\pi}\ln\left(\frac{12.0\,\mathrm{cm}}{16.0\,\mathrm{cm}-12.0\,\mathrm{cm}}\right) \\
&= -5.98\times 10^{-7}\,\mathrm{V}\,.
\end{aligned}$$

(b) From Lenz's law, the induced current in the loop is counterclockwise.

28P

Use Faraday's law to find an expression for the emf induced by the changing magnetic field. First find an expression for the magnetic flux through the loop. Since the field depends on y but not on x, divide the area into strips of length L and width dy, parallel to the x axis. Here L is the length of one side of the square. At time t the flux through a strip with coordinate y is $d\Phi_B = BL\,dy = 4.0Lt^2y\,dy$ and the total flux through the square is

$$\Phi_B = \int_0^L 4.0Lt^2y\,dy = 2.0L^3t^2\,.$$

According to Faraday's law the magnitude of the emf around the square is

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d}{dt}\left(2.0L^3t^2\right) = 4.0L^3t\,.$$

At $t = 2.5\,\mathrm{s}$ this is $4.0(0.020\,\mathrm{m})^3(2.5\,\mathrm{s}) = 8.0\times 10^{-5}\,\mathrm{V}$.

The externally-produced magnetic field is out of the page and is increasing with time. The induced current produces a field that is into the page, so it must be clockwise. The induced emf is also clockwise.

29P

(a) Analogous to 27P, part (a), we have

$$\begin{aligned}
\Phi_B &= \int_{\text{loop}} B(r)dA = \int_{r-\frac{b}{2}}^{r+\frac{b}{2}} B(r)a\,dr \\
&= \int_{r-\frac{b}{2}}^{r+\frac{b}{2}} \frac{\mu_0 ia}{2\pi r}dr = \frac{\mu_0 ia}{2\pi}\ln\left(\frac{2r+b}{2r-b}\right).
\end{aligned}$$

(b) Now

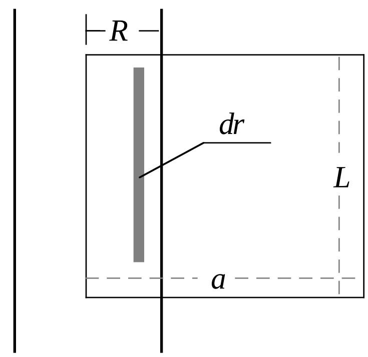
$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\left(\frac{\partial \Phi_B}{\partial r}\right)\left(\frac{dr}{dt}\right) = -v\frac{\partial}{\partial r}\left[\frac{\mu_0 i a}{2\pi}\ln\left(\frac{2r+b}{2r-b}\right)\right]$$
$$= -\frac{\mu_0 i a w}{\pi}\left(\frac{1}{2r+b} - \frac{1}{2r-b}\right) = \frac{2\mu_0 i a b v}{\pi(4r^2 - b^2)}\,.$$

Thus

$$i = \frac{\mathcal{E}}{R} = \frac{2\mu_0 i a b v}{\pi R(4r^2 - b^2)}\,.$$

30P*

(a) Suppose each wire has radius R and the distance between their axes is a. Consider a single wire and calculate the flux through a rectangular area with the axis of the wire along one side. Take this side to have length L and the other dimension of the rectangle to be a. The magnetic field is everywhere perpendicular to the rectangle. First consider the part of the rectangle that is inside the wire. The field a distance r from the axis is given by $B = \mu_0 i r/2\pi R^2$ and the flux through the strip of length L and width dr at that distance is $(\mu_0 i r/2\pi R^2)L\,dr$. Thus the flux through the area inside the wire is



$$\Phi_{\text{in}} = \int_0^R \frac{\mu_0 i L}{2\pi R^2}\, r\, dr = \frac{\mu_0 i L}{4\pi}\,.$$

Now consider the region outside the wire. There the field is given by $B = \mu_0 i/2\pi r$ and the flux through an infinitesimally thin strip is $(\mu_0 i/2\pi r)L\,dr$. The flux through the whole region is

$$\Phi_{\text{out}} = \int_R^a \frac{\mu_0 i L}{2\pi}\frac{dr}{r} = \frac{\mu_0 i L}{2\pi}\ln\left(\frac{a}{R}\right)\,.$$

The total flux through the area bounded by the dotted lines is the sum of the two contributions:

$$\Phi = \frac{\mu_0 i L}{4\pi}\left[1 + 2\ln\left(\frac{a}{R}\right)\right]\,.$$

Now include the contribution of the other wire. Since the currents are in the same direction the two contributions have the same sign. They also have the same magnitude, so

$$\Phi_{\text{total}} = \frac{\mu_0 i L}{2\pi}\left[1 + 2\ln\left(\frac{a}{R}\right)\right]\,.$$

The total flux per unit length is

$$\frac{\Phi_{\text{total}}}{L} = \frac{\mu_0 i}{2\pi}\left[1 + 2\ln\left(\frac{a}{R}\right)\right] = \frac{(4\pi \times 10^{-7}\,\text{T·m/A})(10\,\text{A})}{2\pi}\left[1 + 2\ln\left(\frac{20\,\text{mm}}{1.25\,\text{mm}}\right)\right]$$
$$= 1.3 \times 10^{-5}\,\text{Wb/m}\,.$$

(b) Again consider the flux of a single wire. The flux inside the wire itself is again $\Phi_{\text{in}} = \mu_0 iL/4\pi$. The flux inside the region of the other wire is

$$\Phi_{\text{out}} = \int_{a-R}^{a} \frac{\mu_0 iL}{2\pi}\frac{dr}{r} = \frac{\mu_0 iL}{2\pi}\ln\left(\frac{a}{a-R}\right)\,.$$

Double this to include the flux of the other wire (inside the first wire) and divide by L to obtain the flux per unit length. The total flux per unit length that is inside the wires is

$$\frac{\Phi_{\text{wires}}}{L} = \frac{\mu_0 i}{2\pi}\left[1 + 2\ln\left(\frac{a}{a-R}\right)\right]$$
$$= \frac{(4\pi \times 10^{-7}\,\text{T·m/A})(10\,\text{A})}{2\pi}\left[1 + 2\ln\left(\frac{20\,\text{mm}}{20\,\text{mm} - 1.25\,\text{mm}}\right)\right]$$
$$= 2.26 \times 10^{-6}\,\text{Wb/m}\,.$$

The fraction of the total flux that is inside the wires is

$$\frac{2.26 \times 10^{-6}\,\text{Wb/m}}{1.31 \times 10^{-5}\,\text{Wb/m}} = 0.17\,.$$

(c) The contributions of the two wires to the total flux have the same magnitudes but now the currents are in opposite directions, so the contributions have opposite signs. This means $\Phi_{\text{total}} = 0$.

31E

$$U_{\text{thermal}} = P_{\text{thermal}}\Delta t = \frac{\mathcal{E}^2\Delta t}{R} = \frac{1}{R}\left(-\frac{d\Phi_B}{dt}\right)^2\Delta t = \frac{1}{R}\left(-A\frac{\Delta B}{\Delta t}\right)^2\Delta t = \frac{A^2B^2}{R\Delta t}\,.$$

32E

Thermal energy is generated at the rate $\mathcal{E}^2/R$, where $\mathcal{E}$ is the emf in the wire and R is the resistance of the wire. The resistance is given by $R = \rho L/A$, where ρ is the resistivity of copper, L is the length of the wire, and A is the cross-sectional area of the wire. The resistivity can be found in Table 27–1. Thus

$$R = \frac{(1.69 \times 10^{-8}\,\Omega\text{·m})(0.500\,\text{m})}{\pi(0.500 \times 10^{-3}\,\text{m})^2} = 1.076 \times 10^{-2}\,\Omega\,.$$

Faraday's law is used to find the emf. If B is the magnitude of the magnetic field through the loop, then $\mathcal{E} = A\,dB/dt$, where A is the area of the loop. The radius r of the loop is $r = L/2\pi$ and its area is $\pi r^2 = \pi L^2/4\pi^2 = L^2/4\pi$. Thus

$$\mathcal{E} = \frac{L^2}{4\pi}\frac{dB}{dt} = \frac{(0.500\,\text{m})^2}{4\pi}(10.0 \times 10^{-3}\,\text{T/s}) = 1.989 \times 10^{-4}\,\text{V}\,.$$

The rate of thermal energy generation is

$$P = \frac{\mathcal{E}^2}{R} = \frac{(1.989 \times 10^{-4}\,\text{V})^2}{1.076 \times 10^{-2}\,\Omega} = 3.68 \times 10^{-6}\,\text{W}\,.$$

33E

(a) The flux changes because the area bounded by the rod and rails increases as the rod moves. Suppose that at some instant the rod is a distance x from the right-hand end of the rails and has speed v. Then the flux through the area is $\Phi_B = BA = BLx$, where L is the distance between the rails. According to Faraday's law the magnitude of the emf induced is $\mathcal{E} = d\Phi_B/dt = BL\,(dx/dt) = BLv = (0.350\,\text{T})(0.250\,\text{m})(0.550\,\text{m/s}) = 4.81 \times 10^{-2}\,\text{V}$.

