CHAPTER 32

Answer to Checkpoint Questions

1. (d), (b), (c), (a) (zero)
2. (a) 2; (b) 1
3. (a) away; (b) away; (c) less
4. (a) toward; (b) toward; (c) less
5. a, c, b, d (zero)
6. tie of b, c, and d, then a

Answer to Questions

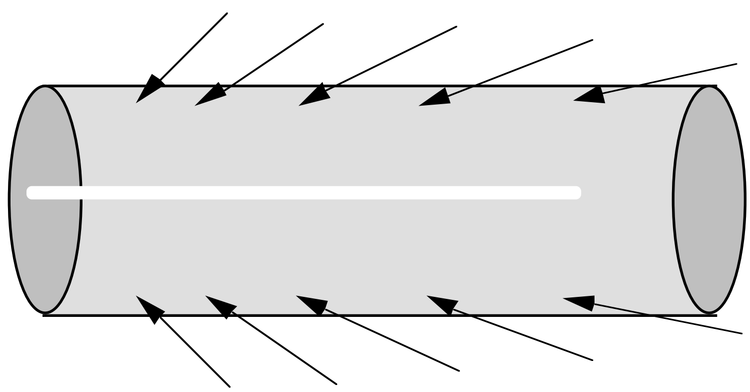
1. (a) a, c, f; (b) bar gh
2. (a) and (c)
3. supplied
4. b
5. (a) all down; (b) 1 up, 2 down, 3 zero
6. (a) increase; (b) increase
7. (a) 1 up, 2 up, 3 down; (b) 1 down, 2 up, 3 zero
8. (a) 1 down, 2 down, 3 up; (b) 1 up, 2 up, 3 zero
9. (a) rightward; (b) leftward
10. counterclockwise
11. (a) decreasing; (b) decreasing
12. (a) counterclockwise; (b) clockwise; (c) no direction (no induced $\mathbf{B}$)
13. (a) tie of a and b, then c, d; (b) none (plate lacks circular symmetry, so $\mathbf{B}$ is not tangent to a circular loop); (c) none
14. (a) rightward; (b) leftward; (c) into
15. $1/4$

16. 1, a; 2, b; 3, c and d

Solutions to Exercises & Problems

1E

(a)



(b) The sign of $\mathbf{B}\cdot d\mathbf{A}$ for every $d\mathbf{A}$ on the side of the paper cylinder is negative.
(c) No, because Gauss's law for magnetism applies to an *enclosed* surface only. In fact if we include the top and bottom of the cylinder to form an enclosed surface S then $\oint_s \mathbf{B}\cdot d\mathbf{A} = 0$ will be valid, as the flux through the open end of the cylinder near the magnet is positive.

2E

Use $\sum_{n=1}^{6} \Phi_{Bn} = 0$ to obtain

$$\Phi_{B6} = -\sum_{n=1}^{5} \Phi_{Bn} = -(-1\,\mathrm{Wb} + 2\,\mathrm{Wb} - 3\,\mathrm{Wb} + 4\,\mathrm{Wb} - 5\,\mathrm{Wb}) = +3\,\mathrm{Wb}\,.$$

3P

Use Gauss' law for magnetism: $\oint \mathbf{B}\cdot d\mathbf{A} = 0$. Write $\oint \mathbf{B}\cdot d\mathbf{A} = \Phi_1 + \Phi_2 + \Phi_C$, where Φ_1 is the magnetic flux through the first end mentioned, Φ_2 is the magnetic flux through the second end mentioned, and Φ_C is the magnetic flux through the curved surface. Over the first end the magnetic field is inward, so the flux is $\Phi_1 = -25.0\,\mu\,\mathrm{Wb}$. Over the second end the magnetic field is uniform, normal to the surface, and outward, so the flux is $\Phi_2 = AB = \pi r^2 B$, where A is the area of the end and r is the radius of the cylinder. It value is

$$\Phi_2 = \pi(0.120\,\mathrm{m})^2(1.60\times 10^{-3}\,\mathrm{T}) = +7.24\times 10^{-5}\,\mathrm{Wb} = +72.4\,\mu\mathrm{Wb}\,.$$

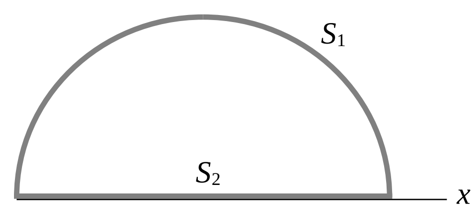
Since the three fluxes must sum to zero,

$$\Phi_C = -\Phi_1 - \Phi_2 = 25.0\,\mu\text{Wb} - 72.4\,\mu\text{Wb} = -47.4\,\mu\text{Wb}\,.$$

The minus sign indicates that the flux is inward through the curved surface.

4P*

From Gauss' law for magnetism the flux through S_1 is equal to that through S_2, the portion of the xz plane that lies within the cylinder. Here the normal direction of S_2 is in the positive y-axis. So



$$\begin{aligned}\Phi_B(S_1) &= \Phi_B(S_2)\\ &= \int_{-r}^{r} B(x)L\,dx\\ &= 2\int_{-r}^{r} B_{\text{left}}(x)L\,dx\\ &= 2\int_{-r}^{r} \frac{\mu_0 i}{2\pi}\frac{1}{2r-x}L\,dx\\ &= \frac{\mu_0 iL}{\pi}\ln 3\,.\end{aligned}$$

5E

The horizontal component of the Earth's magnetic field is given by $B_h = B\cos\phi_i$, where B is the magnitude of the field and ϕ_i is the inclination angle. Thus

$$B = \frac{B_h}{\cos\phi_i} = \frac{16\,\mu\text{T}}{\cos 73°} = 55\,\mu\text{T}\,.$$

6E

The flux through Arizona is

$$\Phi = -B_r A = -(43\times10^{-6}\,\text{T})(295,000\,\text{km}^2)(10^3\,\text{m/km})^2 = -1.3\times10^7\,\text{Wb}\,,$$

inward. By Gauss' law this is equal to the negative value of the flux Φ' through the rest of the surface of the Earth. So $\Phi' = 1.3\times10^7$ Wb, outward.

7E

(a) From $\mu = iA = i\pi R_e^2$ we get

$$i = \frac{\mu}{\pi R_e^2} = \frac{8.0\times10^{22}\,\text{J/T}}{\pi(6.37\times10^6\,\text{m})^2} = 6.3\times10^8\,\text{A}\,.$$

(*b*) Yes, because far away from the Earth the fields of both the Earth itself and the current loop are dipole fields. If these two dipoles cancel out then the net field will be zero.
(*c*) No, because the field of the current loop is not that of a magnetic dipole in the region close to the loop.

8P
(*a*)

$$B = \sqrt{B_h^2 + B_v^2} = \sqrt{\left(\frac{\mu_0\mu}{4\pi r^3}\cos\lambda_m\right)^2 + \left(\frac{\mu_0\mu}{2\pi r^3}\sin\lambda_m\right)^2}$$
$$= \frac{\mu_0\mu}{4\pi r^3}\sqrt{\cos^2\lambda_m + 4\sin^2\lambda_m} = \frac{\mu_0\mu}{4\pi r^3}\sqrt{1 + 3\sin^2\lambda_m}\,,$$

where $\cos^2\lambda_m + \sin^2\lambda_m = 1$ was used.
(*b*)

$$\tan\phi_i = \frac{B_v}{B_h} = \frac{(\mu_0\mu/2\pi r^3)\sin\lambda_m}{(\mu_0\mu/4\pi r^3)\cos\lambda_m} = 2\tan\lambda_m\,.$$

9P
(*a*) At the magnetic equator $\lambda_m = 0$, so

$$B = \frac{\mu_0\mu}{4\pi r^3} = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(8.00\times10^{22}\,\mathrm{A\cdot m^2})}{4\pi(6.37\times10^6\,\mathrm{m})^3} = 3.10\times10^{-5}\,\mathrm{T}$$

and $\phi_i = \tan^{-1}(2\tan\lambda_m) = \tan^{-1}(0) = 0$.
(*b*) At $\lambda_m = 60°$:

$$B = \frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m} = (3.10\times10^{-5})\sqrt{1+3\sin^2 60°} = 5.6\times10^{-5}\,\mathrm{T}$$

and $\phi_i = \tan^{-1}(2\tan 60°) = 74°$.
(*c*) At the north magnetic pole $\lambda_m = 90.0°$, so

$$B = \frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m} = (3.1\times10^{-5})\sqrt{1+3(1.00)^2} = 6.20\times10^{-5}\,\mathrm{T}$$

and $\phi_i = \tan^{-1}(2\tan 90°) = 90°$.

10P
(*a*) At a distance r from the center of the Earth, the magnitude of the magnetic field is given by

$$B = \frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m}\,,$$

where μ is the Earth's dipole moment and λ_m is the magnetic latitude. The ratio of the field magnitudes for two different distances at the same latitude is

$$\frac{B_2}{B_1} = \frac{r_1^3}{r_2^3}\,.$$

Take B_1 to be at the surface and B_2 to be half of B_1. Set r_1 equal to the radius R_e of the Earth and r_2 equal to $R_e + h$, where h is altitude at which B is half its value at the surface. Thus

$$\frac{1}{2} = \frac{R_e^3}{(R_e + h)^3}\,.$$

Take the cube root of both sides and solve for h. You should get

$$h = \left(2^{1/3} - 1\right) R_e = \left(2^{1/3} - 1\right)(6370\,\mathrm{km}) = 1660\,\mathrm{km}\,.$$

(b) Use the expression for B obtained in 8P, part (a). For maximum B set $\sin\lambda_m = 1$. Also, $r = 6370\,\mathrm{km} - 2900\,\mathrm{km} = 3470\,\mathrm{km}$. Thus

$$\begin{aligned} B_{\max} &= \frac{\mu_0\mu}{4\pi r^3}\sqrt{1 + 3\sin^2\lambda_m} \\ &= \frac{(4\pi\times 10^{-7}\,\mathrm{T\cdot m/A})(8.00\times 10^{22}\,\mathrm{A\cdot m^2})}{4\pi(3.47\times 10^6\,\mathrm{m})^3}\sqrt{1 + 3(1)^2} = 3.83\times 10^{-4}\,\mathrm{T}\,. \end{aligned}$$

(c) The angle between the magnetic axis and the rotational axis of the Earth is 11.5°, so $\lambda_m = 90.0° - 11.5° = 78.5°$ at the Earth's north geographic pole. Also $r = R_e = 6370\,\mathrm{km}$. Thus

$$\begin{aligned} B &= \frac{\mu_0\mu}{4\pi R_E^3}\sqrt{1 + 3\sin^2\lambda_m} \\ &= \frac{(4\pi\times 10^{-7}\,\mathrm{T\cdot m/A})(8.0\times 10^{22}\,\mathrm{J/T})\sqrt{1 + 3\sin^2 78.5°}}{4\pi(6.37\times 10^6\,\mathrm{m})^3} = 6.11\times 10^{-5}\,\mathrm{T}\,, \end{aligned}$$

and $\phi_i = \tan^{-1}(2\tan 78.5°) = 84.2°$.
A plausible explaination to the discrepancy between the calculated and measured values of the Earth's magnetic field is that the formulas we obtained in 8P are based on dipole approximation, which does not accurately represent the Earth's actual magnetic field distribution on or near its surface. (Incidentally, the dipole approximation does get better when we calculate the Earth's magnetic field far from its center.)

11E

Use Eq. 32-7 to obtain $\Delta U = -\Delta(\mu_{s,z}B) = -B\Delta\mu_{s,z}$, where $\mu_{s,z} = \pm eh/4\pi m_e = \pm\mu_B$ (see Eqs. 32-4 and 32-5). Thus

$$\Delta U = -B[\mu_B - (-\mu_B)] = 2\mu_B B = 2(9.27\times 10^{-24}\,\mathrm{J/T})(0.25\,\mathrm{T}) = 4.6\times 10^{-24}\,\mathrm{J}\,.$$

12E

Use Eq. 32-11: $\mu_{\text{orb},z} = -m_l \mu_B$.
(a) For $m_l = 1$, $\mu_{\text{orb},z} = -(1)(9.27 \times 10^{-24}\,\text{J/T}) = -9.27 \times 10^{-24}\,\text{J/T}$.
(b) For $m_l = -2$, $\mu_{\text{orb},z} = -(-2)(9.27 \times 10^{-24}\,\text{J/T}) = 1.85 \times 10^{-23}\,\text{J/T}$.

13E

(a)

$$E = \frac{e}{4\pi\epsilon_0 r^2} = \frac{(1.60 \times 10^{-19}\,\text{C})(8.99 \times 10^9\,\text{N}\cdot\text{m}^2/\,\text{C}^2)}{(5.2 \times 10^{-11}\,\text{m})^2} = 5.3 \times 10^{11}\,\text{N/C}\,.$$

(b) Use Eq. 30-29:

$$B = \frac{\mu_0}{2\pi}\frac{\mu_p}{r^3} = \frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(1.4 \times 10^{-26}\,\text{J/T})}{2\pi(5.2 \times 10^{-11}\,\text{m})^3} = 2.0 \times 10^{-2}\,\text{T}.$$

(c) From Eq. 32-10,

$$\frac{\mu_{\text{orb}}}{\mu_p} = \frac{eh/4\pi m_e}{\mu_p} = \frac{\mu_B}{\mu_p} = \frac{9.27 \times 10^{-24}\,\text{J/T}}{1.4 \times 10^{-26}\,\text{J/T}} = 6.6 \times 10^2.$$

14E

(a) Since $m_l = 0$, $L_{\text{orb},z} = m_l h/2\pi = 0$.
(b) Since $m_l = 0$, $\mu_{\text{orb},z} = -m_l \mu_B = 0$.
(c) Since $m_l = 0$, from Eq. 32-12 $U = -\mu_{\text{orb},z} B_{\text{ext}} = -m_l \mu_B B_{\text{ext}} = 0$.
(d) Regardless of the value of m_l, for the spin part

$$U = -\mu_{s,z} B = \pm \mu_B B = \pm(9.27 \times 10^{-24}\,\text{J/T})(35\,\text{mT}) = \pm 3.2 \times 10^{-25}\,\text{J}\,.$$

(d) Now $m_l = -3$ so

$$L_{\text{orb},z} = \frac{m_l h}{2\pi} = \frac{(-3)(6.626 \times 10^{-34}\,\text{J}\cdot\text{s})}{2\pi} = -3.16 \times 10^{-34}\,\text{J}\cdot\text{s}$$

and

$$\mu_{\text{orb},z} = -m_l \mu_B = -(-3)(9.27 \times 10^{-24}\,\text{J/T}) = 2.78 \times 10^{-23}\,\text{J/T}\,.$$

The potential energy associated with the electron's orbital magnetic moment is now

$$U = -\mu_{\text{orb},z} B_{\text{ext}} = -(2.78 \times 10^{-23}\,\text{J/T})(35 \times 10^{-3}\,\text{T}) = -9.73 \times 10^{-25}\,\text{J}\,;$$

while the potential energy associated with the electron spin, being independent of m_l, remains the same: $\pm 3.2 \times 10^{-25}$ J.