(b) Use Ohm's law. If R is the resistance of the rod then the current in the rod is $i = \mathcal{E}/R = (4.81 \times 10^{-2}\,\text{V})/(18.0\,\Omega) = 2.67 \times 10^{-3}\,\text{A}$.

(c) The rate at which thermal energy is generated is

$$P = i^2R = (2.67 \times 10^{-3}\,\text{A})^2(18.0\,\Omega) = 1.28 \times 10^{-4}\,\text{W}\,.$$

34E

(a) Let x be the distance from the right end of the rails to the rod. The area enclosed by the rod and rails is Lx and the magnetic flux through the area is $\Phi_B = BLx$. The emf induced is $\mathcal{E} = d\Phi_B/dt = BL\,dx/dt = BLv$, where v is the speed of the rod. Thus $\mathcal{E} = (1.2\,\text{T})(0.10\,\text{m})(5.0\,\text{m/s}) = 0.60\,\text{V}$.

(b) If R is the resistance of the rod, the current in the loop is $i = \mathcal{E}/R = (0.60\,\text{V})/(0.40\,\Omega) = 1.5\,\text{A}$. Since the rod moves to the left in the diagram, the flux increases. The induced current must produce a magnetic field that is into the page in the region bounded by the rod and rails. To do this the current must be clockwise.

(c) The rate of generation of thermal energy by the resistance of the rod is $P = \mathcal{E}^2/R = (0.60\,\text{V})^2/(0.40\,\Omega) = 0.90\,\text{W}$.

(d) Since the rod moves with constant velocity the net force on it must be zero. This means the force of the external agent has the same magnitude as the magnetic force but is in the opposite direction. The magnitude of the magnetic force is $F_B = iLB = (1.5\,\text{A})(0.10\,\text{m})(1.2\,\text{T}) = 0.18\,\text{N}$. Since the field is out of the page and the current is upward through the rod the magnetic force is to the right. The force of the external agent must be $0.18\,\text{N}$, to the left.

(e) As the rod moves an infinitesimal distance dx the external agent does work $dW = F\,dx$, where F is the force of the agent. The force is in the direction of motion, so the work done by the agent is positive. The rate at which the agent does work is $dW/dt = F\,dx/dt = Fv = (0.18\,\text{N})(5.0\,\text{m/s}) = 0.90\,\text{W}$, the same as the rate at which thermal energy is generated. The energy supplied by the external agent is converted completely to thermal energy.

35P

Let $F_{\text{net}} = BiL - mg = 0$ and solve for i:

$$i = \frac{mg}{BL} = \frac{|\mathcal{E}|}{R} = \frac{1}{R}\left|\frac{d\Phi_B}{dt}\right| = \frac{B}{R}\left|\frac{dA}{dt}\right| = \frac{B(v_t L)}{R}\,,$$

so

$$v_t = \frac{mgR}{B^2L^2}\,.$$

36P

(a) At time t the area of the closed triangular loop is $A(t) = \frac{1}{2}(vt)(2vt) = v^2t^2$. Thus

$$\Phi_B|_{t=3.00\,\text{s}} = BA(t) = (0.350\,\text{T})[(5.20\,\text{m/s})(3.00\,\text{s})]^2 = 85.2\,\text{T}\cdot\text{m}^2\,.$$

(b)

$$\begin{aligned}\mathcal{E} &= -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -B\frac{dA}{dt} = -B\frac{d(v^2t^2)}{dt} = -2Bv^2t \\ &= -2(0.350\,\text{T})(5.20\,\text{m/s})^2(3.00\,\text{s}) = -56.8\,\text{V}\,.\end{aligned}$$

(c) From part (b) above we see that $\mathcal{E} = -2Bv^2t \propto t^1$. Thus $n = 1$.

37P

From

$$[\sin^2(2\pi ft)]_{\text{av}} = \frac{1}{T}\int_0^T \sin^2(2\pi ft)dt = \frac{1}{2}$$

we obtain

$$P_{\text{av}} = \frac{\mathcal{E}_{\text{av}}^2}{R} = \frac{\mathcal{E}_0^2[\sin^2(2\pi ft)]_{\text{av}}}{R} = \frac{\mathcal{E}_0^2}{2R} = \frac{(150\,\text{V})^2}{2(42.0\,\Omega)} = 268\,\text{W}\,.$$

38P

(a) Apply Newton's second law to the rod:

$$m\frac{dv}{dt} = iBL\,.$$

Integrate to obtain

$$v = \frac{iBLt}{m}\,.$$

$\mathbf{v}$ points away from the generator G.
(b) When the current i in the rod becomes zero, the rod will no longer be accelerated by a force $F = iBL$ and will therefore reach a constant terminal velocity. This happens when $|\mathcal{E}_{\text{induced}}| = \mathcal{E}$, i.e.,

$$|\mathcal{E}_{\text{induced}}| = \left|\frac{d\Phi_B}{dt}\right| = \left|\frac{d(BA)}{dt}\right| = B\left|\frac{dA}{dt}\right| = BvL = \mathcal{E}\,,$$

or $v = \mathcal{E}/BL$, to the left.
(c) In case (a) above electric energy is supplied by the generator and is transferred into the kinetic energy of the rod. In the current case the battery initially supplies electric energy to the rod, causing its kinetic energy to increase to a maximum value of $\frac{1}{2}mv^2 = \frac{1}{2}(\mathcal{E}/BL)^2$. Afterwards, there is no further energy transfer from the battery to the rod, and the kinetic energy of the rod remains constant.

39P

(a) Let x be the distance from the right end of the rails to the rod and find an expression for the magnetic flux through the area enclosed by the rod and rails. The magnetic field is not uniform but varies with distance from the long straight wire. The field is normal to the area and has magnitude $B = \mu_0 i/2\pi r$, where r is the distance from the wire and i is the current in the wire. Consider an infinitesimal strip of length x and width dr, parallel to the wire and a distance r from it. The area of this strip is $A = x\,dr$ and the flux through it is $d\Phi_B = (\mu_0 ix/2\pi r)dr$. The total flux through the area enclosed by the rod and rails is

$$\Phi_B = \frac{\mu_0 ix}{2\pi}\int_a^{a+L}\frac{dr}{r} = \frac{\mu_0 ix}{2\pi}\ln\left(\frac{a+L}{a}\right)\,.$$

According to Faraday's law the emf induced in the loop is

$$\begin{aligned}\mathcal{E} &= \frac{d\Phi}{dt} = \frac{\mu_0 i}{2\pi}\frac{dx}{dt}\ln\left(\frac{a+L}{a}\right) = \frac{\mu_0 iv}{2\pi}\ln\left(\frac{a+L}{a}\right)\\ &= \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(100\,\mathrm{A})(5.00\,\mathrm{m/s})}{2\pi}\ln\left(\frac{1.00\,\mathrm{cm}+10.0\,\mathrm{cm}}{1.00\,\mathrm{cm}}\right)\\ &= 2.40\times10^{-4}\,\mathrm{V}\,.\end{aligned}$$

(b) If R is the resistance of the rod then the current in the conducting loop is $i_\ell = \mathcal{E}/R = (2.40\times10^{-4}\,\mathrm{V})/(0.400\,\Omega) = 6.00\times10^{-4}\,\mathrm{A}$. Since the flux is increasing the magnetic field produced by the induced current must be into the page in the region enclosed by the rod and rails. This means the current is clockwise.

(c) Thermal energy is being generated at the rate $P = i_\ell^2 R = (6.00\times10^{-4}\,\mathrm{A})^2(0.400\,\Omega) = 1.44\times10^{-7}\,\mathrm{W}$.

(d) Since the rod moves with constant velocity the net force on it is zero. The force of the external agent must have the same magnitude as the magnetic force and must be in the opposite direction. The magnitude of the magnetic force on an infinitesimal segment of the rod, with length dr and a distance r from the long straight wire, is $dF_B = i_\ell B\,dr = (\mu_0 i_\ell i/2\pi r)\,dr$. The total magnetic force on the rod has magnitude

$$\begin{aligned} F_B &= \frac{\mu_0 i_\ell i}{2\pi}\int_a^{a+L}\frac{dr}{r} = \frac{\mu_0 i_\ell i}{2\pi}\ln\left(\frac{a+L}{a}\right) \\ &= \frac{(4\pi\times 10^{-7}\,\mathrm{T\cdot m/A})(6.00\times 10^{-4}\,\mathrm{A})(100\,\mathrm{A})}{2\pi}\ln\left(\frac{1.00\,\mathrm{cm}+10.0\,\mathrm{cm}}{1.00\,\mathrm{cm}}\right) \\ &= 2.87\times 10^{-8}\,\mathrm{N}\,. \end{aligned}$$

Since the field is out of the page and the currentin the rod is upward in the diagram, this force is toward the right. The external agent must apply a force of 2.87×10^{-8} N, to the left.

(e) The external agent does work at the rate $P = Fv = (2.87\times 10^{-8}\,\mathrm{N})(5.00\,\mathrm{m/s}) = 1.44\times 10^{-7}\,\mathrm{W}$. This is the same as the rate at which thermal energy is generated in the rod. All the energy supplied by the agent is converted to thermal energy.