15E

For a given value of l, m_l varies from $-l$ to $+l$. Thus in our case $l = 3$, and the number of different m_l's is $2l + 1 = 2(3) + 1 = 7$.
(*a*) Since $L_{\text{orb},z} \propto m_l$, there are a total of seven different values of $L_{\text{orb},z}$.
(*b*) Similarly, since $\mu_{\text{orb},z} \propto m_l$, there are also a total of seven different values of $\mu_{\text{orb},z}$.
(*c*) Since $L_{\text{orb},z} = m_l h/2\pi$, the greatest allowed value of $L_{\text{orb},z}$ is given by $|m_l|_{\max} h/2\pi = 3h/2\pi$; while the least allowed value is given by $|m_l|_{\min} h/2\pi = 0$.
(*d*) Similar to part (*c*), since $\mu_{\text{orb},z} = -m_l \mu_B$, the greatest allowed value of $\mu_{\text{orb},z}$ is given by $|m_l|_{\max} \mu_B = 3eh/4\pi m_e$; while the least allowed value is given by $|m_l|_{\min} \mu_B = 0$.
(*e*) From Eqs. 32-3 and 32-9 the z component of the net angular momentum of the electron is given by

$$L_{\text{net},z} = L_{\text{orb},z} + L_{s,z} = \frac{m_l h}{2\pi} + \frac{m_s h}{2\pi}.$$

For the maximum value of $L_{\text{net},z}$ let $m_l = [m_l]_{\max} = 3$ and $m_s = \frac{1}{2}$. Thus

$$[L_{\text{net},z}]_{\max} = \left(3 + \frac{1}{2}\right)\frac{h}{2\pi} = \frac{3.5h}{2\pi}.$$

(*f*) Since the maximum value of $L_{\text{net},z}$ is given by $[m_J]_{\max} h/2\pi$ with $[m_J]_{\max} = 3.5$ (see the last part above), the number of allowed values for the z component of $L_{\text{net},z}$ is given by $2[m_J]_{\max} + 1 = 2(3.5) + 1 = 8$.

16P

(*a*) and (*b*) The potential energy of the atom in association with the presence an external magnetic field $\mathbf{B}_{\text{ext}}$ is given by Eqs. 32-11 and 32-12:

$$U = -\mu_{\text{orb}} \cdot \mathbf{B}_{\text{ext}} = -\mu_{\text{orb},z} B_{\text{ext}} = -m_l \mu_B B_{\text{ext}}.$$

For level E_1 there is no change in energy as a result of the introduction of $\mathbf{B}_{\text{ext}}$ so $U \propto m_l = 0$, meaning that $m_l = 0$ for this level. For level E_2 the single level splits into a triplet (i.e., three separate ones) in the presence of $\mathbf{B}_{\text{ext}}$, meaning that there are three different values of m_l. The middle one in the triplet is unshifted from the original value of E_2 so its m_l must be equal to 0. The other two in the triplet then correspond to $m_l = -1$ and $m_1 = +1$, respectively.
(*c*) For any pair of adjacent levels in the triplet $|\Delta m_l| = 1$. Thus the spacing is given by

$$\begin{aligned}\Delta U &= |\Delta(-m_l \mu_B B)| = |\Delta m_l| \mu_B B = \mu_B B \\ &= (5.79 \times 10^{-5}\,\text{eV/T})(0.50\,\text{T}) = 2.9 \times 10^{-5}\,\text{eV},\end{aligned}$$

which is equivalent to 4.6×10^{-24} J.

17P

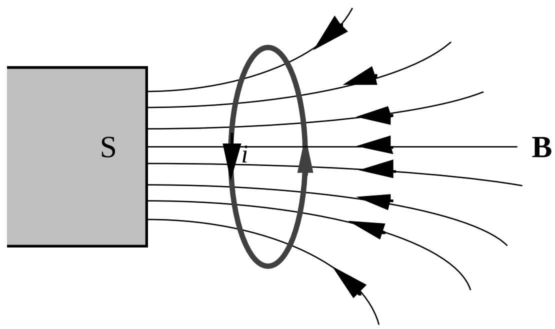
(*a*) The period of rotation is $T = 2\pi/\omega$ and in this time all the charge passes any fixed point near the ring. The average current is $i = q/T = q\omega/2\pi$ and the magnitude of the magnetic dipole moment is

$$\mu = iA = \frac{q\omega}{2\pi}\pi r^2 = \frac{1}{2} q\omega r^2.$$

(*b*) Curl the fingers of your right hand in the direction of rotation. Since the charge is positive your thumb points in the direction of the dipole moment. It is the same as the direction of the angular momentum of the ring.

18E

(*a*)



(*b*) Since the material is diamagnetic its magnetic moment μ is opposed to the direction of $\mathbf{B}$, so μ points away from the bar magnet.
(*c*) In order to produce a magnetic moment which points away from the bar magnet the current in the ring must be running clockwise when viewed along the symmetry axis from the location of the bar magnet.
(*d*) Consider the magnetic force $d\mathbf{F}_B$ exerted by the bar magnet on an infinitesimal segment $d\mathbf{L}$ of the ring. Use $d\mathbf{F}_B = i d\mathbf{L} \times \mathbf{B}$ to convince yourself that the net magnetic force on the ring points away from the magnet.

19P

An electric field with circular field lines is induced as the magnetic field is turned on. Suppose the magnetic field increases linearly from zero to B in time t. According to Eq. 32-24 the magnitude of the electric field at the orbit is given by

$$E = \left(\frac{r}{2}\right)\frac{dB}{dt} = \left(\frac{r}{2}\right)\frac{B}{t},$$

where r is the radius of the orbit. The induced electric field is tangent to the orbit and changes the speed of the electron, the change in speed being given by

$$\Delta v = at = \frac{eE}{m_e}t = \left(\frac{e}{m_e}\right)\left(\frac{r}{2}\right)\left(\frac{B}{t}\right)t = \frac{erB}{2m_e}.$$

The average current associated with the circulating electron is $i = ev/2\pi r$ and the dipole moment is

$$\mu = Ai = (\pi r^2)\left(\frac{ev}{2\pi r}\right) = \frac{1}{2}evr.$$

The change in the dipole moment is

$$\Delta\mu = \frac{1}{2}er\,\Delta v = \frac{1}{2}er\left(\frac{erB}{2m_e}\right) = \frac{e^2r^2B}{4m_e}\,.$$

20E

Let $E = \frac{3}{2}kT = |\boldsymbol{\mu}\cdot\mathbf{B} - (-\boldsymbol{\mu}\cdot\mathbf{B})| = 2\mu B$ to obtain

$$T = \frac{4\mu B}{3k} = \frac{4(1.0\times10^{-23}\,\mathrm{J/T})(0.50\,\mathrm{T})}{3(1.38\times10^{-23}\,\mathrm{J/K})} = 0.48\,\mathrm{K}\,.$$

21E

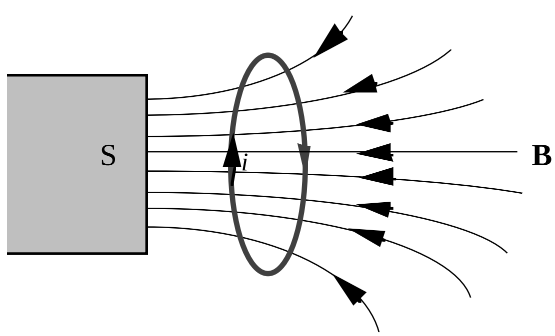
The magnetization is the dipole moment per unit volume, so the dipole moment is given by $\mu = MV$, where M is the magnetization and V is the volume of the cylinder. Use $V = \pi r^2 L$, where r is the radius of the cylinder and L is its length. Thus

$$\mu = M\pi r^2 L = (5.30\times10^3\,\mathrm{A/m})\pi(0.500\times10^{-2}\,\mathrm{m})^2(5.00\times10^{-2}\,\mathrm{m}) = 2.08\times10^{-2}\,\mathrm{J/T}\,.$$

22E

This problem is analogous to 18E.

(*a*)



(*b*) Since the material is paramagnetic its magnetic moment μ is in the same direction with $\mathbf{B}$, so μ points toward the bar magnet.

(*c*) In order to produce a magnetic moment which points toward the bar magnet the current in the ring must be running counterclockwise when viewed along the axis from the location of the bar magnet.

(d) Consider the magnetic force $d\mathbf{F}_B$ exerted by the bar magnet on an infinitesimal segment $d\mathbf{L}$ of the ring. Use $d\mathbf{F}_B = i d\mathbf{L} \times \mathbf{B}$ to convince yourself that the net magnetic force on the ring points toward the magnet.

23P

For the measurements carried out the largest ratio of the magnetic field to the temperature is $(0.50\,\mathrm{T})/(10\,\mathrm{K}) = 0.050\,\mathrm{T/K}$. Look at Fig. 32-10 to see if this is in the region where the magnetization is a linear function of the ratio. It is quite close to the origin, so we conclude that the magnetization obeys Curie's law.

24P

(a) From Fig. 32-10 we see that $B/T = 0.50$ when $M/M_{\mathrm{max}} = 50\%$. So $B = 0.50\,\mathrm{T} = (0.50)(300\,\mathrm{K}) = 150\,\mathrm{K}$.
(b) Similarly, now $B/T = 2.0$ so $B = (2.0)(300) = 600\,\mathrm{T}$.
(c) Not yet.

25P

(a) Again from Fig. 32-10, for $M/M_{\mathrm{max}} = 50\%$ we have $B/T = 0.50$. So $T = B/0.50 = 2/0.50 = 4\,\mathrm{K}$.
Now $B/T = 2.0$ so $T = 2/2.0 = 1\,\mathrm{K}$.

26P

(a) A charge e traveling with uniform speed v around a circular path of radius r takes time $T = 2\pi r/v$ to complete one orbit, so the average current is

$$i = \frac{e}{T} = \frac{ev}{2\pi r}\,.$$

The magnitude of the dipole moment is this times the area of the orbit:

$$\mu = \frac{ev}{2\pi r}\,\pi r^2 = \frac{evr}{2}\,.$$

Since the magnetic force, with magnitude evB, is centripetal, Newton's law yields $evB = m_e v^2/r$, so

$$r = \frac{m_e v}{eB}\,.$$

Thus

$$\mu = \frac{1}{2}(ev)\left(\frac{m_e v}{eB}\right) = \left(\frac{1}{B}\right)\left(\frac{1}{2}m_e v^2\right) = \frac{K_e}{B}\,.$$

The magnetic force $-e\mathbf{v} \times \mathbf{B}$ must point toward the center of the circular path. If the magnetic field is into the page, for example, the electron will travel clockwise around the circle. Since the electron is negative, the current is in the opposite direction, counterclockwise and, by the right-hand rule for dipole moments, the dipole moment is out of the page. That is, the dipole moment is directed opposite to the magnetic field vector.

(b) Notice that the charge canceled in the derivation of $\mu = K_e/B$. Thus the relation $\mu = K_i/B$ holds for a positive ion. If the magnetic field is into the page, the ion travels counterclockwise around a circular orbit and the current is in the same direction. Thus the dipole moment is again out of the page, opposite to the magnetic field.

(c) The magnetization is given by $M = \mu_e n_e + \mu_i n_i$, where μ_e is the dipole moment of an electron, n_e is the electron concentration, μ_i is the dipole moment of an ion, and n_i is the ion concentration. Since $n_e = n_i$, we may write n for both concentrations. Substitute $\mu_e = K_e/B$ and $\mu_i = K_i/B$ to obtain

$$M = \frac{n}{B}(K_e + K_i) = \frac{5.3\times10^{21}\,\mathrm{m}^{-3}}{1.2\,\mathrm{T}}\left[6.2\times10^{-20}\,\mathrm{J} + 7.6\times10^{-21}\,\mathrm{J}\right] = 310\,\mathrm{A/m}\,.$$

27P

(a)

$$M = \frac{N\mu P(\mu) - N\mu P(-\mu)}{P(\mu) + P(-\mu)} = \frac{N\mu\left(e^{\mu B/KT} - e^{-\mu B/KT}\right)}{e^{\mu B/KT} + e^{-\mu B/KT}} = N\mu\tanh\left(\frac{\mu B}{kT}\right).$$

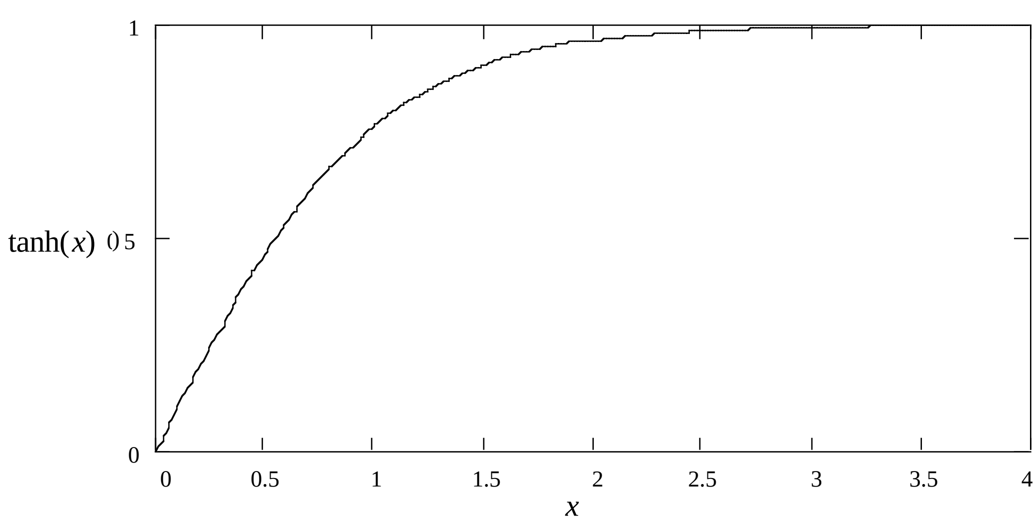
(b) For $\mu B \ll kT$ (i.e., $\mu B/kT \ll 1$) we have $e^{\pm\mu B/kT} \approx 1 \pm \mu B/kT$, so

$$M = N\mu\tanh\left(\frac{\mu B}{kT}\right) \approx \frac{N\mu[(1+\mu B/kT) - (1-\mu B/kT)]}{(1+\mu B/kT) + (1-\mu B/kT)} = \frac{N\mu^2 B}{kT}\,.$$

(c) For $\mu B \gg kT$ we have $\tanh(\mu B/kT) \approx 1$, so

$$M = N\mu\tanh\left(\frac{\mu B}{kT}\right) \approx N\mu\,.$$

(d) The following is a plot of $\tanh(\alpha x)$ as a function of x for $\alpha = 1$. Notice the resemblence between this plot and Fig. 32-10. By adjusting α properly you can fit the curve in Fig. 3-10 with a tanh function.