40E

(a) The field point is inside the solenoid, so Eq. 31-27 applies. The magnitude of the induced electric field is

$$E = \frac{1}{2}\frac{dB}{dt}r = \frac{1}{2}(6.5\times 10^{-3}\,\mathrm{T/s})(0.0220\,\mathrm{m}) = 7.15\times 10^{-5}\,\mathrm{V/m}\,.$$

(b) Now the field point is outside the solenoid and Eq. 31–29 applies. The magnitude ofthe induced field is

$$E = \frac{1}{2}\frac{dB}{dt}\frac{R^2}{r} = \frac{1}{2}(6.5\times 10^{-3}\,\mathrm{T/s})\frac{(0.0600\,\mathrm{m})^2}{(0.0820\,\mathrm{m})} = 1.43\times 10^{-4}\,\mathrm{V/m}\,.$$

41E

$$\begin{aligned} \oint_1 \mathbf{E}\cdot d\mathbf{s} &= -\frac{d\Phi_{B1}}{dt} = \frac{d}{dt}(B_1A_1) = A_1\frac{dB_1}{dt} = \pi r_1^2\frac{dB_1}{dt} \\ &= \pi(0.200\,\mathrm{m})^2(-8.50\times 10^{-3}\,\mathrm{T/s}) = -1.07\times 10^{-3}\,\mathrm{V}\,, \end{aligned}$$

$$\begin{aligned} \oint_2 \mathbf{E}\cdot d\mathbf{s} &= -\frac{d\Phi_{B2}}{dt} = \pi r_2^2\frac{dB_2}{dt} \\ &= \pi(0.300\,\mathrm{m})^2(-8.50\times 10^{-3}\,\mathrm{T/s}) = -2.40\times 10^{-3}\,\mathrm{V}\,, \end{aligned}$$

and

$$\oint_3 \mathbf{E}\cdot d\mathbf{s} = \oint_1 \mathbf{E}\cdot d\mathbf{s} - \oint_2 \mathbf{E}\cdot d\mathbf{s} = -1.07 \times 10^{-3}\,\mathrm{V} - (-2.4 \times 10^{-3}\,\mathrm{V}) = 1.33 \times 10^{-3}\,\mathrm{V}\,.$$

42P

The magnetic field B can be expressed as

$$B(t) = B_0 + B_1 \sin(\omega t + \phi_0)\,,$$

where $B_0 = (30.0\,\mathrm{T} + 29.6\,\mathrm{T})/2 = 29.8\,\mathrm{T}$ and $B_1 = (30.0\,\mathrm{T} - 29.6\,\mathrm{T})/2 = 0.200\,\mathrm{T}$. Then from Eq. 31-27

$$E = \frac{1}{2}\left(\frac{dB}{dt}\right) r = \frac{r}{2}\frac{d}{dt}[B_0 + B_1 \sin(\omega t + \phi_0)] = \frac{1}{2} B_1 \omega r \cos(\omega t + \phi_0)\,.$$

Thus

$$E_{\max} = \frac{1}{2} B_1 (2\pi f) r = \frac{1}{2}(0.200\,\mathrm{T})(2\pi)(15\,\mathrm{Hz})(1.6 \times 10^{-2}\,\mathrm{m} = 0.15\,\mathrm{V/m}\,.$$

43P

The induced electric field E as a function of r is given by $E(r) = (r/2)(dB/dt)$. So

$$\begin{aligned} a_c = a_a &= \frac{eE}{m} = \frac{er}{2m}\left(\frac{dB}{dt}\right) \\ &= \frac{(1.60 \times 10^{-19}\,\mathrm{C})(5.0 \times 10^{-2}\,\mathrm{m})(10 \times 10^{-3}\,\mathrm{T/s})}{2(9.11 \times 10^{-27}\,\mathrm{kg})} = 4.4 \times 10^{7}\,\mathrm{m/s^2}\,. \end{aligned}$$

$\mathbf{a}_a$ points to the right and $\mathbf{a}_c$ points to the left. At point b we have $a_b \propto r_b = 0$.

44P

Use Faraday's law in the form $\oint \mathbf{E}\cdot d\mathbf{s} = -(d\Phi_B/dt)$. Integrate around the dotted path shown in Fig. 31–61. At all points on the upper and lower sides the electric field is either perpendicular to the side or else it vanishes. Assume it vanishes at all points on the right side (outside the capacitor). On the left side it is parallel to the side and has constant magnitude. Thus direct integration yields $\oint \mathbf{E}\cdot d\mathbf{s} = EL$, where L is the length of the left side of the rectangle. The magnetic field is zero and remains zero, so $d\Phi_B/dt = 0$. Faraday's law leads to a contradiction: $EL = 0$, but neither E nor L is zero. There must be an electric field along the right side of the rectangle.

45E

Since $N\Phi_B = Li$, where N is the number of turns, L is the inductance, and i is the current,

$$\Phi_B = \frac{Li}{N} = \frac{(8.0 \times 10^{-3}\,\mathrm{H})(5.0 \times 10^{-3}\,\mathrm{A})}{400} = 1.0 \times 10^{-7}\,\mathrm{Wb}\,.$$

46E

(*a*)

$$\Phi_B = NBA = NB(\pi r^2) = (30.0)(2.60 \times 10^{-3}\,\mathrm{T})(\pi)(0.100\,\mathrm{m})^2 = 2.45 \times 10^{-3}\,\mathrm{Wb}\,.$$

(*b*)

$$L = \frac{\Phi_B}{i} = \frac{2.45 \times 10^{-3}\,\mathrm{Wb}}{3.80\,\mathrm{A}} = 6.45 \times 10^{-4}\,\mathrm{H/m}\,.$$

47E

(*a*) $N = 2.0\,\mathrm{m}/2.5\,\mathrm{mm} = 800$.
(*b*) $L/l = \mu_0 n^2 A = (4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A})(800/2.0\,\mathrm{m})^2(\pi)(0.040\,\mathrm{m})^2/4 = 2.5 \times 10^{-4}\,\mathrm{H}$.

48P

If the solenoid is long and thin, then when it is bent into a toroid $(b-a)/a \ll 1$. Thus

$$L_{\text{toroid}} = \frac{\mu_0 N^2 h}{2\pi}\ln\frac{b}{a} = \frac{\mu_0 N^2 h}{2\pi}\ln\left(1 + \frac{b-a}{b}\right) \approx \frac{\mu_0 N^2 h(b-a)}{2\pi b}\,.$$

Since $A = h(b-a)$ is the cross-sectional area and $l = 2\pi b$ is the length of the toroid, we may rewrite the expression for L toroid above as

$$\frac{L_{\text{toroid}}}{l} \approx \frac{\mu_0 N^2 A}{l^2} = \mu_0 n^2 A\,,$$

which indeed reduces to that of a long solenoid. Note that the approximation $\ln(1+x) \approx x$ was used for $|x| \ll 1$.

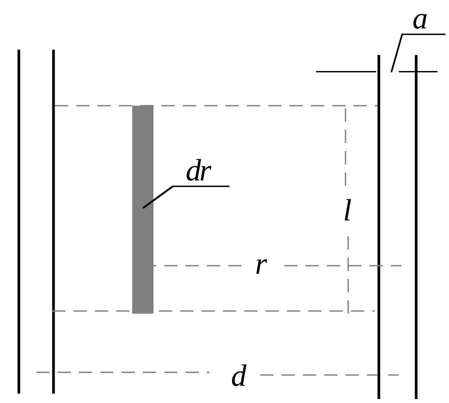
49P

(*a*) Suppose we divide the one-turn solenoid into N small circular loops placed along the width W of the copper strip. Each loop carries a current $\Delta i = i/N$. Then the magnetic field inside the solenoid is $B = \mu_0 n \Delta i = \mu_0(N/W)(i/N) = \mu_0 i/W$.
(*b*)

$$L = \frac{\Phi_B}{i} = \frac{\pi R^2 B}{i} = \frac{\pi R^2(\mu_0 i/W)}{i} = \frac{\pi\mu_0 R^2}{W}\,.$$

50P

The area of integartion for the calculation of the magnetic flux is bounded by the two dotted lines and the boundaries of the wires. If the origin is taken to be on the axis of the right-hand wire and r measures distance from that axis, it extends from $r = a$ to $r = d - a$. Consider the right-hand wire first. In the region of integation the field it produces is into the page and has magnitude $B = \mu_0 i/2\pi r$. Divide the region into strips of length l and width dr, as shown. The flux through the strips a distnace r from the axis of the wire is $d\Phi = Bl\,dr$ adn the flux through the entire region is



$$\Phi = \frac{\mu_0 i l}{2\pi}\int_a^{d-a}\frac{dr}{r} = \frac{\mu_0 i l}{2\pi}\ln\left(\frac{d-a}{a}\right).$$

The other wire produces the same result, so the total flux through the dotted rectangle is

$$\Phi_{\text{total}} = \frac{\mu_0 i l}{\pi}\ln\left(\frac{d-a}{a}\right).$$

The inductance is Φ_{total} diveded by i:

$$L = \frac{\Phi_{\text{total}}}{i} = \frac{\mu_0 l}{\pi}\ln\left(\frac{d-a}{a}\right).$$

51E

(a) Since $\mathcal{E}$ enhances i, i must be decreasing.
(b) From $\mathcal{E} = L\,di/dt$ we get

$$L = \frac{\mathcal{E}}{di/dt} = \frac{17\,\text{V}}{2.5\,\text{kA/s}} = 6.8\times 10^{-4}\,\text{H}.$$

52E

Since $\mathcal{E} = -L(di/dt)$, we may obtain the desired induced emf by setting

$$\frac{di}{dt} = \frac{\mathcal{E}}{L} = \frac{60\,\text{V}}{12\,\text{H}} = 5.0\,\text{A/s}.$$

You might, for example, uniformly reduce the current from 2.0 A to zero in 40 ms.