28E

The Curie temperature for iron is 770 ℃. If x is the depth at which the temperature has this value, then $10^\circ\,\mathrm{C} + (30^\circ\,\mathrm{C/km})x = 770\,℃$ or

$$x = \frac{770\,℃ - 10\,℃}{30\,℃/\mathrm{km}} = 25\,\mathrm{km}\,.$$

29E

(a) The field of a dipole along its axis is given by Eq. 30–29:

$$B = \frac{\mu_0}{2\pi}\frac{\mu}{z^3}\,,$$

where μ is the dipole moment and z is the distance from the dipole. Thus

$$B = \frac{(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m}/A)(1.5 \times 10^{-23}\,\mathrm{J/T})}{2\pi(10 \times 10^{-9}\,\mathrm{m})} = 3.0 \times 10^{-6}\,\mathrm{T}\,.$$

(b) The energy of a magnetic dipole $\boldsymbol{\mu}$ in a magnetic field $\mathbf{B}$ is given by $U = \boldsymbol{\mu}\cdot\mathbf{B} = \mu B\cos\phi$, where ϕ is the angle between the dipole moment and the field. The energy required to turn it end-for-end (from $\phi = 0^\circ$ to $\phi = 180^\circ$) is

$$\Delta U = 2\mu B = 2(1.5 \times 10^{-23}\,\mathrm{J/T})(3.0 \times 10^{-6}\,\mathrm{T}) = 9.0 \times 10^{-29}\,\mathrm{J} = 5.6 \times 10^{-10}\,\mathrm{eV}\,.$$

The mean kinetic energy of translation at room temperature is about 0.04 eV. Thus if dipole-dipole interactions were responsible for aligning dipoles, collisions would easily randomize the directions of the moments and they would not remain aligned.

30E

The saturation magnetization corresponds to complete alignment of all atomic dipoles and is given by $M_{\mathrm{sat}} = \mu n$, where n is the number of atoms per unit volume and μ is the magnetic dipole moment of an atom. The number of nickel atoms per unit volume is $n = \rho/m$, where ρ is the density of nickel and m is the mass of a single nickel atom, calculated using $m = M/N_A$, where M is the atomic mass of nickel and N_A is the Avogadro constant. Thus

$$\begin{aligned} n = \frac{\rho N_A}{M} &= \frac{(8.90\,\mathrm{g/cm^3})(6.02 \times 10^{23}\,\mathrm{atoms/mol})}{58.71\,\mathrm{g/mol}} \\ &= 9.126 \times 10^{22}\,\mathrm{atoms/cm^3} = 9.126 \times 10^{28}\,\mathrm{atoms/m^3}\,. \end{aligned}$$

The dipole moment of a single atom of nickel is

$$\mu = \frac{M_{\mathrm{sat}}}{n} = \frac{4.70 \times 10^5\,\mathrm{A/m}}{9.126 \times 10^{28}\,\mathrm{m^3}} = 5.15 \times 10^{-24}\,\mathrm{A{\cdot}m^2}\,.$$

31E

(a) The number of iron atoms in the iron bar is

$$N = \frac{(7.9\,\mathrm{g/cm^3})(5.0\,\mathrm{cm})(1.0\,\mathrm{cm^2})}{(55.847\,\mathrm{g/mol})/(6.022\times10^{23}/\mathrm{mol})} = 4.3\times10^{23}\,.$$

Thus the dipole moment of the iron bar is

$$\mu = (2.1\times10^{-23}\,\mathrm{J/T})(4.3\times10^{23}) = 8.9\,\mathrm{A\cdot m^2}.$$

(b) $\tau = \mu B\sin 90^\circ = (8.9\,\mathrm{A\cdot m^2})(1.57\,\mathrm{T}) = 13\,\mathrm{N\cdot m}\,.$

32P

(a) If the magnetization of the sphere is saturated the total dipole moment is $\mu_{\mathrm{total}} = N\mu$, where N is the number of iron atoms in the sphere and μ is the dipole moment of an iron atom. We wish to find the radius of an iron sphere with N iron atoms. The mass of such a sphere is Nm, where m is the mass of an iron atom. It is also given by $4\pi\rho R^3/3$, where ρ is the density of iron and R is the radius of the sphere. Thus $Nm = 4\pi\rho R^3/3$ and

$$N = \frac{4\pi\rho R^3}{3m}\,.$$

Substitute this into $\mu_{\mathrm{total}} = N\mu$ to obtain

$$\mu_{\mathrm{total}} = \frac{4\pi\rho R^3\mu}{3m}\,.$$

Solve for R and obtain

$$R = \left(\frac{3m\mu_{\mathrm{total}}}{4\pi\rho\mu}\right)^{1/3}\,.$$

The mass of an iron atom is

$$m = 56\,\mathrm{u} = (56\,\mathrm{u})(1.66\times10^{-27}\,\mathrm{kg/u}) = 9.30\times10^{-26}\,\mathrm{kg}\,.$$

So

$$R = \left[\frac{3(9.30\times10^{-26}\,\mathrm{kg})(8.0\times10^{22}\,\mathrm{J/T})}{4\pi(14\times10^{3}\,\mathrm{kg/m^3})(2.1\times10^{-23}\,\mathrm{J/T})}\right]^{1/3} = 1.8\times10^{5}\,\mathrm{m}\,.$$

(b) The volume of the sphere is

$$V_s = \frac{4\pi}{3}R^3 = \frac{4\pi}{3}(1.82\times10^{5}\,\mathrm{m})^3 = 2.53\times10^{16}\,\mathrm{m^3}$$

and the volume of the Earth is

$$V_e = \frac{4\pi}{3}(6.37\times10^{6}\,\mathrm{m})^3 = 1.08\times10^{21}\,\mathrm{m^3}\,,$$

so the fraction of the Earth's volume that is occupied by the sphere is

$$\frac{2.53\times10^{16}\,\mathrm{m}^3}{1.08\times10^{21}\,\mathrm{m}^3}=2.3\times10^{-5}\,.$$

33P

From the way the wire is wound it is clear that P_2 is the magnetic north pole while P_1 is the south pole.
(a) The deflection will be toward P_1 (away from the magnetic north pole).
(b) As the electromagnet is turned on the magnetic flus Φ through the aluminum changes abruptly, causing a strong induced current which produces a magnetic field opposite to that of the electromagnet. As a result the aluminum sample will be pushed toward P_1, away from the magnetic north pole of the bar magnet. As Φ reaches a constant value, however, the induced current disappears and the aluminum sample, being paramagnetic, will move slightly toward P_2, the magnetic north pole of the electromagnet.
(c) A magnetic north pole will now be induced on the side of the sample closer to P_1, and a magnetic south pole will appear on the other side. If the field of the electromagnet is stonger near P_1 then the sample will move toward P_1.

34P

The interacting potenial energy between the magnetic dipole of the compass and the Earth's magnetic field is $U=-\boldsymbol{\mu}\cdot\mathbf{B}_e=-\mu B_e\cos\theta$, where θ is the angle between $\boldsymbol{\mu}$ and $\mathbf{B}_e$. For small angle θ

$$U(\theta)=-\mu B_e\cos\theta\approx-\mu B_e\left(1-\frac{\theta^2}{2}\right)=\frac{1}{2}k\theta^2-\mu B_e\,,$$

where $k=\mu B_e$. Conservation of energy for the compass then gives

$$\frac{1}{2}I\left(\frac{d\theta}{dt}\right)^2+\frac{1}{2}k\theta^2=\text{const.}.$$

This is to be compared with the following expression for the mechanical energy of a spring-mass system:

$$\frac{1}{2}m\left(\frac{dx}{dt}\right)^2+\frac{1}{2}kx^2=\text{const.},$$

which yields $\omega=\sqrt{k/m}$. So by analogy, in our case

$$\omega=\sqrt{\frac{k}{I}}=\sqrt{\frac{\mu B_e}{I}}=\sqrt{\frac{\mu B_e}{ml^2/12}}\,,$$

which gives

$$\mu=\frac{ml^2\omega^2}{12B_e}=\frac{(0.050\,\mathrm{kg})(4.0\times10^{-2}\,\mathrm{m})^2(45\,\mathrm{rad/s})^2}{12(16\times10^{-6}\,\mathrm{T})}=8.4\times10^2\,\mathrm{J/T}\,.$$

35P

(a) The magnitude of the toroidal field is given by $B_0 = \mu_0 n i_p$, where n is the number of turns per unit length of toroid and i_p is the current required to produce the field (in the absence of the ferromagnetic material). Use the average radius ($\bar{r} = 5.5\,\text{cm}$) to calculate n:

$$n = \frac{N}{2\pi\bar{r}} = \frac{400\,\text{turns}}{2\pi(5.5\times10^{-2}\,\text{m})} = 1.16\times10^{3}\,\text{turns/m}\,.$$

Thus

$$i_p = \frac{B_0}{\mu_0 n} = \frac{0.20\times10^{-3}\,\text{T}}{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(1.16\times10^{3}\,\text{m}^{-1})} = 0.14\,\text{A}\,.$$

(b) If Φ is the magnetic flux through the secondary coil then the magnitude of the emf induced in that coil is $\mathcal{E} = N(d\Phi/dt)$ and the current in the secondary is $i_s = \mathcal{E}/R$, where R is the resistance of the coil. Thus

$$i_s = \left(\frac{N}{R}\right)\frac{d\Phi}{dt}\,.$$

The charge that passes through the secondary when the primary current is turned on is

$$q = \int i_s\,dt = \frac{N}{R}\int\frac{d\Phi}{dt}\,dt = \frac{N}{R}\int_0^{\Phi} d\Phi = \frac{N\Phi}{R}\,.$$

The magnetic field through the secondary coil has magnitude $B = B_0 + B_M = 801B_0$, where B_M is the field of the magnetic dipoles in the magnetic material. The total field is perpendicular to the plane of the secondary coil, so the magnetic flux is $\Phi = AB$, where A is the area of the Rowland ring (the field is inside the ring, not in the region between the ring and coil). If r is the radius of the ring's cross section then $A = \pi r^2$. Thus

$$\Phi = 801\pi r^2 B_0\,.$$

The radius r is $(6.0\,\text{cm} - 5.0\,\text{cm})/2 = 0.50\,\text{cm}$ and

$$\Phi = 801\pi(0.50\times10^{-2}\,\text{m})^2(0.20\times10^{-3}\,\text{T}) = 1.26\times10^{-5}\,\text{Wb}\,.$$

Thus

$$q = \frac{50(1.26\times10^{-5}\,\text{Wb})}{8.0\,\Omega} = 7.9\times10^{-5}\,\text{C}\,.$$

36E

Let R be the radius of a capacitor plate and r be the distance from axis of the capacitor. For points with $r \le R$ the magnitude of the magnetic field is given by

$$B = \frac{\mu_0\epsilon_0 r}{2}\frac{dE}{dt}$$

and for $r \geq R$ it is

$$B = \frac{\mu_0 \epsilon_0 R^2}{2r} \frac{dE}{dt}\,.$$

The maximum magnetic field occurs at points for which $r = R$ and its value is given by either of the formulas above:

$$B_{\text{max}} = \frac{\mu_0 \epsilon_0 R}{2} \frac{dE}{dt}\,.$$

There are two values of r for which $B = B_{\text{max}}/2$: one less than R and one greater. To find the one that is less than R, solve

$$\frac{\mu_0 \epsilon_0 r}{2} \frac{dE}{dt} = \frac{\mu_0 \epsilon_0 R}{4} \frac{dE}{dt}$$

for r. The result is $r = R/2 = (55.0\,\text{mm})/2 = 27.5\,\text{mm}$.

To find the one that is greater than R, solve

$$\frac{\mu_0 \epsilon_0 R^2}{2r} \frac{dE}{dt} = \frac{\mu_0 \epsilon_0 R}{4} \frac{dE}{dt}$$

for r. The result is $r = 2R = 2(55.0\,\text{mm}) = 110\,\text{mm}$.

37E

Use the result of part (b) in Sample Problem 32-4:

$$B = \frac{\mu_0 \epsilon_0 R^2}{2r} \frac{dE}{dt} \qquad \text{(for } r \geq R\text{)}$$

to solve for dE/dt:

$$\begin{aligned} \frac{dE}{dt} &= \frac{2Br}{\mu_0 \epsilon_0 R^2} \\ &= \frac{2(2.0 \times 10^{-7}\ T)(6.0 \times 10^{-3}\,\text{m})}{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(3.0 \times 10^{-3}\,\text{m})^2} \\ &= 2.4 \times 10^{13}\,\text{V/m}\cdot\text{s}\,. \end{aligned}$$

38P

(a) Use the result of part (a) in Sample Problem 32-4:

$$B = \frac{\mu_0 \epsilon_0 r}{2} \frac{dE}{dt} \qquad \text{(for } r \leq R\text{)}\,,$$

where $r = 0.80R$ and

$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{V}{d}\right) = \frac{1}{d}\frac{d}{dt}\left(V_0 e^{-t/\tau}\right) = -\frac{V_0}{\tau d} e^{-t/\tau}\,.$$

Here $V_0 = 100\,\text{V}$. Thus

$$\begin{aligned} B(t) &= \left(\frac{\mu_0\epsilon_0 r}{2}\right)\left(-\frac{V_0}{\tau d}e^{-t/\tau}\right) = -\frac{\mu_0\epsilon_0 V_0 r}{2\tau d}\,e^{-t/\tau} \\ &= -\frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(100\,\text{V})(0.80)(16\,\text{mm})}{2(12\times10^{-3}\,\text{s})(5.0\,\text{mm})}\,e^{-t/12\,\text{ms}} \\ &= -(1.2\times10^{-13}\,\text{T})\,e^{-t/12\,\text{ms}}\,. \end{aligned}$$

The minus sign here is insignificant.
(*b*) At time $t = 3\tau$, $B(t) = -(1.2\times10^{-13}\,\text{T})e^{-3\tau/\tau} = -5.9\times10^{-15}\,\text{T}$.