53E

(a)

$$\frac{L}{l}=\mu_0 n^2 A=(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(100\,\mathrm{turns/cm})^2(\pi)(1.6\,\mathrm{cm})^2=0.10\,\mathrm{H/m}\,.$$

(b) The induced emf per meter is

$$\mathcal{E}=L\frac{di}{dt}=(0.10\,\mathrm{H/m})(13\,\mathrm{A/s})=1.3\,\mathrm{V/m}\,.$$

54E

(a)

$$L=\frac{\mathcal{E}}{di/dt}=\frac{3.0\times10^{-3}\,\mathrm{V}}{5.0\,\mathrm{A/s}}=6.0\times10^{-4}\,\mathrm{H}\,.$$

(b) From $L=N\Phi_B/i$ we get

$$N=\frac{iL}{\Phi_B}=\frac{(8.0\,\mathrm{A})(6.0\times10^{-4}\,\mathrm{H})}{40\times10^{-6}\,\mathrm{Wb}}=120\,.$$

55P

Use $\mathcal{E}=L\,di/dt$.

(a) For $0<t<2\,\mathrm{ms}$

$$\mathcal{E}=L\frac{\Delta i}{\Delta t}=\frac{(4.6\,\mathrm{H})(7.0\,\mathrm{A}-0)}{2.0\times10^{-3}\,\mathrm{s}}=1.6\times10^{4}\,\mathrm{V}\,.$$

(b) For $2\,\mathrm{ms}<t<5\,\mathrm{ms}$

$$\mathcal{E}=L\frac{\Delta i}{\Delta t}=\frac{(4.6\,\mathrm{H})(5.0\,\mathrm{A}-7.0\,\mathrm{A})}{(5.0-2.0)10^{-3}\,\mathrm{s}}=3.1\times10^{3}\,\mathrm{V}\,.$$

(c) For $5\,\mathrm{ms}<t<6\,\mathrm{ms}$

$$\mathcal{E}=L\frac{\Delta i}{\Delta t}=\frac{(4.6\,\mathrm{H})(0-5.0\,\mathrm{A})}{(6.0-5.0)10^{-3}\,\mathrm{s}}=2.3\times10^{4}\,\mathrm{V}\,.$$

56P

(a)

$$L_{\mathrm{eq}}=\frac{\Phi_{\mathrm{total}}}{i}=\frac{\Phi_1+\Phi_2}{i}=\frac{\Phi_1}{i}+\frac{\Phi_2}{i}=L_1+L_2\,.$$

(b) If the separation is not large enough then the magnetic field in one inductor may produce a flux in the other, which would render the expression $\Phi = Li$ invalid.
(c) For N inductors in series

$$L_{\text{eq}} = \frac{\Phi_{\text{total}}}{i} = \frac{1}{i}\sum_{n=1}^{N}\Phi_n = \sum_{n=1}^{N}\frac{\Phi_n}{i} = \sum_{n=1}^{N}L_n\,.$$

57P
(a) In this case $i_{\text{total}} = i_1 + i_2$. But $V_{\text{total}} = \Phi/L_{\text{eq}}$ and $i_{1,2} = \Phi/L_{1,2}$ so $\Phi/L_{\text{eq}} = \Phi/L_1 + \Phi/L_2$, or

$$\frac{1}{L_{\text{eq}}} = \frac{1}{L_1} + \frac{1}{L_2}\,.$$

(b) The reason is the same as in part (b) of the previous problem.
(c) For N inductors in parallel you can easily generize the expression above to

$$\frac{1}{L_{\text{eq}}} = \sum_{n=1}^{N}\frac{1}{L_n}\,.$$

58E
Suppose that $i(t_0) = i_0/3$ at $t = t_0$. Write $i(t_0) = i_0(1 - e^{-t_0/\tau_L})$, which gives

$$\tau_L = -\frac{t_0}{\ln(1 - i/i_0)} = -\frac{5.00\,\text{s}}{\ln(1 - 1/3)} = 12.3\,\text{s}\,.$$

59E
Starting with zero current at time $t = 0$, when the switch is closed, the current in an RL series circuit at a later time t is given by

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right)\,,$$

where τ_L is the inductive time constant, $\mathcal{E}$ is the emf, and R is the resistance. You want to calculate the time t for which $i = 0.9990\mathcal{E}/R$. This means

$$0.9990\frac{\mathcal{E}}{R} = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right)\,,$$

so

$$0.9990 = 1 - e^{-t/\tau_L}$$

or

$$e^{-t/\tau_L} = 0.0010\,.$$

Take the natural logarithm of both sides to obtain $-(t/\tau) = \ln(0.0010) = -6.91$. Thus $t = 6.91\tau_L$. That is, 6.91 inductive time constants must elapse.

60E

The current in the circuit is given by

$$i = i_0 e^{-t/\tau_L}\,,$$

where i_0 is the initial current (at time $t = 0$) and τ_L ($= L/R$) is the inductive time constant. Solve for τ_L. Divide by i_0 and take the natural logarithm of both sides of the resulting equation to obtain

$$\ln\left(\frac{i}{i_0}\right) = -\frac{t}{\tau_L}\,.$$

This yields

$$\tau_L = -\frac{t}{\ln[i/i_0]} = -\frac{1.0\,\mathrm{s}}{\ln[(10\times10^{-3}\,\mathrm{A})/(1.0\,\mathrm{A})]} = 0.217\,\mathrm{s}\,.$$

Thus $R = L/\tau_L = (10\,\mathrm{H})/(0.217\,\mathrm{s}) = 46\,\Omega$.

61E

Write $i = i_0 e^{-t/\tau_L}$ and note that $i = 10\%\, i_0$. Solve for t:

$$t = \tau_L \ln\frac{i_0}{i} = \frac{L}{R}\ln\frac{i_0}{i} = \frac{2.00\,\mathrm{H}}{3.00\,\Omega}\ln\left(\frac{i_0}{10.0\%\, i_0}\right) = 1.54\,\mathrm{s}\,.$$

62E

(a) Immediately after the switch is closed $\mathcal{E} - \mathcal{E}_L = iR$. But $i = 0$ at this instant so $\mathcal{E}_L = \mathcal{E}$.
(b) $\mathcal{E}_L(t) = \mathcal{E}e^{-t/\tau_L} = \mathcal{E}e^{-2.0\tau_L/\tau_L} = \mathcal{E}e^{-2.0} = 0.135\mathcal{E}$.
(c) Solve $\mathcal{E}_L(t) = \mathcal{E}e^{-t/\tau_L}$ for t/τ_L: $t/\tau_L = \ln(\mathcal{E}/\mathcal{E}_L) = \ln 2$. So $t = \tau_L \ln 2 = 0.693\tau_L$.

63E

(a) If the battery is switched into the circuit at time $t = 0$, then the current at a later time t is given by

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right)\,,$$

where $\tau_L = L/R$. You want to find the time for which $i = 0.800\mathcal{E}/R$. This means

$$0.800 = 1 - e^{-t/\tau_L}$$

or

$$e^{-t/\tau_L} = 0.200 \,.$$

Take the natural logarithm of both sides to obtain $-(t/\tau_L) = \ln(0.200) = -1.609$. Thus

$$t = 1.609\tau_L = \frac{1.609L}{R} = \frac{1.609(6.30 \times 10^{-6}\,\mathrm{H})}{1.20 \times 10^3\,\Omega} = 8.45 \times 10^{-9}\,\mathrm{s}\,.$$

(b) At $t = 1.0\tau_L$ the current in the circuit is

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-1.0}\right) = \left(\frac{14.0\,\mathrm{V}}{1.20 \times 10^3\,\Omega}\right)\left(1 - e^{-1.0}\right) = 7.37 \times 10^{-3}\,\mathrm{A}\,.$$

64E

(a) $L = \Phi/i = 26 \times 10^{-3}\,\mathrm{Wb}/5.5\,\mathrm{A} = 4.7 \times 10^{-3}\,\mathrm{H}$.
(b) Use Eq. 31-45 to solve for t:

$$\begin{aligned} t &= -\tau_L \ln\left(1 - \frac{iR}{\mathcal{E}}\right) = -\frac{L}{R}\ln\left(1 - \frac{iR}{\mathcal{E}}\right) \\ &= -\frac{4.7 \times 10^{-3}\,\mathrm{H}}{0.75\,\Omega}\ln\left[1 - \frac{(2.5\,\mathrm{A})(0.75\,\Omega)}{6.0\,\mathrm{V}}\right] = 2.4 \times 10^{-3}\,\mathrm{s}\,. \end{aligned}$$

65P

Apply the loop theorem $\mathcal{E}(t) - L(di/dt) = iR$ and solve for $\mathcal{E}(t)$:

$$\begin{aligned} \mathcal{E}(t) &= L\frac{di}{dt} + iR = L\frac{d}{dt}(3.0 + 5.0t) + (3.0 + 5.0t)R \\ &= (6.0\,\mathrm{H})(5.0\,\mathrm{A/s}) + (3.0\,\mathrm{A} + 5.0t)(4.0\,\Omega) \\ &= (42 + 20t)\,\mathrm{V}\,, \end{aligned}$$

where t is in seconds.

66P

(a) and (b) Write $V_L(t) = \mathcal{E}e^{-t/\tau_l}$. Consider the first two data points, (V_{L1}, t_1) and (V_{L2}, t_2), satisfying $V_{Li} = \mathcal{E}e^{-t_i/\tau_L}$ $(i = 1, 2)$. We have $V_{L1}/V_{L2} = \mathcal{E}e^{-(t_1 - t_2)/\tau_L}$, which gives

$$\tau_L = \frac{t_1 - t_2}{\ln(V_2/V_1)} = \frac{1.0\,\mathrm{ms} - 2.0\,\mathrm{ms}}{\ln(13.8/18.2)} = 3.6\,\mathrm{ms}\,.$$

So $\mathcal{E} = V_{L1}e^{t_1/\tau_L} = (18.2\,\mathrm{V})e^{1.0\,\mathrm{ms}/3.6\,\mathrm{ms}} = 24\,\mathrm{V}$. You can easily check that the values of τ_L and $\mathcal{E}$ are consistent with the rest of the date points.

67P

Take the time derivative of both sides of Eq. 31-45:

$$\begin{aligned}\frac{di}{dt} &= \frac{d}{dt}\left[\frac{\mathcal{E}}{R}\left(1-e^{-Rt/\tau}\right)\right] = \frac{\mathcal{E}}{L}e^{-RT/L} \\ &= \left(\frac{45.0\,\mathrm{V}}{50.0\times10^{-3}\,\mathrm{H}}\right)e^{-(180\,\Omega)(1.20\times10^{-3}\,\mathrm{s})/(50.0\times10^{-3}\,\mathrm{H})} = 12.0\,\mathrm{A/s}\,.\end{aligned}$$

68P

(a) Since the inner circumference of the toroid is $l = 2\pi a = 2\pi(10\,\mathrm{cm}) = 62.8\,\mathrm{cm}$, the number fo turns of the toroid is roughly $N = 62.8\,\mathrm{cm}/1.0\,\mathrm{mm} = 628$. Thus

$$\begin{aligned}L &= \frac{\mu_0 N^2 H}{2\pi}\ln\frac{b}{a} \approx \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(628)^2(0.12\,\mathrm{m}-0.10\,\mathrm{m})}{2\pi}\ln\left(\frac{12\,\mathrm{cm}}{10\,\mathrm{cm}}\right) \\ &= 2.9\times10^{-4}\,\mathrm{H}\,.\end{aligned}$$

(b) Since the total length l of the wire is $l = (628)4(2.0\,\mathrm{cm}) = 50\,\mathrm{m}$, the resistance of the wire is $R = (50\,\mathrm{m})(0.02\,\Omega/\,\mathrm{m}) = 1.0\,\Omega$. Thus

$$\tau_L = \frac{L}{R} = \frac{2.9\times10^{-4}\,\mathrm{H}}{1.0\,\Omega} = 2.9\times10^{-4}\,\mathrm{s}\,.$$

69P

(a) The inductor prevents a fast build-up of the current through it, so immediately after the switch is closed the current in the inductor is zero. This means

$$i_1 = i_2 = \frac{\mathcal{E}}{R_1+R_2} = \frac{100\,\mathrm{V}}{10.0\,\Omega+20.0\,\Omega} = 3.33\,\mathrm{A}\,.$$

(b) A long time later the current reaches steady state and no longer changes. The emf across the inductor is zero and the circuit behaves as if it were replaced by a wire. The current in R_3 is $i_1 - i_2$. Kirchhoff's loop rule gives

$$\mathcal{E} - i_1R_1 - i_2R_2 = 0$$

and

$$\mathcal{E} - i_1R_1 - (i_1-i_2)R_3 = 0\,.$$

Solve these simultaneously for i_1 and i_2. The results are

$$\begin{aligned}i_1 &= \frac{\mathcal{E}(R_2+R_3)}{R_1R_2+R_1R_3+R_2R_3} \\ &= \frac{(100\,\mathrm{V})(20.0\,\Omega+30.0\,\Omega)}{(10.0\,\Omega)(20.0\,\Omega)+(10.0\,\Omega)(30.0\,\Omega)+(20.0\,\Omega)(30.0\,\Omega)} \\ &= 4.55\,\mathrm{A}\end{aligned}$$

and

$$\begin{aligned} i_2 &= \frac{\mathcal{E}R_3}{R_1R_2 + R_1R_3 + R_2R_3} \\ &= \frac{(100\,\mathrm{V})(30.0\,\Omega)}{(10.0\,\Omega)(20.0\,\Omega) + (10.0\,\Omega)(30.0\,\Omega) + (20.0\,\Omega)(30.0\,\Omega)} \\ &= 2.73\,\mathrm{A}\,. \end{aligned}$$

(*c*) The left-hand branch is now broken. If its inductance is zero the current immediately drops to zero when the switch is opened. That is, $i_1 = 0$. The current in R_3 changes only slowly because there is an inductor in its branch. Immediately after the switch is opened it has the same value as it had just before the switch was opened. That value is $4.55\,\mathrm{A} - 2.73\,\mathrm{A} = 1.82\,\mathrm{A}$. The current in R_2 is the same as that in R_3, $1.82\,\mathrm{A}$.

(*d*) There are no longer any sources of emf in the circuit, so all currents eventually drop to zero.

70P

(*a*) When swith S is just closed (case I), $V_1 = \mathcal{E}$ and $i_1 = \mathcal{E}/R_1 = 10\,\mathrm{V}/5.0\,\Omega = 2.0\,\mathrm{A}$. After a long time (case II) we still have $V_1 = \mathcal{E}$, so $i_1 = 2.0\,\mathrm{A}$.
(*b*) Case I: since now $\mathcal{E}_L = \mathcal{E}$, $i_2 = 0$; case II: since now $\mathcal{E}_L = 0$, $i_2 = \mathcal{E}/R_2 = 10\,\mathrm{V}/10\,\Omega = 1.0\,\mathrm{A}$.
(*c*) Case I: $i = i_1 + i_2 = 2.0\,\mathrm{A} + 0 = 2.0\,\mathrm{A}$; case II: $i = i_1 + i_2 = 2.0\,\mathrm{A} + 1.0\,\mathrm{A} = 3.0\,\mathrm{A}$.
(*d*) Case I: since $\mathcal{E}_L = \mathcal{E}$, $V_2 = \mathcal{E} - \mathcal{E}_L = 0$; case II: since $\mathcal{E}_L = 0$, $V_2 = \mathcal{E} - \mathcal{E}_L = \mathcal{E} = 10\,\mathrm{V}$.
(*e*) Case I: $\mathcal{E}_L = \mathcal{E} = 10\,\mathrm{V}$; case II: $\mathcal{E}_L = 0$.
(*f*) Case I: $di_2/dt = \mathcal{E}_L/L = \mathcal{E}/L = 10\,\mathrm{V}/5.0\,\mathrm{H} = 2.0\,\mathrm{A/s}$; case II: $di_2/dt = \mathcal{E}_L/L = 0$.

71P

For $t < 0$, no current goes through L_2, so $i_2 = 0$ and $i_1 = \mathcal{E}/R$. As the switch is opened there will be a very brief sparking across the gap. i_1 drops while i_2 increases, both very quickly. The loop rule can be written as

$$\mathcal{E} - i_1R - L_1\frac{di_1}{dt} - L_2R - L_2\frac{di_2}{dt} = 0\,,$$

where the initial value of i_1 at $t = 0$ is given by $\mathcal{E}/R$ and that of i_2 at $t = 0$ is 0. Now consider the situation shortly after $t = 0$. Since the sparking is very brief, we can reasonably assume that both i_1 and i_2 get equalized quickly, before they can change appreciably from their respective initial values. Thus the loop rule requires that $L_1(di_1/dt)$, which is large and negative, must roughly cancel $L_2(di_2/dt)$, which is large and positive:

$$L_1\frac{di_1}{dt} \approx -L_2\frac{di_2}{dt}\,.$$

Let the common value reached by i_1 and i_2 be i, then

$$\frac{di_1}{dt} \approx \frac{\Delta i_1}{\Delta t} = \frac{i - \mathcal{E}/R}{\Delta t}$$

and

$$\frac{di_2}{dt} \approx \frac{\Delta i_2}{\Delta t} = \frac{i - 0}{\Delta t} .$$