39P

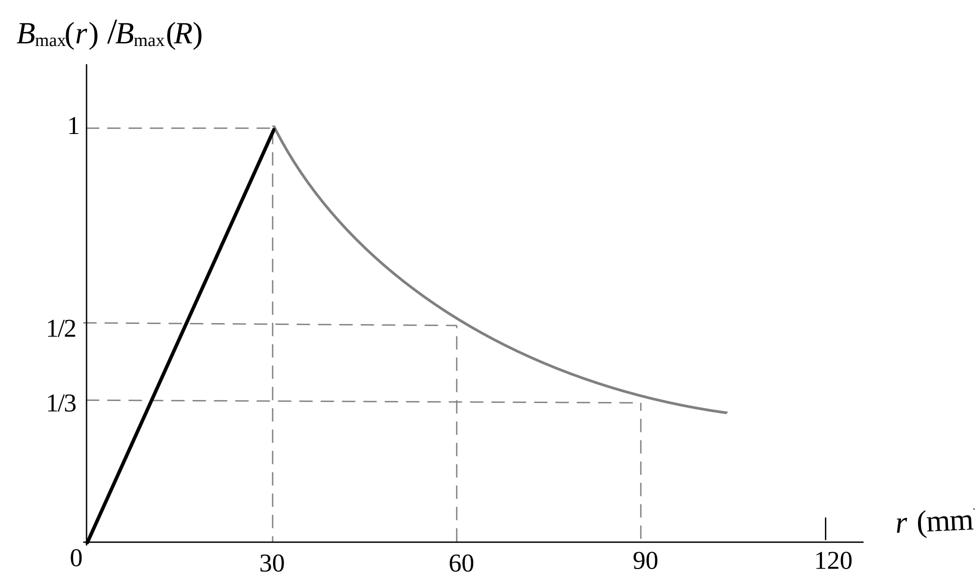
Apply Eq. 32-28 to a circular loop of radius R centered at the center of the plates:

$$\begin{aligned} \oint \mathbf{B}\cdot d\mathbf{s} = 2\pi RB &= \mu_0\epsilon_0\frac{d\Phi_E}{dt} = \mu_0\epsilon_0\frac{d}{dt}(\pi R^2 E) = \mu_0\epsilon_0\pi R^2\frac{dE}{dt} \\ &= \frac{\mu_0\epsilon_0\pi R^2}{d}\frac{d}{dt}[V_\text{m}\cos(\omega t+\phi)] = \frac{\mu_0\epsilon_0\pi R^2 V_\text{m}\omega}{d}\sin(\omega t+\phi)\,. \end{aligned}$$

Thus

$$\begin{aligned} B_\text{max} &= \frac{\mu_0\epsilon_0\pi R^2 V_\text{m}\omega}{2\pi R d}[\sin(\omega t+\phi)]_\text{max} = \frac{\pi\mu_0\epsilon_0 R V_\text{m} f}{d} = \frac{R V_\text{m} f}{c^2 d} \\ &= \frac{(30\times10^{-3}\,\text{m})(150\,\text{V})(60\,\text{Hz})}{(3.0\times10^8\,\text{m/s})^2(5.0\times10^{-3}\,\text{m})} = 1.9\times10^{-12}\,\text{T}\,. \end{aligned}$$

(*b*) For $r \le R$ $B(r) \propto r$, and for $r \ge R$ $B(r) \propto r^{-1}$ (see Eqs. 32-38 and 32-39). The plot is shown below.



40E

The displacement current is given by

$$i_d = \epsilon_0 A \frac{dE}{dt},$$

where A is the area of a plate and E is the magnitude of the electric field between the plates. The field between the plates is uniform, so $E = V/d$, where V is the potential difference across the plates and d is the plate separation. Thus

$$i_d = \frac{\epsilon_0 A}{d} \frac{dV}{dt}.$$

Now $\epsilon_0 A/d$ is the capacitance C of a parallel-plate capacitor without a dielectric, so

$$i_d = C \frac{dV}{dt}.$$

41E

Let the area plate be A and the plate separation be d. Use Eq. 32-32:

$$i_d = \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 \frac{d}{dt}(AE) = \epsilon_0 A \frac{d}{dt}\left(\frac{V}{d}\right) = \frac{\epsilon_0 A}{d}\left(\frac{dV}{dt}\right),$$

or

$$\frac{dV}{dt} = \frac{i_d d}{\epsilon_0 A} = \frac{i_d}{C} = \frac{1.5\,\text{A}}{2.0 \times 10^{-6}\,\text{F}} = 7.5 \times 10^5\,\text{V/s}.$$

So we need to change the voltage difference across the capacitor at the rate of 7.5×10^5 V/s.

42E

Consider an area A, normal to a uniform electric field **E**. The displacement current density is uniform and normal to the area. Its magnitude is given by $J_d = i_d/A$. For this situation

$$i_d = \epsilon_0 A \frac{dE}{dt},$$

so

$$J_d = \frac{1}{A} \epsilon_0 A \frac{dE}{dt} = \epsilon_0 \frac{dE}{dt}.$$

43E

Use Eq. 32-36:

$$i_d = \epsilon_0 A \frac{dE}{dt}.$$

Note that here A is the area over which a changing electric field is present. In this case $r > R$ so $A = \pi R^2$. Thus

$$\frac{dE}{dt} = \frac{i_d}{\epsilon_0 A} = \frac{i_d}{\epsilon_0 \pi R^2} = \frac{2.0\,\mathrm{A}}{\pi(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})(0.10\,\mathrm{m})^2} = 7.2\times10^{12}\,\mathrm{V/m\cdot s}\,.$$

44E

Let the area of each circular plate be A and that of the central circular section be a, then

$$\frac{A}{a} = \frac{\pi R^2}{\pi(R/2)^2} = 4\,.$$

Thus from Eqs. 32-36 and 32-37 the total discharge current is given by $i = i_d = 4(2.0\,\mathrm{A}) = 8.0\,\mathrm{A}$.

45P

From Eq. 32-32

$$\begin{aligned} i_d &= \epsilon_0\frac{d\Phi_E}{dt} = \epsilon_0 A\frac{dE}{dt} = \epsilon_0 A\frac{d}{dt}\left[(4.0\times10^5) - (6.0\times10^4 t)\right] \\ &= -\epsilon_0 A(6.0\times10^4\,\mathrm{V/m\cdot s}) \\ &= -(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})(4.0\times10^{-2}\,\mathrm{m^2})(6.0\times10^4\,\mathrm{V/m\cdot s}) \\ &= -2.1\times10^{-8}\,\mathrm{A}\,. \end{aligned}$$

(*b*) If one draws a counterclockwise circular loop s around the plates then according to Eq. 32-23

$$\oint_s \mathbf{B}\cdot d\mathbf{s} = \mu_0 i_d < 0\,,$$

which means that $\mathbf{B}\cdot d\mathbf{s} < 0$. Thus $\mathbf{B}$ must be clockwise.

46P

(*a*) From Sample Problem 32-4 we know that $B \propto r$ for $r \le R$ and $B \propto r^{-1}$ for $r \ge R$. So the maximum value of B occurs at $r = R$, and there are two possible values of r at which the magnetic field is 75% of $B_{\max}$. Denote these two values as r_1 and r_2, where $r_1 < R$ and $r_2 > R$. Then $0.75B_{\max}/B_{\max} = r_1/R$, or $r_1 = 0.75R$; and $0.75B_{\max}/B_{\max} = (r_2/R)^{-1}$, or $r_2 = R/0.75 = 1.3R$.