The equations above yield

$$L_1\left(i - \frac{\mathcal{E}}{R}\right) = -L_2(i - 0) ,$$

or

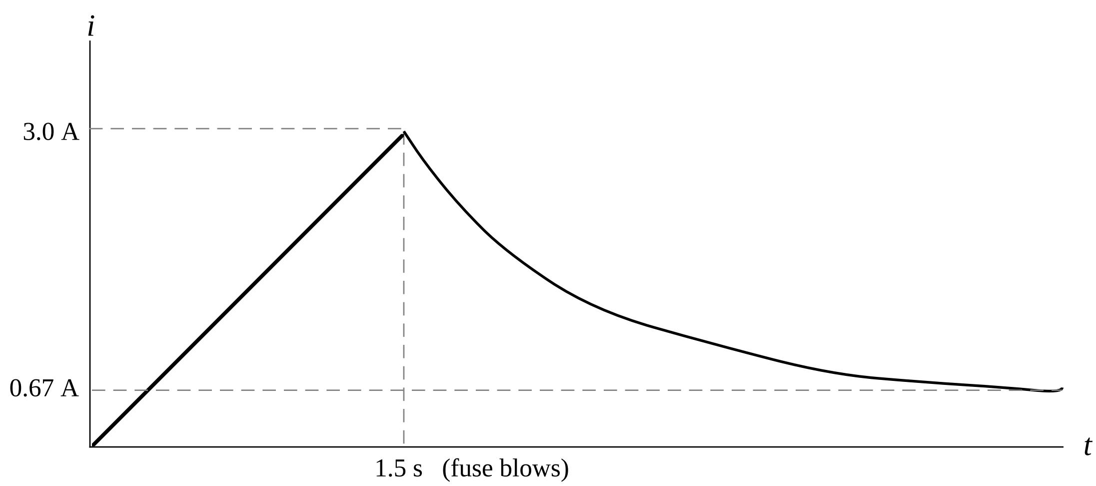
$$i = \frac{\mathcal{E}L_1}{L_2R_1 + L_1R_2} = \frac{L_1}{L_1 + L_2}\frac{\mathcal{E}}{R} .$$

72P

(a) Before the fuse blows, the current through the resistor remains zero. Apply the loop theorem to the battery-fuse-inductor loop: $\mathcal{E} - L\,di/dt = 0$. So $i = \mathcal{E}t/L$. As the fuse blows at $t = t_0$, $i = i_0 = 3.0\,\text{A}$. Thus

$$t_0 = \frac{i_0 L}{\mathcal{E}} = \frac{(3.0\,\text{A})(5.0\,\text{H})}{10\,\text{V}} = 1.5\,\text{s} .$$

(b)



73P*

(a) Assume i is from left to right through the closed switch. Let i_1 be the current in the resistor and take it to be downward. Let i_2 be the current in the inductor and also take it to be downward. The junction rule gives $i = i_1 + i_2$ and the loop rule gives $i_1R - L(di_2/dt) = 0$.

According to the junction rule, $(di_1/dt) = -(di_2/dt)$. Substitute into the loop equation to obtain

$$L\frac{di_1}{dt} + i_1 R = 0\,.$$

This equation is similar to Eq. 31-48, and its solution is the function given as Eq. 31-49:

$$i_1 = i_0 e^{-Rt/L}\,,$$

where i_0 is the current through the resistor at $t = 0$, just after the switch is closed. Now just after the switch is closed the inductor prevents the rapid build-up of current in its branch, so at that time $i_2 = 0$ and $i_1 = i$. Thus $i_0 = i$, so

$$i_1 = i e^{-Rt/L}$$

and

$$i_2 = i - i_1 = i\left(1 - e^{-Rt/L}\right)\,.$$

(*b*) When $i_2 = i_1$,

$$e^{-Rt/L} = 1 - e^{-Rt/L}\,,$$

so

$$e^{-Rt/L} = \frac{1}{2}\,.$$

Take the natural logarithm of both sides and use $\ln(1/2) = -\ln 2$ to obtain $(Rt/L) = \ln 2$ or

$$t = \frac{L}{R}\ln 2\,.$$

74P

(*a*) Use $U_B = \frac{1}{2}Li^2$:

$$L = \frac{2U_B}{i^2} = \frac{2(25.0\times10^{-3}\,\text{J})}{(60.0\times10^{-3}\,\text{A})^2} = 13.9\,\text{H}.$$

(*b*) Since $U_B \propto i^2$, for U_B to increase by a factor of 4 i must increase by a factor of 2, i.e., i should be increased to $2(60.0\,\text{mA}) = 120\,\text{mA}$.

75E

Let $U_B(t) = \frac{1}{2}Li^2(t) = \frac{1}{2}U_B(t\to\infty) = \frac{1}{4}Li_0^2$. This gives $i(t) = i_0/\sqrt{2}$. But $i(t) = i_0(1-e^{-t/\tau_L})$, so $1 - e^{-t/\tau_L} = 1/\sqrt{2}$. Solve for t:

$$t = -\tau_L \ln\left(1 - \frac{1}{\sqrt{2}}\right) = 1.23\tau_L\,.$$

76E

(a) $i_0 = \mathcal{E}/R = 100\,\mathrm{V}/10\,\Omega = 10\,\mathrm{A}$.
(b) $U_B = \frac{1}{2}Li_0^2 = \frac{1}{2}(2.0\,\mathrm{H})(10\,\mathrm{A})^2 = 100\,\mathrm{J}$.

77E

(a)

$$\begin{aligned}
\frac{dU_B}{dt} &= \frac{d}{dt}\left(\frac{1}{2}Li^2\right) = Li\frac{di}{dt} = Li_0\left(1-e^{-t/\tau_L}\right)\frac{d}{dt}\left[i_0\left(1-e^{-t/\tau_L}\right)\right] \\
&= \frac{Li_0^2}{\tau_L}e^{-t/\tau_L}\left(1-e^{-t/\tau_L}\right) = \frac{L\mathcal{E}^2}{(L/R)R^2}e^{-Rt/L}\left(1-e^{-t/\tau_L}\right) \\
&= \frac{(100\,\mathrm{V})^2}{10\,\Omega}e^{-(10\,\Omega)(0.10\,\mathrm{s})/2.0\,\mathrm{H}}\left[1-e^{-(10\,\Omega)(0.10\,\mathrm{s})/2.0\,\mathrm{H}}\right] \\
&= 2.4\times10^2\,\mathrm{W}\,.
\end{aligned}$$

(b)

$$\begin{aligned}
P_{\text{thermal}} &= i^2R = i_0^2\left(1-e^{-t/\tau_L}\right)^2R = \frac{\mathcal{E}^2}{R}\left(1-e^{-Rt/L}\right)^2 \\
&= \frac{(100\,\mathrm{V})^2}{10\,\Omega}\left[1-e^{-(10\,\Omega)(0.10\,\mathrm{s})/2.0\,\mathrm{H}}\right]^2 \\
&= 1.5\times10^2\,\mathrm{W}\,.
\end{aligned}$$

(c)

$$\begin{aligned}
P_{\text{battery}} &= i\mathcal{E} = \mathcal{E}i_0\left(1-e^{-t/\tau_L}\right) = \frac{\mathcal{E}^2}{R}\left(1-e^{-Rt/L}\right) \\
&= \frac{(100\,\mathrm{V})^2}{10\,\Omega}\left[1-e^{-(10\,\Omega)(0.10\,\mathrm{s})/2.0\,\mathrm{H}}\right] \\
&= 3.9\times10^2\,\mathrm{W}\,.
\end{aligned}$$

Note that the law of conservation of energy requirement $P_{\text{battery}} = dU_B/dt + P_{\text{thermal}}$ is satisfied.

78P

Use the results of the last problem. Let $P_{\text{thermal}} = dU_B/dt$, or

$$\frac{\mathcal{E}^2}{R}\left(1-e^{-Rt/L}\right)^2 = \frac{\mathcal{E}^2}{R}e^{-Rt/L}\left(1-e^{-Rt/L}\right)$$

and solve for t:

$$t = \frac{L}{R}\ln 2 = \tau_L\ln 2 = (37.0\,\mathrm{ms})\ln 2 = 25.6\,\mathrm{ms}\,.$$

79P

(*a*) If the battery is applied at time $t = 0$ the current is given by

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right),$$

where $\mathcal{E}$ is the emf of the battery, R is the resistance, and τ_L is the inductive time constant. In terms of R and the inductance L, $\tau_L = L/R$. Solve the current equation for the time constant. First obtain

$$e^{-t/\tau_L} = 1 - \frac{iR}{\mathcal{E}},$$

then take the natural logarithm of both sides to obtain

$$-\frac{t}{\tau_L} = \ln\left(1 - \frac{iR}{\mathcal{E}}\right).$$

Since

$$\ln\left(1 - \frac{iR}{\mathcal{E}}\right) = \ln\left[1 - \frac{(2.00\times10^{-3}\,\mathrm{A})(10.0\times10^{3}\,\Omega)}{50.0\,\mathrm{V}}\right] = -.5108\,,$$

the inductive time constant is $\tau_L = t/0.5108 = (5.00\times10^{-3}\,\mathrm{s})/(0.5108) = 9.79\times10^{-3}\,\mathrm{s}$ and the inductance is

$$L = \tau_L R = (9.79\times10^{-3}\,\mathrm{s})(10.0\times10^{3}\,\Omega) = 97.9\,\mathrm{H}\,.$$

(*b*) The energy stored in the coil is

$$U_B = \frac{1}{2}Li^2 = \frac{1}{2}(97.9\,\mathrm{H})(2.00\times10^{-3}\,\mathrm{A})^2 = 1.96\times10^{-4}\,\mathrm{J}\,.$$

80P

(*a*)

$$\begin{aligned}
E_{\text{battery}} &= \int_0^t P_{\text{battery}}\,dt = \int_0^t \frac{\mathcal{E}^2}{R}\left(1 - e^{-Rt/L}\right)dt = \frac{\mathcal{E}^2}{R}\left[t + \frac{L}{R}\left(e^{-Rt/L} - 1\right)\right] \\
&= \frac{(10.0\,\mathrm{V})^2}{6.70\,\Omega}\left[2.00\,\mathrm{s} + \frac{(5.50\,\mathrm{H})\left[e^{-(6.70\,\Omega)(2.00\,\mathrm{s})/5.50\,\mathrm{H}} - 1\right]}{6.70\,\Omega}\right] \\
&= 18.7\,\mathrm{J}\,.
\end{aligned}$$

(*b*)

$$\begin{aligned}
E_{\text{inductor}} &= U_B(t) = \frac{1}{2}Li^2(t) = \frac{1}{2}L\left(\frac{\mathcal{E}}{R}\right)^2(1 - e^{-Rt/L})^2 \\
&= \frac{1}{2}(5.50\,\mathrm{H})\left(\frac{10.0\,\mathrm{V}}{6.70\,\Omega}\right)^2\left[1 - e^{-(6.70\,\Omega)(2.00\,\mathrm{s})/5.50\,\mathrm{H}}\right]^2 \\
&= 5.10\,\mathrm{J}\,.
\end{aligned}$$

(c) $E_{\text{dissipated}} = E_{\text{battery}} - E_{\text{inductor}} = 18.7\,\text{J} - 5.10\,\text{J} = 13.6\,\text{J}$.

81P

(a) The inductance of the solenoid is

$$\begin{aligned} L &= \mu_0 n^2 A l \\ &= (4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(3000\,\text{turns}/0.800\,\text{m})^2(\pi)(5.00\times 10^{-2}\,\text{m})^2(0.800\,\text{m}) \\ &= 0.111\,\text{H}\,. \end{aligned}$$

Thus

$$\begin{aligned} U_B(t) &= \frac{1}{2}Li^2(t) = \frac{1}{2}L\left(\frac{\mathcal{E}}{R}\right)^2\left(1 - e^{-Rt/L}\right)^2 \\ &= \frac{(0.111\,\text{H})}{2}\left(\frac{12.0\,\text{V}}{10.0\,\Omega}\right)^2\left[1 - e^{-(10.0\,\Omega)(5.00\times 10^{-3}\,\text{s})/0.111\,\text{H}}\right]^2 \\ &= 1.05\times 10^{-2}\,\text{J}\,. \end{aligned}$$

(b)

$$\begin{aligned} E_{\text{battery}} &= \frac{\mathcal{E}^2}{R}\left[t + \frac{L}{R}\left(e^{-Rt/L} - 1\right)\right] \\ &= \frac{(12.0\,\text{V})^2}{10.0\,\Omega}\left[5.00\times 10^{-3}\,\text{s} + \left(\frac{0.111\,\text{H}}{10.0\,\Omega}\right)\left[e^{-(10.0\,\Omega)(5.00\times 10^{-3}\,\text{s})/0.111\,\text{H}} - 1\right]\right] \\ &= 1.41\times 10^{-2}\,\text{J}\,. \end{aligned}$$

82P

Suppose that the switch has been in position a for a long time, so the current has reached the steady-state value i_0. The energy stored in the inductor is $U_B = \frac{1}{2}Li_0^2$. Now the switch is thrown to position b at time $t = 0$. Thereafter the current is given by

$$i = i_0 e^{-t/\tau_L}\,,$$

where τ_L is the inductive time constant, given by $\tau = L/R$. The rate at which thermal energy is generated in the resistor is given by

$$P = i^2 R = i_0^2 R e^{-2t/\tau_L}\,.$$

Over a long time period the energy dissipated is

$$E = \int_0^\infty P\,dt = i_0^2 R \int_0^\infty e^{-2t/\tau_L}\,dt = -\frac{1}{2}i_0^2 R\tau_L e^{-2t/\tau_L}\Big|_0^\infty = \frac{1}{2}i_0^2 R\tau_L\,.$$

Substitute $\tau_L = L/R$ to obtain

$$E = \frac{1}{2} L i_0^2 \,,$$

the same as the total energy originally stored in the inductor.

83E

(a) At any point the magnetic energy density is given by $u_B = B^2/2\mu_0$, where B is the magnitude of the magnetic field at that point. Inside a solenoid $B = \mu_0 n i$, where n is the number of turns per unit length and i is the current. For the solenoid of this problem $n = (950)/(0.850\,\mathrm{m}) = 1.118 \times 10^3\,\mathrm{m}^{-1}$. The magnetic energy density is

$$u_B = \frac{1}{2}\mu_0 n^2 i^2 = \frac{1}{2}(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A})(1.118 \times 10^3\,\mathrm{m}^{-1})^2(6.60\,\mathrm{A})^2 = 34.2\,\mathrm{J/m^3} \,.$$

(b) Since the magnetic field is uniform inside an ideal solenoid, the total energy stored in the field is $U_B = u_B V$, where V is the volume of the solenoid. V is calculated as the product of the cross-sectional area and the length. Thus

$$U_B = (34.2\,\mathrm{J/m^3})(17.0 \times 10^{-4}\,\mathrm{m}^2)(0.850\,\mathrm{m}) = 4.94 \times 10^{-2}\,\mathrm{J} \,.$$

84E

The magnetic energy stored in the toroid is given by $U_B = \frac{1}{2} L i^2$, where L is its inductance and i is the current. The energy is also given by $U_B = u_B V$, where u_B is the average energy density and V is the volume. Thus

$$i = \sqrt{\frac{2 u_B V}{L}} = \sqrt{\frac{2(70.0\,\mathrm{J/m^3})(0.0200\,\mathrm{m}^3)}{90.0 \times 10^{-3}\,\mathrm{H}}} = 5.58\,\mathrm{A} \,.$$

85E

Let $u_E = \frac{1}{2}\epsilon_0 E^2 = u_B = \frac{1}{2} B^2/\mu_0$ and solve for E:

$$E = \frac{B}{\sqrt{\epsilon_0 \mu_0}} = \frac{0.50\,\mathrm{T}}{\sqrt{(8.85 \times 10^{-12}\,\mathrm{F/m})(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A})}} = 1.5 \times 10^8\,\mathrm{V/m}.$$

86E

Use $1\,\mathrm{ly} = 9.46 \times 10^{15}\,\mathrm{m}$.

$$U_B = V u_B = \frac{V B^2}{2\mu_0} = \frac{(9.46 \times 10^{15})^3 (10^{-10}\,\mathrm{T})^2}{2(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A})} = 3 \times 10^{36}\,\mathrm{J} \,.$$

87E

Use Eq. 31-53 to solve for L:

$$L = \frac{2U}{i^2} = \frac{2}{i^2}\frac{\mu_0 i^2 l}{4\pi}\ln\frac{b}{a} = \frac{\mu_0 l}{2\pi}\ln\frac{b}{a}\,.$$

88E

(a) The energy needed is

$$U_E = u_E V = \frac{1}{2}\epsilon_0 E^2 V = \frac{1}{2}(8.85\times 10^{-12}\,\mathrm{F/m})(100\,\mathrm{kV/m})^2(10\,\mathrm{cm})^3 = 4.4\times 10^{-5}\,\mathrm{J}\,.$$

(b) The energy needed is

$$U_B = u_B V = \frac{1}{2\mu_0}B^2 V = \frac{(1.0\,\mathrm{T})^2}{2(4\pi\times 10^{-7}\,\mathrm{T\cdot m/A})}(10\,\mathrm{cm})^3 = 4.0\times 10^2\,\mathrm{J}\,.$$

(b) Obviously, since $U_B > U_E$ greater amounts of energy can be stored in the magneticc field.