(*b*) From Eqs. 32-37 and 32-43

$$B_{\max} = \frac{\mu_0 i_d}{2\pi R} = \frac{\mu_0 i}{2\pi R} = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(6.0\,\mathrm{A})}{2\pi(0.040\,\mathrm{m})} = 3.0\times10^{-5}\,\mathrm{T}\,.$$

47P

(a) Use $\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I_{\text{enclosed}}$ to find

$$B = \frac{\mu_0 I_{\text{enclosed}}}{2\pi r} = \frac{\mu_0 (J_d \pi r^2)}{2\pi r} = \frac{1}{2}\mu_0 J_d r$$
$$= \frac{1}{2}(1.26\times 10^{-6}\,\text{H/m})(20\,\text{A/m}^2)(50\times 10^{-3}\,\text{m}) = 6.3\times 10^{-7}\,\text{T}\,.$$

(b) From

$$i_d = J_d \pi r^2 = \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 \pi r^2 \frac{dE}{dt}$$

we get

$$\frac{dE}{dt} = \frac{J_d}{\epsilon_0} = \frac{20\,\text{A/m}^2}{8.85\times 10^{-12}\,\text{F/m}} = 2.3\times 10^{12}\,\text{V/m}\cdot\text{s}\ .$$

48P

(a)

$$|i_d| = \epsilon_0 \left|\frac{d\Phi_E}{dt}\right| = \epsilon_0 A \left|\frac{dE}{dt}\right|$$
$$= (8.85\times 10^{-12}\,\text{F/m})(1.6\,\text{m}^2)\left|\frac{4.5\times 10^5\,\text{N/C} - 6.0\times 10^5\,\text{N/C}}{4.0\times 10^{-6}\,\text{s}}\right| = 0.71\,\text{A}.$$

(b) $i_d \propto dE/dt = 0$.

(c)

$$|i_d| = \epsilon_0 A \left|\frac{dE}{dt}\right| = (8.85\times 10^{-12}\,\text{F/m})(1.6\,\text{m}^2)\left|\frac{-4.0\times 10^5\,\text{N/C}}{15\times 10^{-6}\,\text{s} - 10\times 10^{-6}\,\text{s}}\right| = 1.1\,\text{A}\,.$$

49P

(a) At any instant the displacement current i_d in the gap between the plates equals the conduction current i in the wires. Thus $i_d = i = 2.0\,\text{A}$.

(b)

$$\frac{dE}{dt} = \frac{1}{\epsilon_0 A}\left(\epsilon_0 \frac{d\Phi_E}{dt}\right) = \frac{i_d}{\epsilon_0 A} = \frac{2.0\,\text{A}}{(8.85\times 10^{-12}\,\text{F/m})(1.0\,\text{m})^2} = 2.3\times 10^{11}\,\text{V/m}\cdot\text{s}\,.$$

(c)

$$i'_d = i_d \times \left(\frac{\text{area enclosed by the path}}{\text{area of each plate}}\right) = (2.0\,\text{A})\left(\frac{0.50\,\text{m}}{1.0\,\text{m}}\right)^2 = 0.50\,\text{A}\,.$$

(d)

$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 i_d' = (1.26\times10^{-6}\,\mathrm{H/m})(0.50\,\mathrm{A}) = 6.3\times10^{-7}\,\mathrm{T\cdot m}\,.$$

50P

(a)

$$E = \frac{J}{\sigma} = \rho J = \frac{\rho i}{A} = \frac{(1.62\times10^{-8}\,\Omega\cdot\mathrm{m})(100\,\mathrm{A})}{5.00\times10^{-6}\,\mathrm{m}^2} = 0.324\,\mathrm{V/m}\,.$$

(b)

$$\begin{aligned} i_d &= \epsilon_0\frac{d\Phi_E}{dt} = \epsilon_0 A\frac{dE}{dt} = \epsilon_0 A\frac{d}{dt}\left(\frac{\rho i}{A}\right) = \epsilon_0\rho\frac{di}{dt} \\ &= (8.85\times10^{-12}\,\mathrm{F})(1.62\times10^{-8}\,\Omega)(2000\,\mathrm{A/s}) = 2.87\times10^{-16}\,\mathrm{A}\,. \end{aligned}$$

(c)

$$\frac{B(\text{ due to } i_d)}{B(\text{ due to } i)} = \frac{\mu_0 i_d/2\pi r}{\mu_0 i/2\pi r} = \frac{i_d}{i} = \frac{2.87\times10^{-16}\,\mathrm{A}}{100\,\mathrm{A}} = 2.87\times10^{-18}\,.$$

51P

(a) At any instant the displacement current i_d in the gap between the plates equals the conduction current i in the wires. Thus $i_{\max} = i_{d\ \max} = 7.60\,\mu\mathrm{A}$.

(b) Since $i_d = \epsilon_0\,(d\Phi_E/dt)$,

$$\left(\frac{d\Phi_E}{\mathrm{d}t}\right)_{\max} = \frac{i_{d\ \max}}{\epsilon_0} = \frac{7.60\times10^{-6}\,\mathrm{A}}{8.85\times10^{-12}\,\mathrm{F/m}} = 8.59\times10^5\,\mathrm{V\cdot m/s}\,.$$

(c) According to 40E

$$i_d = C\frac{dV}{dt} = \frac{\epsilon_0 A}{d}\frac{dV}{dt}\,.$$

Now the potential difference across the capacitor is the same in magnitude as the emf of the generator, so $V = \mathcal{E}_\mathrm{m}\sin\omega t$ and $dV/dt = \omega\mathcal{E}_\mathrm{m}\cos\omega t$. Thus

$$i_d = \frac{\epsilon_0 A\omega\mathcal{E}_\mathrm{m}}{d}\cos\omega t$$

and

$$i_{d\ \max} = \frac{\epsilon_0 A\omega\mathcal{E}_\mathrm{m}}{d}\,.$$

This means

$$\begin{aligned} d &= \frac{\epsilon_0 A\omega\mathcal{E}_\mathrm{m}}{i_{d\max}} = \frac{(8.85\times10^{-12}\,\mathrm{F/m})\pi(0.180\,\mathrm{m})^2(130\,\mathrm{rad/s})(220\,\mathrm{V})}{7.60\times10^{-6}\,\mathrm{A}} \\ &= 3.39\times10^{-3}\,\mathrm{m}\,, \end{aligned}$$

where $A = \pi R^2$ was used.

(d) Use the Ampere-Maxwell law in the form $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_d$, where the path of integration is a circle of radius r between the plates and parallel to them. I_d is the displacement current through the area bounded by the path of integration. Since the displacement current density is uniform between the plates $I_d = (r^2/R^2)i_d$, where i_d is the total displacement current between the plates and R is the plate radius. The field lines are circles centered on the axis of the plates, so $\mathbf{B}$ is parallel to d$\mathbf{s}$. The field has constant magnitude around the circular path, so $\oint \mathbf{B} \cdot d\mathbf{s} = 2\pi r B$. Thus

$$2\pi r B = \mu_0 \left(\frac{r^2}{R^2}\right) i_d$$

and

$$B = \frac{\mu_0 i_d r}{2\pi R^2}\,.$$

The maximum magnetic field is given by

$$B_{\text{max}} = \frac{\mu_0 i_{d\ \text{max}}\, r}{2\pi R^2} = \frac{(4\pi \times 10^{-7}\,\mathrm{T \cdot m/A})(7.6 \times 10^{-6}\,\mathrm{A})(0.110\,\mathrm{m})}{2\pi (0.180\,\mathrm{m})^2} = 5.16 \times 10^{-12}\,\mathrm{T}\,.$$