89E

(a)

$$B = \frac{\mu_0 i}{2R} = \frac{(4\pi\times 10^{-7}\,\mathrm{T\cdot m/A})(100\,\mathrm{A})}{2(50\times 10^{-3}\,\mathrm{m})} = 1.3\times 10^{-3}\,\mathrm{T}\,.$$

(b)

$$u_B = \frac{B^2}{2\mu_0} = \frac{(1.3\times 10^{-3}\,\mathrm{T})^2}{2(4\pi\times 10^{-7}\,\mathrm{T\cdot m/A})} = 0.63\,\mathrm{J/m^3}\,.$$

90P

(a) The energy density at any point is given by $u_B = B^2/2\mu_0$, where B is the magnitude of the magnetic field. The magnitude of the field inside a toroid, a distance r from the center, is given by Eq. 30–26: $B = \mu_0 iN/2\pi r$, where N is the number of turns and i is the current. Thus

$$u_B = \frac{1}{2\mu_0}\left(\frac{\mu_0 iN}{2\pi r}\right)^2 = \frac{\mu_0 i^2 N^2}{8\pi^2 r^2}\,.$$

(b) Evaluate the integral $U_B = \int u_B\,dV$ over the volume of the toroid. A circular strip with radius r, height h, and thickness dr has volume $dV = 2\pi rh\,dr$, so

$$U_B = \frac{\mu_0 i^2 N^2}{8\pi^2}2\pi h\int_a^b \frac{dr}{r} = \frac{\mu_0 i^2 N^2 h}{4\pi}\ln\left(\frac{b}{a}\right)\,.$$

Numerically,

$$U_B = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(0.500\,\mathrm{A})^2(1250)^2(13 \times 10^{-3}\,\mathrm{m})}{4\pi} \ln\left(\frac{95\,\mathrm{mm}}{52\,\mathrm{mm}}\right)$$
$$= 3.06 \times 10^{-4}\,\mathrm{J}\,.$$

(c) The inductance is given by Eq. 31–37:

$$L = \frac{\mu_0 N^2 h}{2\pi} \ln\left(\frac{b}{a}\right)$$

so the energy is given by

$$U_B = \frac{1}{2}Li^2 = \frac{\mu_0 N^2 i^2 h}{4\pi} \ln\left(\frac{b}{a}\right)\,.$$

This the exactly the same as the expression found in part (b) and yields the same numerical result.

91P

(a)

$$u_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 i}{2R}\right)^2 = \frac{\mu_0 i^2}{R^2} = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(10\,\mathrm{A})^2}{8(2.5 \times 10^{-3}\,\mathrm{m}/2)^2} = 1.0\,\mathrm{J/m^3}\,.$$

(b)

$$u_E = \frac{1}{2}\epsilon_0 E^2 = \frac{\epsilon_0}{2}\left(\frac{J}{\sigma}\right)^2 = \frac{\epsilon_0}{2}\left(\frac{iR}{l}\right)^2$$
$$= \frac{1}{2}(8.85 \times 10^{-12}\,\mathrm{F/m})[(10\,\mathrm{A})(3.3\,\Omega/10^3\,\mathrm{m})]^2$$
$$= 4.8 \times 10^{-15}\,\mathrm{J/m^3}\,.$$

Here we used $J/\sigma = i/\sigma A$ and $R/l = \rho/A = 1/\sigma A$ to obtain $J/\sigma = iR/l$.

92P

(a)

$$u_B = \frac{B_e^2}{2\mu_0} = \frac{(50 \times 10^{-6}\,\mathrm{T})^2}{2(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})} = 1.0 \times 10^{-3}\,\mathrm{J/m^3}\,.$$

(b) The volume of the shell of thickness h is $V \approx 4\pi R_e^2 h$, where R_e is the radius of the Earth. So

$$U_B \approx V u_B \approx 4\pi(6.4 \times 10^6\,\mathrm{m})^2(16 \times 10^3\,\mathrm{m})(1.0 \times 10^{-3}\,\mathrm{J/m^3}) = 8.4 \times 10^{15}\,\mathrm{J}\,.$$

93E

(a)

$$M=\frac{\mathcal{E}_1}{|di_1/dt|}=\frac{25.0\,\mathrm{mV}}{15.0\,\mathrm{A/s}}=1.67\,\mathrm{mH}\,.$$

(b)

$$\Phi_2=Mi_1=(1.67\,\mathrm{mH})(3.60\,\mathrm{A})=6.00\,\mathrm{mWb}\,.$$

94E

(a)

$$\Phi_{12}=\frac{L_1 i_1}{N_1}=\frac{(25\,\mathrm{mH})(6.0\,\mathrm{mA})}{100}=1.5\,\mu\mathrm{Wb}\,.$$

$$\mathcal{E}_1=L_1\frac{di_1}{dt}=(25\,\mathrm{mH})(4.0\,\mathrm{A/s})=100\,\mathrm{mV}\,.$$

(b)

$$\Phi_{21}=\frac{Mi_1}{N_2}=\frac{(3.0\,\mathrm{mH})(6.0\,\mathrm{mA})}{200}=90\,\mathrm{nWb}\,.$$

$$\mathcal{E}_{21}=M\frac{di_1}{dt}=(3.0\,\mathrm{mH})(4.0\,\mathrm{A/s})=12\,\mathrm{mV}\,.$$

95E

Use $\mathcal{E}_2=M\,di_1/dt$ to find M:

$$M=\frac{\mathcal{E}}{di_1/dt}=\frac{30\times10^3\,\mathrm{V}}{6.0\,\mathrm{A}/(2.5\times10^{-3}\,\mathrm{s})}=13\,\mathrm{H}\,.$$

96P

(a) Assume the current is changing at the rate di/dt and calculate the total emf across both coils. First consider the left-hand coil. The magnetic field due to the current in that coil points to the left. So does the magnetic field due to the current in coil 2. When the current increases both fields increase and both changes in flux contribute emf's in the same direction. Thus the emf in coil 1 is

$$\mathcal{E}_1=-(L_1+M)\,\frac{di}{dt}\,.$$

The magnetic field in coil 2 due to the current in that coil points to the left, as does the field in coil 2 due to the current in coil 1. The two sources of emf are again in the same direction and the emf in coil 2 is

$$\mathcal{E}_2=-(L_2+M)\,\frac{di}{dt}\,.$$

The total emf across both coils is

$$\mathcal{E}=\mathcal{E}_1+\mathcal{E}_2=-(L_1+L_2+2M)\,\frac{di}{dt}\,.$$

This is exactly the emf that would be produced if the coils were replaced by a single coil with inductance $L_{\mathrm{eq}}=L_1+L_2+2M$.

(b) Reverse the leads of coil 2 so the current enters at the back of coil rather than the front as pictured in the diagram. Then the field produced by coil 2 at the site of coil 1 is opposite the field produced by coil 1 itself. The fluxes have opposite signs. An increasing current in coil 1 tends to increase the flux in that coil but an increasing current in coil 2 tends to decrease it. The emf across coil 1 is

$$\mathcal{E}_1 = -(L_1 - M)\,\frac{di}{dt}\,.$$

Similarly the emf across coil 2 is

$$\mathcal{E}_2 = -(L_2 - M)\,\frac{di}{dt}\,.$$

The total emf across both coils is

$$\mathcal{E} = -(L_1 + L_2 - 2M)\,\frac{di}{dt}\,.$$

This the same as the emf that would be produced by a single coil with inductance $L_{\rm eq} = L_1 + L_2 - 2M$.

97P

$$M = M_{cs} = \frac{N\Phi_{cs}}{i_s} = \frac{N(\mu_0 i_s n\pi R^2)}{i_s} = \mu_0 \pi R^2 nN.$$

As long as the magnetic field of the solenoid is entirely contained within the cross-section of the coil we always have $\Phi_{sc} = B_s A_s = B_s \pi R^2$, regardless of the shape, size, or possible lack of close-packing of the coil.

98P

Use the expression for the flux Φ over the toroid cross-section derived in Sample Problem 31-6 to obtain

$$M = M_{ct} = \frac{N_c\Phi_{ct}}{i_t} = \frac{N_c}{i_t}\,\frac{\mu_0 i_t N_t h}{2\pi}\ln\frac{b}{a} = \frac{\mu_0 N_1 N_2 h}{2\pi}\ln\frac{b}{a}\,,$$

where $N_t = N_1$ and $N_c = N_2$.

99P

Assume the current in solenoid 1 is i and calculate the flux linkage in solenoid 2. The mutual inductance is this flux linkage divided by i. The magnetic field inside solenoid 1 is parallel to the axis and has uniform magnitude $B = \mu_0 i n_1$, where n_1 is the number of turns per unit length of the solenoid. The cross-sectional area of the solenoid is πR_1^2 and since the field is normal to a cross section the flux through a cross section is

$$\Phi = AB = \pi R_1^2 \mu_0 n_1 i\,.$$

Since the magnetic field is zero outside the solenoid, this is also the flux through a cross section of solenoid 2. The number of turns in a length l of solenoid 2 is $N_2 = n_2 l$ and the flux linkage is

$$N_2\Phi = n_2 l\pi R_1^2 \mu_0 n_1 i\,.$$

The mutual inductance is

$$M = \frac{N_2\Phi}{i} = \pi R_1^2 l \mu_0 n_1 n_2\,.$$

M does not depend on R_2 because there is no magnetic field in the region between the solenoids. Changing R_2 does not change the flux through solenoid 2, but changing R_1 does.

100P

(a) The flux over the loop cross section due to the current i in the wire is given by

$$\Phi = \int_a^{a+b} B_{\mathrm{wire}}(r) l\,dr = \int_a^{a+b} \frac{\mu_0 i l}{2\pi r}\,dr = \frac{\mu_0 i l}{2\pi}\ln\left(1+\frac{b}{a}\right).$$

Thus

$$M = \frac{N\Phi}{i} = \frac{N\mu_0 l}{2\pi}\ln\left(1+\frac{b}{a}\right).$$

(b) From the formula for M obtained above

$$M = \frac{(100)(4\pi\times 10^{-7}\,\mathrm{T\cdot m/A})(0.30\,\mathrm{m})}{2\pi}\ln\left(1+\frac{8.0}{1.0}\right) = 1.3\times 10^{-5}\,\mathrm{H}\,.$$